Multiphonon resonant Raman scattering in a strong magnetic field

C. Trallero-Giner,* F. Iikawa,† and M. Cardona
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany
(Received 16 July 1991)

Explicit expressions for multiphonon resonant Raman scattering processes in a strong magnetic field are derived in the framework of a quasiclassical picture, where the photoexcited electron and hole make real transitions through the LO-phonon Fröhlich interaction. It is shown that spin quantization produces a strong selection rule in different scattering configurations and the calculated magneto-Raman profile shows a set of sharp peaks that correspond to outgoing resonances. The principal features of the magnetomultiphonon spectra observed recently in GaAs and InP are discussed. These results are compared with experimental magneto-Raman profiles for 2-LO-phonon emission in two scattering configurations. From this comparison a value of the light-hole g factor equal to +22 is obtained.

I. INTRODUCTION

Landau levels in semiconductors have been intensely investigated recently by means of resonant Raman scattering.[1–3] Using this technique band nonparabolicities,[1] resonant magnetopolarons,[3] magnetic-field-induced double resonances,[4] or selection rules for electron-phonon interactions in a magnetic field (H) (Ref. 4) have been studied. An interesting problem of magneto-Raman scattering in semiconductors concerns multiphonon scattering. It was shown[5,6] that the intensity of the multiphonon process in a magnetic field is proportional to the Fröhlich interaction constant C_F whereas in zero magnetic field this intensity is proportional to higher powers of the Fröhlich constant which depend on the number of emitted phonons.[7] The latter conclusion explains why higher-order resonant Raman scattering in III-V semiconductors ($C_F \ll 1$) without magnetic field can only be seen for two or, at most, three phonons. This is in contrast to observations in other more polar semiconductors (larger C_F) where multiphonon peaks up to 9 (e.g., II-VI compounds)[8] or up to 20 (e.g., III-VII compounds)[9] have been reported. When a high magnetic field H is applied to a III-V semiconductor a sharp intensity increase is expected for secondary radiation processes.[5,6] Besides, the Raman efficiency is an oscillating function of H and the incident light frequency ω_L. These predictions were confirmed recently in GaAs and InP. Multiphonon scattering up to 9 LO phonons has been observed in GaAs,[10] and up to 4 LO phonons in InP.[11] The measured Raman efficiency of the multiphonon peaks shows resonant behavior as a function of H. This fact permits an identification of the Landau levels involved in the observed resonance. In Refs. 11 and 12 fan plots of magneto-Raman resonances for 2-LO-, 3-LO-, and 4-LO-phonon scattering in GaAs and InP have been given and the dependence of the peaks on magnetic field in terms of electronic interband gaps has also been analyzed. From the experimental fan plots the authors of Refs. 11 and 12 have concluded that (a) the phonon scattering processes involve interband and intraband transitions between Landau levels, (b) outgoing resonances are much stronger than incoming ones, and (c) magneto-optical transitions involving light mass valence levels predominate. Moreover, the investigation of the multiphonon magneto-Raman spectra unraveled different features which demand a more comprehensive theoretical treatment. The spectra, taken in backscattering configurations with **H** perpendicular to the surface using circularly polarized light [$\sigma^{\pm} = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$ referred to **H**], exhibit pronounced selection rules for the intensities of the multiphonon lines (see Fig. 1). The $\bar{z}(\sigma^+,\sigma^+)$ and $\bar{z}(\sigma^-,\sigma^-)z$ geometries ($\bar{z}$ and z indicate the propagation directions of incident and scattered photon, respectively) yield spectra with mutually exclusive peaks for both configurations and a large separation between the peaks of the two spectra. Strong enhancements of the multiphonon Raman efficiencies occur when the photon frequency is in resonance with an inter-Landau-level transition. In Refs. 5 and 6 only Fröhlich-interaction-mediated scattering in the conduction-band Landau levels was considered (i.e., is the limit of hole mass $m_h \rightarrow \infty$) in the multiphonon process. Nevertheless, it is clear from the experimental observation in GaAs (Ref. 10) and InP (Ref. 11) that phonon scattering processes involving intervalence and intravalence levels are of importance for the explanation of the different features of the Raman profile.

The analysis of fan lines obtained for second to fourth order Raman scattering in Refs. 11 and 12 shows that the same magneto-optical interband transitions are involved in the multiphonon magneto-Raman profile. The aim of this paper is to develop a semiclassical formalism which should permit a more quantitative interpretation of the principal features in the magneto-multiphonon Raman processes.

 ©1991 The American Physical Society

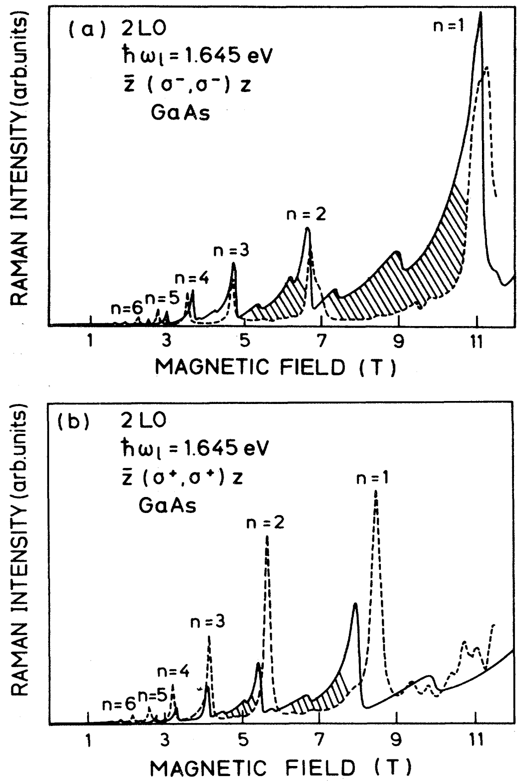

FIG. 1. Calculated 2-LO-phonon Raman intensities (solid lines) vs H compared to magneto-2-LO-phonon Raman spectra of GaAs measured (dashed lines) for an incident laser energy $\hbar\omega_l = 1.645$ eV in $\bar{z}(\sigma^-,\sigma^-)z$ (a) and $\bar{z}(\sigma^+,\sigma^+)z$ (b) configurations. The theoretical outgoing resonances are labeled with the corresponding conduction-band Landau level n.

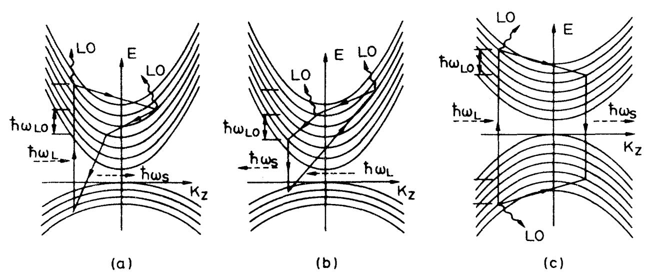

FIG. 2. Schematic representation of (a) the 3-LO-phonon Raman scattering in a strong magnetic field with direct creation and indirect annihilation of electron-hole pair ($m_e \neq m_h$), (b) with indirect creation and direct annihilation of electron-hole pair ($m_e \neq m_h$), and (c) the second-order Raman scattering for $m_e = m_h$ case when the direct creation and direct annihilation is allowed. See text for details about transitions.

II. MULTIPHONON RESONANT RAMAN SCATTERING EFFICIENCY

Multiphonon Raman scattering processes involving free electron-hole pairs (without Coulomb interaction) in a high magnetic field are shown schematically in Fig. 2. The electron energy is represented by

$$E_e = \frac{\hbar^2 k_{ez}^2}{2m_e} + \hbar\omega_e(n+\tfrac{1}{2}) + \mu_B H g_e m_{se} \quad \text{with } n = 0,1,\ldots\,, \qquad (1)$$

where m_e, ω_e, and g_e are the effective mass, the cyclotron frequency, and the g factor of the electron, respectively, μ_B is the Bohr magneton, k_{ez} the wave-vector projection on the direction of the magnetic field, and $m_{se} = \pm\frac{1}{2}$ for the two different spin states. For III-V semiconductors in a magnetic field the valence-band structure is a complicated function of H and k_z due to the hybridization of light and heavy holes.[13] For simplicity in the following we consider a parabolic model for the light and heavy valence bands. Thus, an energy dependence similar to that of Eq. (1) is obtained for holes in the valence band. The multiphonon process is represented qualitatively as follows: (a) the photon creates an electron and a hole with energies $E_e(n,k_z,m_s)$ and $E_h(n,k_z,m_s)$ [*direct* creation, see Fig. 2(a)], respectively, then the electron and/or hole are scattered $N-1$ times by LO phonons into states separated by the $\omega_{\rm LO}$ phonon frequency (in Fig. 2 only the electron scattering is presented in a 3-LO-phonon process), finally the electron-hole pair annihilates *indirectly* with emission of one more LO phonon and a secondary photon with energy $\hbar\omega_S = \hbar\omega_L - N\hbar\omega_{\rm LO}$; (b) similarly, an electron-hole pair is created indirectly by the incident light with the emission of one LO phonon [*indirect* creation, see Fig. 2(b)], and after the emission of $(N-1)$ LO phonons by the electron and/or hole, the pair annihilates *directly*. As was pointed out in Refs. 5 and 6 the difference between the two processes (a) and (b) is that either the first pair creation (a) or the final annihilation (b) is direct.

The calculation of the efficiency for high-order phonon processes in semiconductors is a rather complicated task. The Nth-order resonant Raman scattering via iterated electron–one-phonon interaction can be treated in $(N+2)$th-order perturbation theory. Depending on the incident laser frequency and applied magnetic field, different energy denominators in the expression of the Raman scattering efficiency can vanish (real transitions) when the incoming or outgoing resonance condition is fulfilled or for apppropriate wave vector of the scattering phonons. If the laser photon energy satisfies

$$\hbar\omega_L \geq E_g + N\hbar\omega_{\rm LO} + \frac{\hbar\omega_e}{2} + \frac{\hbar\omega_h}{2}, \qquad (2)$$

where E_g is the gap, the N electron and/or hole transitions can be real. Each real transition corresponds to a vanishing denominator in the Nth-order Raman scattering and should provide the dominant contributions to the scattering cross section.

Under condition (2) the multiphonon Raman process in a magnetic field can be described as a cascade of real intermediate transitions. This permits a "classical" representation of the high-order scattering ($N \geq 2$) in semiconductors in a magnetic field. Following the above discussion the two main contributions to the N-LO phonon Raman scattering cross-section can be written in the form

$$\frac{\partial^2\sigma^N}{\partial\Omega\,\partial\omega_S} = \frac{V_0^2\omega_S^2 n(\omega_S)}{8\pi^3c^3n(\omega_L)} \sum_{n,k_{ez}} \sum_{m,k_{hz}} [P_{n,m}^{N-1,D}(k_{ez},k_{hz})W_{n,m}^{SI}(\hbar\omega_S;k_{ez},k_{hz}) + P_{n,m}^{N,I}(k_{ez},k_{hz})W_{n,m}^{SD}(\hbar\omega_S;k_{ez},k_{hz})] \ , \quad (3)$$

where $n(\omega)$ is the refractive index, Ω the solid angle, c the light velocity, V_0 the crystal volume, and ω_S the scattered photon frequency. In Eq. (3) $P_{n,m}^{N-1,D}(k_{ez},k_{hz})$ $[P_{n,m}^{N,I}(k_{ez},k_{hz})]$ represents the fraction of electrons and holes in the state $n,k_{ez};\ m,k_{hz}$, respectively, after the direct (indirect) creation by the incident laser with energy $\hbar\omega_L$ and emission of $N-1$ (or N) LO phonons. $W_{n,m}^{SI}(k_{ez},k_{hz})$ ($W_{n,m}^{SD}$) is the indirect (direct) annihilation probability per unit time and solid angle due to simultaneous emission of an LO phonon and a photon with energy $\hbar\omega_S$. The physical meaning of Eq. (3) is the following: the contribution of electrons and holes in $n,k_{ez};\ m,k_{hz}$ states to $\partial^2\sigma^{N\omega_{\rm LO}}/\partial\Omega\partial\omega_S$ is proportional to the number of electron-hole pairs in this state $[P_{n,m}(k_{ez},k_{hz})]$ multiplied by their probability of emitting a photon of energy $\hbar\omega_S$ $[W_{n,m}^S(\hbar\omega_S;k_{ez},k_{hz})]$. It is clear that for momentum and energy conservation when the electron effective mass is different from its hole counterpart m_h, if Eq. (2) holds one of the N intermediate states necessarily needs to be a virtual state. In this case direct creation accompanied by an indirect annihilation of the electron-hole pair, as much as indirect creation followed by direct annihilation have to be considered in the scattering efficiency. These two processes correspond to the first and the second term in the right-hand side of Eq. (3) and to the generalized schemes of Figs. 2(a) and 2(b), respectively.

Within this classical picture, the distribution function $P_{n,m}^N$ is obtained from the Boltzmann equation. For an isotropic distribution of electron-hole pairs it is possible to show that the function $P_{n,m}^K$ can be obtained from the following recurrent equations

$$P_{n,m}^K(k_{ez},k_{hz})\gamma_{n,m}(k_{ez},k_{hz}) = \sum_{n',k'_{ez}} P_{n',m}^{K-1}(k'_{ez},k_{hz})W_{n'\to n}^e(k'_{ez})\delta\left(\frac{\hbar^2k_{ez}^2}{2m_e} + \hbar\omega_{\rm LO} + \hbar\omega_e(n-n') - \frac{\hbar^2k_{ez}'^2}{2m_e}\right)$$
$$+ \sum_{m',k'_{hz}} P_{n,m'}^{K-1}(k_{ez},k'_{hz})W_{m'\to m}^h(k'_{hz})\delta\left(\frac{\hbar^2k_{hz}^2}{2m_h} + \hbar\omega_{\rm LO} + \hbar\omega_h(m-m') - \frac{\hbar^2k_{hz}'^2}{2m_h}\right),$$
$$K = p, p+1, \ldots \quad (4)$$

and

$$P_{n,m}^p(k_{ez},k_{hz})\gamma_{n,m}(k_{ez},k_{hz}) = W_{n,m}^{Lp}(k_{ez},k_{hz};\hbar\omega_L)\delta\bigg(\hbar\omega_L - E_g - \frac{\hbar^2k_{ez}^2}{2m_e} - \frac{\hbar^2k_{hz}^2}{2m_h} - \hbar\omega_e(n+\tfrac{1}{2}) - \hbar\omega_h(m+1)$$
$$-\mu_B H(m_{se}g_e - m_{sh}g_h) - p\hbar\omega_{\rm LO}\bigg). \quad (5)$$

In Eq. (4) $W_{n'\to n}^e$ ($W_{m'\to m}^h$) is the probability per unit time that the electron (hole) makes a transition from the state n',k'_{ez} (m',k'_{hz}) to the state n,k_{ez} (m,k_{hz}) accompained by LO-phonon emission. In Eq. (5) $W_{n,m}^{Lp}$ is the direct ($p=0$) or indirect ($p=1$) electron-hole pair creation probability in the state $n,k_{ez},\ m,k_{hz}$ by a photon with frequency ω_L. In the indirect creation an LO phonon is emitted. The function $\gamma_{n,m}(k_e,k_n)$ is written as

$$\gamma_{n,m}(k_e,k_n) = W_n^e(k_e) + W_m^h(k_n)\ . \quad (6)$$

$1/W_n(k_z)$ being the electron lifetime in the state n,k_z. For $\hbar^2k_z^2/2m + \hbar\omega(n+\frac{1}{2}) > \hbar\omega_{\rm LO}$ the electron lifetime is determined by the LO-phonon interaction, then

$$W_n(k_z) = \sum_{n',k'_z} W_{n\to n'}(k'_z)\delta\left(\frac{\hbar^2k_z^2}{2m} + \hbar\omega_{\rm LO} + \hbar\omega(n-n') - \frac{\hbar^2k_z'^2}{2m}\right). \quad (7)$$

For the transition rate $W_{n\to n'}$ the following explicit expression is obtained:[14]

$$W_{n\to n'}(k_{iz}) = \alpha_i \omega_{\rm LO}\sqrt{\hbar\omega_{\rm LO}/a_i}\int_0^\infty e^{-t}F_{n',m}(t)\{\,[t+(\sqrt{\hbar k_{iz}^2/2m_i\omega_i}+\sqrt{a_i/\hbar\omega_i})^2]^{-1} \times[t+(\sqrt{\hbar k_{iz}^2/2m_i\omega_i}-\sqrt{a_i/\hbar\omega_i})^2]^{-1}\}dt, \quad i=(e,h), \tag{8}$$

where

$$a_i = \frac{\hbar^2 k_{iz}^2}{2m_i} + \hbar\omega_i(n-n') - \hbar\omega_{\rm LO}\ , \tag{9}$$

$$F_{n',n}(t) = \begin{cases} \dfrac{n'!}{n!}t^{n-n'}[L_{n'}^{n-n'}(t)]^2, & n \ge n' \\ \dfrac{n!}{n'!}t^{n'-n}[L_{n}^{n'-n}(t)]^2, & n' > n, \end{cases} \tag{10}$$

$L_n^{n-n'}$ being the Laguerre polynominal and α_e (α_n) the Fröhlich electron (hole) –LO-phonon coupling constant. The first and second terms in square brackets in Eq. (8) correspond to the dependence of the Fröhlich interaction constant on the phonon wave vector $\mathbf{q}$ ($C_{\mathbf{q}} \sim 1/q$) evaluated at the maximum ($q_z = |k_z| + |k_z'|$) and minimum ($q_z = |k_z| - |k_z'|$) for the transition between n and n' with the emission of one LO phonon.

The value of $W_{n\to n'}$ is different from zero if

$$\frac{\hbar^2 k_{iz}'}{2m_i} = \frac{\hbar^2 k_{iz}^2}{2m_i} + \hbar\omega_i(n-n') - \hbar\omega_{\rm LO} > 0. \tag{11}$$

Transitions into the state $k_z' = 0$ give the maximum value of $W_{n\to n'}$, i.e., when the wave-vector projection k_{iz} satisfies the condition $\hbar k_{iz}^2/2m_i = \hbar\omega_{\rm LO} + \hbar\omega_i(n'-n)$.

If $\hbar^2 k_z^2/2m + \hbar\omega(n+\frac{1}{2}) \le \hbar\omega_{\rm LO}$ the electron–LO-phonon coupling is switched off by conservation of energy and other processes dominate the lifetime (e.g., LA phonons, impurities, etc.). This means that the lifetime is greatly increased, in this case we will assume $W_n(k_z) = \gamma_0 =$ const, independent of k_z and n.

Because we are dealing with LO phonons, the electron and hole populations will be in a set of energy intervals separated exactly by $\hbar\omega_{\rm LO}$. In the case of $\bar{z}(\sigma^\pm,\sigma^\pm)z$ configurations in Faraday geometry the most important contribution comes from the Fröhlich interaction,[4] and the corresponding electron-phonon Hamiltonian couples intraband, i.e., conduction to conduction bands, heavy-to heavy-hole bands, or light- to light-hole bands. In a multiphonon process, however, the phonon wave vectors involved have nonzero q's and they can couple states with different Landau quantum numbers n. This fact is contained implicitly in Eq. (8). For $\bar{z}(\sigma^\pm,\sigma^\pm)z$ geometries and after light absorption all electron and hole states involved in the Raman process will be populated mainly via Fröhlich interaction. The $\bar{z}(\sigma^\pm,\sigma^\mp)z$ configurations involve mainly interband transitions via deformation potential interaction.[4] In this paper we only consider multiphonon processes in $\bar{z}(\sigma^\pm,\sigma^\pm)z$ scattering configurations. The evaluation of Eqs. (4)–(8), and the electron-hole creation and annihilation probabilities will be published elsewhere.[14] Using Eqs. (3)–(8) it is possible to calculate the Raman efficiency for an N-phonon process.

In Fig. 3 the Raman scattering efficiency for a 2-LO-phonon process is shown as a function of the dimensionless magnetic field H/H_0 ($H_0 = cm_0\omega_{\rm LO}/e$) for different ratios of electron and hole effective masses m_e/m_h. The main maxima in the displayed oscillations are due to direct electron-hole annihilation from states with conduction Landau levels $n = 1,2,3$, the secondary structures correspond to incoming resonances with higher index Landau levels. The incoming resonances are weaker in comparison with the outgoing ones. The dashed lines show the evolution in magnetic field of the main resonant peaks with varying m_e/m_h ratio. It can be seen from Fig. 3 that for the smaller m_e/m_h the contribution to $d\sigma/d\Omega$ becomes very small. This means that in GaAs ($m_e/m_h = 0.15$) and InP ($m_e/m_h = 0.175$) the contribution of the heavy-hole ladders can be neglected and light holes should give the main contribution to the Raman scattering efficiency. Another general feature obtained from Eqs. (3)–(8), illustrated in Fig. 3 for the 2-LO-phonon case, is the fact that the principal structures for a fixed laser frequency and sweeping the magnetic field correspond to outgoing resonance with different Landau levels. These resonances are related to *direct* annihilation of the electron-hole pair in their corresponding Landau

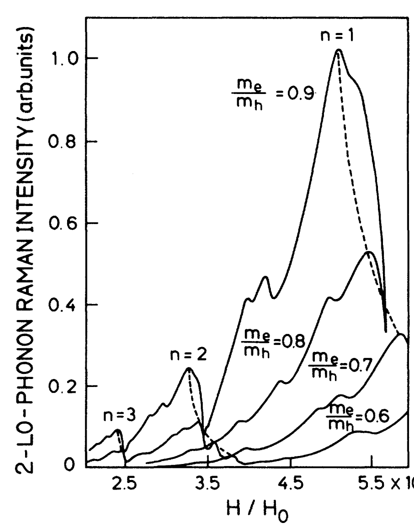

FIG. 3. Results of numerical calculations according to Eq. (3) for the second-order Raman intensity vs H/H_0 at $(\hbar\omega_l - E_g)/\hbar\omega_{\rm LO} = 3.7$ for different ratios of m_e/m_h. The theoretical outgoing resonances are labeled with the corresponding Landau quantum number n in the conduction band. The change of the peak positions with m_e/m_h is represented by dashed lines.

level, i.e., when the electron and hole kinetic energies are equal [see Eq. (3)] the direct annihilation probability is maximum due to the singularity in the one-dimensional density of states. In the same way, the process which corresponds to indirect annihilation $W^{SI}_{n,m}$ presents no singularity like $W^{SD}_{n,n}$, because either the electron or the hole are separated in energy with respect to the Landau band bottom (if $\hbar^2 k_e^2/2m_e \simeq 0$ then $\hbar^2 k_h^2/2m_h \neq 0$ and vice versa). It is therefore clear why only outgoing resonances with only "light-mass" levels are observed in magneto-multiphonon Raman profiles for GaAs and InP.[10,11] Under condition (2) incoming resonances are obtained with higher Landau levels, thus their relative contribution to the cross section are weaker than that corresponding to outgoing ones.

If $m_e \simeq m_h$ the Landau levels have the same separation in the conduction and valence bands, this means that the electron and hole are both in resonance at the same time.[10] In this case all intermediate states in the process are real if the number of emitted phonons is even. For the odd processes one of the several intermediate states has to be virtual even when $m_e = m_h$.[15] In Fig. 2(c) a schematic representation of the Raman scattering by 2 LO phonons is shown for $m_e = m_h$. The physical picture when N is even is as follows: (i) the incident light creates directly an electron-hole pair; (ii) the electron and hole perform $\frac{N}{2}$ real transitions each by emitting LO phonons; (iii) finally, the pair annihilates directly creating a quantum of secondary light $\hbar\omega_s = \hbar\omega_l - N\hbar\omega_{\rm LO}$. The differential cross section can be written in the form

$$\frac{\partial^2\sigma^N}{\partial\Omega\partial\omega_s} = \frac{V_0^2\omega_s^2 n(\omega_s)}{(2\pi)^3 c^4 n(\omega_l)} \sum_{n,k_{ez}} \sum_{m,k_{hz}} P^{N,D}_{n,m}(k_{ez},k_{hz}) \times W^{SD}_{n,m}(\hbar\omega_s;k_{ez},k_{hz}). \quad (12)$$

Following Eqs. (4) and (12) we can obtain the scattering efficiency for $N = 2,4,\ldots$, e.g., for 2 LO phonons:

$$\frac{\partial^2\sigma^N}{\partial\Omega\partial\omega_s} = \frac{V_0^2\omega_s^2 n(\omega_s)}{(2\pi)^3 c^4 n(\omega_l)} \sum_{n,n_1} \frac{W^{LD}_{n_1,n_1}\left(E^0_{n_1,n_1}\right)}{W_n\left(\frac{E^0_{n_1,n_1}}{2}\right)} \times \frac{\left[W_{n_1\to n}\left(\frac{E^0_{n_1,n_1}}{2}\right)\right]^2}{\gamma_{n_1,n}\left(\frac{E^0_{n_1,n_1}}{2};\frac{E^0_{n,n}-2\hbar\omega_{\rm LO}}{2}\right)} \frac{W^{SD}\left(E^0_{n,n}\right)}{2W_n\left(\frac{E^0_{n,n}-2\hbar\omega_{\rm LO}}{2}\right)} \delta\left(\hbar\omega_l - \hbar\omega_s - 2\hbar\omega_{\rm LO}\right), \quad (13)$$

where

$$E^0_{n,n} = \hbar\omega_l - E_g - \mu_B H(g_e m_{se} - g_h m_{sh}) - 2\hbar\omega_e(n+\tfrac{1}{2}). \quad (14)$$

A comparison between the Raman efficiency obtained with Eq. (13) and Eq. (3) for two-LO-phonon processes is shown in Fig. 4. In InP and GaAs the light hole and electron conduction masses are rather similar. The parameters used in this case correspond to GaAs (see Table I). An average mass between those of the electron and the light hole was used in Eq. (13). From Fig. 4 immediately follows the fact that the processes involving direct-direct transitions, where all intermediate states are real, make the strongest contribution and the indirect-direct transitions can be neglected even for $m_e \simeq m_h$.

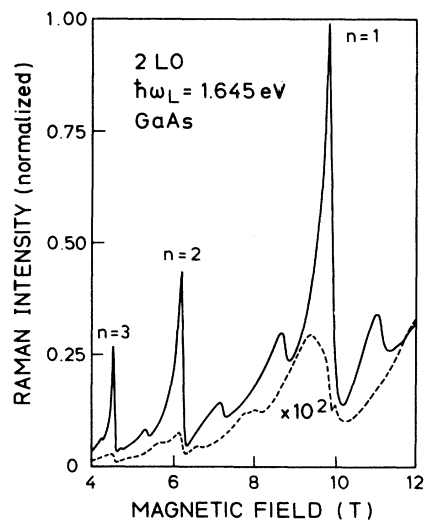

FIG. 4. Second-order Raman intensity vs magnetic field according to Eq. (13) (solid line) and Eq. (3) (dashed line) at $\hbar\omega_l = 1.645$ eV. The calculation has been done with the GaAs parameters and we assume the g factors to be equal to zero. The curve from Eq. (3) has been multiplied by 10^2. In Eq. (13) an average mass between those of the electron and the light hole is used.

III. COMPARISON WITH THE EXPERIMENTAL RESULTS

A comparison with the experimental results for GaAs (Ref. 10) with the calculation is shown in Fig. 1. In $\bar{z}(\sigma^+,\sigma^+)z$ configuration [Fig. 1(b)] the coupling is between the $|n,\frac{3}{2},+\frac{3}{2}\rangle$ hole ladders and an electron

TABLE I. Numerical values of the GaAs parameters used for the calculation of Figs. 1 and 4.

$m_e = 0.0665m_0$ [a]	$g_e = -0.32$ [b]	$E_g = 1520$ meV [a]
$m_{\rm lh} = 0.087m_0$ [a]	$g_{\rm lh} = 22$ [c]	$E_g + \Delta_0 = 1861$ meV [a]
$\epsilon_0 = 12.5$ [a]	$g_{\rm hh} = -4.5$ [b]	$\hbar\omega_{\rm LO} = 37$ meV [a]
$\epsilon_\infty = 10.9$ [a]		

[a] *Landolt-Börnstein Tables*, edited by O. Madelung, M. Schulz, and H. Weiss (Springer-Verlag, Berlin, 1987), Vol. 17a, ϵ_0 and ϵ_∞ are the static and optical dielectric constants, respectively.
[b] This value was calculated according to Ref. 16.
[c] From the best fit to the experiments.

with spin up $|n,s\uparrow\rangle$ in the conduction band,[4] while in $\bar{z}(\sigma^-;\sigma^-)z$ configuration [Fig. 1(a)] the resonant transitions occur between the $|n,\frac{3}{2},-\frac{3}{2}\rangle$ hole ladders and $|n,s\downarrow\rangle$ electron state.[4] This explains the strong selection rule observed experimentally and the shifts between the different peaks, which are proportional to $\mu_B H g_{lh}$. The g factor for the electron and heavy hole were calculated following (Ref. 16). The light-hole g factor is given by $g_{\rm lh}/2 = \lim_{n\to\infty}(E_1^+ - E_2^+) = \gamma_1 + 2\gamma' + 2\kappa$, where γ_1, γ' $[\approx \frac{1}{2}(\gamma_2+\gamma_3)]$, and κ are the Luttinger parameters and E_1^+, E_2^+ are the Landau energies for the "light"-hole ladders reported in Ref. 17. The above equation is obtained by using the optical selection rule $\Delta n = 0$. The value $g_{\rm lh}/2 = \gamma_1 + 2\gamma' - 2\kappa$ given in Ref. 16, which correspond to $\Delta n = 2$, is incorrect for optical experiments. Using typical parameters for GaAs (Ref. 18) the first equation gives $g_{\rm lh} = 26$ while the second one yields $g_{\rm lh} = 18$. In the fitting of Fig. 1 a value of 22 was used. This value is smaller than the calculated one due to the exciton diamagnetic shift.[19]

In order to compare with the experimental data the exciton correction was included in Eq. (13).[3] The peaks (labeled peaks $n = 1,2,3,\ldots$) in Fig. 1 correspond to the Fan plots of Ref. 12 which are in good agreement with our calculation. The differential cross section given by Eq. (13) works well in real semiconductors if the direct creation and direct annihilation is possible near $k_z \simeq 0$. In real III-V semiconductors for $k_z \neq 0$ the hole masses perpendicular and parallel to z are not exactly the same. This explains the discrepancy observed in Fig. 1 between experiment and theory (hatched areas in Fig. 1). Another important result relates to 3-LO-phonon process in GaAs. Within our model the intensity of the 3-LO-phonon process is weaker than that of 2-LO- and 4-LO-phonon overtones, because at least one of the electron and hole intermediate states needs to be virtual. This is in complete agreement with the experimental observations of Refs. 10 and 12.

IV. CONCLUSION

The principal features of the multiphonon processes in a magnetic field can be described through a classical picture where the electron and hole make a number of real transition equal to the order of the process. The validity of this classical picture is governed by the laser frequency and the applied magnetic field whenever

$$\hbar\omega_l > E_g + \mu_B H(g_e m_{se} - g_h m_{sh}) + \frac{\hbar\omega_e}{2} + \frac{\hbar\omega_h}{2} + N\hbar\omega_{\rm LO}.$$

Under this condition the maximum contribution to the scattering efficiency in III-V compounds corresponds to the light-hole valence band and the peak positions in the magneto-Raman profile are due to direct electron-hole recombination from the bottom of Landau bands. Spin quantization introduces a strong selection rule for the Raman efficiency in different scattering configurations.

ACKNOWLEDGMENTS

One of the authors (C.T.-G.) thanks I.G. Lang and S.T. Pavlor for useful discussions in the course of this study. F.I. would like to acknowledge financial support from the Alexander von Humboldt Foundation.

*On leave from Department of Theoretical Physics, Havana University, Havana, Cuba.
†Permanent address: Instituto de Física "Gleb Wataghin," Universidade Estadual de Campinas, Codigo de Endereçamento Postal 13081, Caixa Postal 6165, Campinas, São Paulo, Brazil.

[1] G. Ambrazevičius, M. Cardona, and R. Merlin, Phys. Rev. Lett. **59**, 700 (1989).
[2] F. Iikawa, T. Ruf, and M. Cardona, Phys. Rev. B **43**, 4849 (1991).
[3] T. Ruf, R.T. Phillips, A. Cantarero, G. Ambrazevičius, M. Cardona, J. Schmitz, and U. Rössler, Phys. Rev. B **39**, 13 378 (1989).
[4] C. Trallero-Giner, T. Ruf, and M. Cardona, Phys. Rev. B **41**, 3028 (1990).
[5] V.I. Belitskii, A.V. Gol'tsev, I.G. Lang, and S.T. Pavlov, Zh. Eksp. Teor. Fiz. **86**, 271 (1984) [Sov. Phys. JETP **59**, 155 (1984)].
[6] V.I. Belitsky, A.V. Gol'tsev, I.G. Lang, and S.T. Pavlov, Phys. Status Solidi B **122**, 581 (1984).
[7] R. Zeyher, Solid State Commun. **16**, 49 (1975).
[8] J.F. Scott, R.C.C. Leite, and T.C. Damen, Phys. Rev. **188**, 1285 (1969).
[9] K. Nakamura, N. Ohno, M. Yoshida, and Y. Nakai, Solid State Commun. **36**, 211 (1980).
[10] T. Ruf and M. Cardona, Phys. Rev. Lett. **63**, 2288 (1989).
[11] R.T. Phillips, T. Ruf, and M. Cardona, in *Proceedings of the 20th International Conference on the Physics of Semiconductors*, edited by E. M. Anastassakis and J. D. Joannopouls (World Scientific, Singapore, 1990), p. 1993.
[12] T. Ruf, R.T. Phillips, F. Iikawa, and M. Cardona, in Proceedings of the International Conference on the Application of High Magnetic Fields in Semiconductor Physics, Würzburg, 1990 (to be published).

[13] C.R. Pidgeon and R.N. Brown, Phys. Rev. **146**, 575 (1966).
[14] C. Trallero-Giner, F. Iikawa, and M. Cardona (unpublished).
[15] A detailed calculation of the two-phonon Raman scattering out of resonance in a high magnetic field in semiconductors with equal effective masses for the electron and hole is given recently by V.I. Belitsky, I.G. Lang, and S.T. Pavlov, Physica B (to be published).
[16] L.R. Aggarwal, Phys. Rev. B **2**, 446 (1970).
[17] J.M. Luttinger, Phys. Rev. **102**, 1030 (1956).
[18] M. Cardona, N.E. Christensen, and G. Fasol, Phys. Rev. B **38**, 1806 (1988).
[19] R.J. Elliott and R. Loudon, J. Phys. Chem. Solids **8**, 382 (1959); **15**, 196 (1960).