Tunneling in a periodic array of semimagnetic quantum dots

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We investigate the tunneling of carriers in a quasi-one-dimensional array of semimagnetic quantum dots. The large magnetopolaron effects due to the exchange interaction of carriers with magnetic ions result in a transmission via solitonlike electronic states. The intensity-dependent transmission coefficient shows the opening of intensity-dependent gaps in the transmission spectrum.

INTRODUCTION

Magnetopolaron effects in bulk semimagnetic semiconductors such as $Cd_{1-y}Mn_yTe$ and quantum wells are well documented.[1] Magnetopolarons are free carriers dressed by the induced magnetic polarization field of the magnetic ions. Polaronic effects are weak in three dimensions and can only be observed with carriers bound to impurities. In quantum wells their stability is marginal.[2] In one-dimensional systems these polaronic effects should be strong and lead to localized, solitonlike states.[3] The strength of polaronic effects should increase dramatically in quantum dots. We consider here an array of coupled quantum dots as a model of a periodic nonlinear system. Such a system can be realized in a quantum wire with a strongly varying concentration of magnetic ions.

MODEL

Let us consider a $Cd_{1-y}Mn_yTe$ quantum wire with an effective radius r_0 normal to the growth direction, the z axis. The wire is built with unit cells of periodicity a. The unit cell of this wire consists of a dot of width d with low Mn concentration ($y \sim 0.1$), and a barrier of width b and a high Mn concentration ($y \sim 0.7$). In the barrier, the antiferromagnetic interaction between Mn ions dominates, and we expect the barrier to be in a "spin-glass" phase. The dot with a low Mn concentration is assumed to be in a paramagnetic phase. In the "mean-field" approximation, the effective Hamiltonian for the dot can be written as[4]

$$H=\frac{p^2}{2m}-V_0-V_m B_{5/2}\left[\frac{g_{\rm Mn}\mu_B B_{\rm eff}}{k_B T_{\rm eff}}\right] . \quad (1)$$

The first term is the kinetic energy, V_0 is the barrier height (energy being measured from the top of the barrier), $V_m=\frac{5}{2}\chi\beta N_0 J_z$ is the magnetic potential with βN_0 being the exchange energy (~ 880 meV for the valence band), χ is the average effective concentration of Mn ions, and $J_z=\frac{3}{2}$ is the spin of free carriers (holes). The Brillouin function $B_{5/2}$ describes the paramagnetic susceptibility of the Mn ion with spin $\frac{5}{2}$ in the effective magnetic field $B_{\rm eff}$ at the effective temperature $T_{\rm eff}$. The effective field $B_{\rm eff}$ acting on the ion is a sum of the external field B_0 and the exchange field $B_{\rm ex}=\frac{5}{2}\beta N_0 J_z|\psi(z)|^2/N_0$, which depends on the probability $|\psi(z)|^2$ of the carrier being in the position of the magnetic ion. N_0 is the number of cations per unit volume. The wave function for the carrier in the wire is taken in the form $\psi(r,z)=[1/(\pi r_0^2)^{1/2}]e^{-(r/r_0)^2}\phi(z)$. Substituting the wave function ψ into Eq. (1), and retaining only linear terms in the expansion $B_{5/2}(x)\sim 0.477x$, gives a simple nonlinear Schrödinger equation for $\phi(z)$ that can be written as

$$-\frac{\hbar_2}{2m}\frac{\partial^2}{\partial z^2}\phi(z) -[V_0+V_B+(\tfrac{5}{4}\beta N_0 J_z/n_c)d|\phi(z)|^2]\phi(z)=E\phi(z) \quad (2)$$

in the well, and

$$-\frac{\hbar^2}{2m}\frac{\partial 2}{\partial z^2}\phi(z)=E\phi(z)$$

in the barrier. Here n_c is the number of cations in the well, $V_B=0.467 g_{\rm Mn}\mu_B B_0/k_B T_{\rm eff}$ is the potential due to the external magnetic field, while the last term is the self-consistent potential due to the exchange field. In the following we shall restrict ourselves to zero external magnetic field and narrow dots. To understand the combined effects of nonlinearity and periodicity it is sufficient to replace the effective potential of the well by a suitable chosen δ-function potential. The resulting Schrödinger equation for a wire with N dots now takes the final form:

$$-\frac{\partial^2}{\partial z^2}\phi(z)-\left[\sum_{l=1}^{N}[Vd+\alpha da|\phi(z)|^2]\times\delta(z-la)\right]\phi(z)=E\phi(z)\ . \qquad (3)$$

The energy in Eq. (3) is measured in effective rydbergs (Ry*) and lengths in effective Bohr radii a_0. For holes in CdTe with an effective mass of 0.5 of the electron mass, the effective rydberg Ry* = 72 meV and the Bohr radius $a_0=10$ Å. In Eq. (3) the function ϕ is dimensionless ($\phi\rightarrow\sqrt{a_0}\phi$), and the potential V has been chosen to reproduce the bound state of a single well of width d and barrier height V_0. For $V_0=1$ Ry*, $d=20$ Å, $a=40$ Å, and $k_B T_{\mathrm{eff}}=5$ meV, we estimate $V=0.69$ Ry*. The coupling constant α depends on the number of Mn ions, and for 10 cations per dot we estimate $\alpha=10^{-2}$ Ry*.

Equation (3) describes an array of coupled quantum dots. Because of the nonlinearity, the eigenstates of Eq. (3) cannot be classified according to the Bloch scheme. We proceed by integrating Eq. (3) across the lth singularity. Requiring the continuity of the wave function $\phi(l-)=\phi(l+)$, and writing the energy E of a particle as $E=k^2$, one obtains a nonlinear complex map relating wave functions on successive layers:

$$\varphi_{l+1}+\varphi_{l-1}=2\left[\cos(ka)-\frac{Vda}{2}\frac{\sin(ka)}{ka}-\frac{\alpha da^2}{2}\frac{\sin(ka)}{ka}|\phi_l|^2\right]\ . \qquad (4)$$

In the absence of nonlinearity ($\alpha=0$), the wave functions are Bloch states $=\exp(i\kappa la)$ characterized by a Bloch index κ. The standard band structure is then obtained from Eq. (4) with a single band for negative energies and an infinite number of bands and gaps for positive energies (above the barrier). Such a classification is possible because we only need the relation between the phase of the wave function and its energy. In the nonlinear case we must retain the information about the phase and the amplitude. This problem is simplified due to the global gauge invariance of Eq. (4), i.e., the conservation of the current $J_l=-(i/2)(\phi_l^* l\phi_{l+1}-\phi_{l+1}^*\phi_l)$. Consequently, the four-dimensional map of Eq. (4) can be reduced[5] to a two-dimensional area-preserving map with the current J and the energy E as parameters. The classification of the solutions of Eq. (4) is hence reduced to the study of the fixed points of this nonlinear map, and their basins of stability. This has been discussed in detail in Ref. 5. The main effects are illustrated by considering the tunneling of electrons along a wire.

TUNNELING

Let us consider a situation in which the periodically modulated $Cd_{1-x}Mn_xTe$ section of the wire with N dots is inserted into the CdTe wire. The electrons traveling in a wire will be transmitted with the probability $|T|^2$ and reflected with the probability $|R|^2$. Those electrons that are transmitted have traveled through a self-consistent band structure due to the exchange interaction with magnetic ions. In our model without exchange interaction, tunneling can occur if the energy of the transmitted particle is within the allowed energy spectrum of the Bloch band, irrespective of the amplitude of the wave function or the flux of electrons. This is no longer true in the nonlinear case.

Let us first consider a single dot centered around $z=0$. We are interested in a state trapped in the dot. The wave function for this state has a form $\phi(z)=A\exp(-q|z|)$ with energy $E=-q^2$. It is easy to see from Eq. (3) that the corresponding energy is given by $E=-[Vd/(2-\alpha ad)]^2$. Increasing the nonlinearity α lowers the energy. The first nontrivial case corresponds to two coupled dots. In this case there are two states. In Fig. 1 we show the structure with two dots in the resonant tunneling geometry. The transmission problem is solved in the usual way: we assume that the wave function at the end of the wire has the form of the plane wave $\phi(z)=T\exp(ikz)$, where k is the wave vector corresponding to the energy $E=-q^2$, and T is the amplitude of the transmitted wave. We next iterate the wave function backwards, matching appropriately to the $Cd_{1-x}Mn_xTe$ barriers. Figure 1(a) shows the transmission coefficient as a function of energy of incident particles in the absence of nonlinearity. Two peaks correspond to two resonant states of a coupled-dot system. In the presence of a nonlinear interaction the intensity of transmitted particles $|T|^2$ becomes a multivalued function of the intensity $|I|^2$ of the incident particles, as shown in Fig. 1(b). Hence the usual transmission coefficient $|T|^2/|I|^2$ depends on both the energy of the particle and transmitted intensity. When the number of dots N increases, the energies form a band. The transmission coefficient becomes significant for the incident particle energy within the allowed energy band of the N-dot array. In Fig. 2 we show the transmission spectrum for a linear ($\alpha=0$, dashed line) and non-

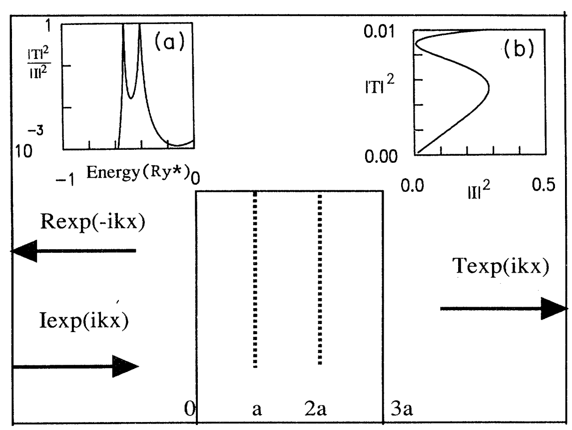

FIG. 1. The schematic picture of the energy diagram of two quantum dots (dashed lines) in a scattering geometry. The inset (a) shows the transmission coefficient $|T|^2/|I|^2$ as a function of energy E. The inset (b) shows the incident intensity $|I|^2$ as a function of transmitted intensity $|T|^2$ in the nonlinear case ($\alpha=0.01$).

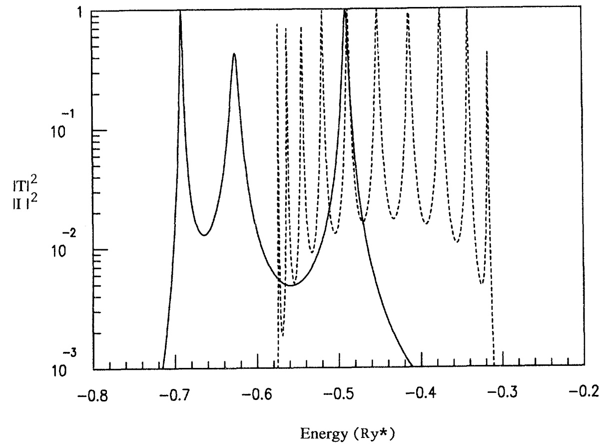

FIG. 2. The transmission coefficient $|T|^2/|I|^2$ as a function of energy E for ten dots: linear spectrum, dashed line; nonlinear spectrum ($\alpha=0.01$), solid line.

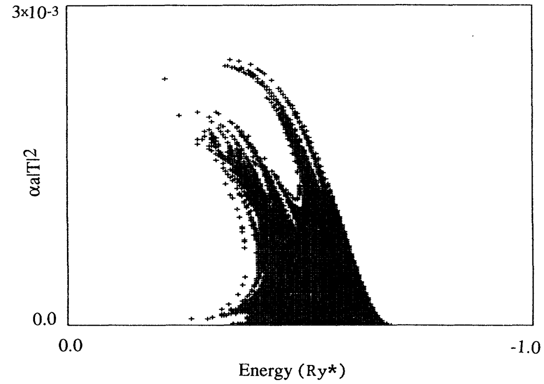

FIG. 3. The "phase diagram" $\alpha a|T|^2$ vs E of parameters corresponding to the transmitted particle with energy E and amplitude $|T|^2$.

linear ($\alpha=0.01$, solid line) array of ten dots. The linear transmission shows an almost uniform spectrum of ten peaks corresponding to the ten possible states. The nonlinear spectrum is shifted to lower energies and shows only three distinct and well-resolved peaks. This illustrates the opening of gaps, i.e., forbidden, nontransmitting energy regions within the linearly allowed band.

The qualitative features of the transmission problem are given by the "phase diagram" in parameter space of the energy E and amplitude $|T|$ of the transmitted particle. To determine whether the particle was transmitted, we iterate the wave function backwards. The solutions that do not grow exponentially are considered to be transmitting. In this way we can map out the regions of parameter space with a finite transmission.

In Fig. 3 we show such a diagram, corresponding to particles incident on the array of ten coupled dots, for which the transmission spectrum is shown in Fig. 2. The dark areas correspond to transmitting states. We see clearly that the transmitting region breaks down into three distinct regions, as shown in Fig. 2. Hence the transmission via flux-carrying states opens gaps in the energy spectrum of the linear theory. The complex nature of this phase diagram is associated with the analysis[5] of the nonlinear dynamical map given by Eq. (4).

In summary, we have shown that semimagnetic semiconductor quantum dots should exhibit interesting features due to strong magnetopolaronic effects. The tunneling studies should demonstrate the multistability, hysteresis, and the opening of gaps in the energy spectrum.

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