Spin correlations and NMR relaxation rates in strongly correlated electron systems

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We consider the magnetic properties of a strongly correlated electron system within the slave-boson approach. A finite concentration of holes destroys the long-range antiferromagnetic order but leaves short-range magnetic correlations intact. These correlations renormalize the interaction vertex between nuclear spin and the spin-carrying quasiparticle; the spin response functions thereby acquire a temperature dependence which is qualitatively similar to that observed in both NMR measurements and neutron-scattering data in high-temperature superconductors. The temperature dependence of $1/T_1T$ is very prominent on the copper site, the NMR relaxation rate on the oxygen site obeys the Korringa law, whereas the spin correlations measured by neutron scattering show little temperature dependence.

I. INTRODUCTION

A growing body of experimental evidence indicates that the layered cuprates display short-range antiferromagnetic spin correlations at any doping and at all reasonable temperatures. These short-range correlations result in several peculiar physical properties of these materials: they are responsible for the NMR, NQR, and Knight shifts, and especially for the non-Korringa temperature dependence of the copper spin-lattice relaxation rate.

The qualitative idea is by no means new: Walstedt and Warren speculated[1] that such correlations play an important role for NMR in these materials; Hammel *et al.* explained[2] qualitatively the difference between Cu and O relaxation rate (this idea was later elaborated by Shastry[3]); Millis, Monien, and Pines[4] developed a quantitative phenomenological one-fluid model which neatly describes the data. The existence of short-range AF correlations is also indicated by available neutron scattering data on lightly-doped cuprates: Birgeneau *et al.* observed[5] antiferromagnet correlations peaked near $(\pi/a,\pi/a)$ in a $La_{1.85}Sr_{0.15}Cu_4$ sample with a correlation length of approximately 3–5 lattice constants; similar results were also obtained[6] by Rossat-Mignon *et al.* on semiconducting samples of $YBa_2Cu_3O_{6.4}$, and more recently by Tranquada *et al.*[7] on oxygen deficient superconducting samples of Y-Ba-Cu-O. Presumably the correlations decrease with doping and so have not yet been observed in $YBa_2Cu_3O_7$.[8]

On the other hand the photoemission data of Ref. 9 indicate the existence of a large well-defined Fermi surface, at least in $YBa_2Cu_3O_7$, in which, we believe, short-range AF correlations coexist with a sharp Fermi surface. We believe this can be understood by realizing that even the most heavily doped materials, such as $YBa_2Cu_3O_7$, are not so far from the metal insulator transition where the material has long-ranged antiferromagnetic order. Even though the low-energy, long-wavelength physics changes drastically with doping, the short-range order should be a smooth function of doping. Thus, as the dopant concentration is decreased, the antiferromagnetic (AF) correlations should become longer and longer ranged in space and time until eventually the material becomes an insulator.

Two alternative scenarios of the metal-insulator transition are known. In the classical scenario proposed by Mott[10] the formation of long-range magnetic order produces a gap in the electronic spectrum. In this scenario the major changes to the electronic system are due to the changes in its spectrum near the Fermi surface. The number of charge carriers coincides with the number of electrons and, thus, depends lightly on doping until the gap is formed. This feature obviously contradicts the experimentally observed doping dependence of the charge carrier density measured by the Hall effect,[11] which shows that at small doping the charge carrier density is close to the density of holes introduced by doping and deviates from this value only at larger doping. (A more sophisticated version of Mott scenario has been developed by Kampf and Schrieffer.[12] They assume the existence of slow antiferromagnetic fluctuations in the doped material, which produce a pseudogap in the electron spectrum even in the absence of true long-range AF order.) Further evidence that the number of charges is equal to the doping concentration comes from magnetic penetration length measurements[13] which directly measures the ratio n/m^* of the charge carriers. Moreover, no substantial doping dependence of the effective mass of the charge

carries was observed in these materials, which also contradicts the classical scenario. In the other scenario proposed by Anderson[14] and developed in a number of other papers (see, for instance, Refs. 15–17) the major changes to the electron system are due to the changes in the overlap between the bare electron and the charge and spin carrying quasiparticles; the overlap goes to zero at the metal-insulator transition. In Fermi-liquid language this means that at the metal-insulator transition the residue of the electron Green function goes to zero, although the bulk of the spectrum varies continuously. In more modern language, this can be reformulated as a spin-charge separation with the spinons carrying the spin and the holons the charge.[14,18] The number of holons in this scenario coincides with the doping concentration x whereas the number of spinons is $1-x$. Since the spinons are Fermi particles in this scenario, they form a large Fermi surface, depending lightly on doping, which is in better agreement with the aforementioned photoemission and penetration-length experiments.

The early discussions of the spin-charge separation were either entirely based on mean-field approximations[15,16] (MFA) or took into account only Gaussian fluctuations around the mean field solution.[17] In these treatments the formation of the short-range AF correlations was missed due to the MFA on which they were based. In this paper we shall try to incorporate the short-range AF correlations into the spinon-holon picture and discuss the resulting physical properties.

We shall argue that short-range AF order survives when the long-range order is destroyed either by frustration or by a light doping in agreement with Refs. 19–22. In spinon language, only the spinons close to the Fermi surface can be regarded as quasiparticles with a large relaxation time. The spinons deeper in the Fermi sea interact strongly with each other and their self-energies acquire large imaginary parts, so the spinons do not exist as well-defined elementary excitations. Instead, at these large energies and momenta, spin-wave excitations are better defined. We shall see that the cross-over scale between these two behaviors is determined by the changes in the effective spin interactions induced by the holons.

To deal with this situation we use an approximation in which we consider the spinons close to the Fermi surface and far from it separately: we construct "spin" operators from the spinons far from the Fermi surface and study the effect of the interaction between these quasispins and the spinons in perturbation theory. Finally, we estimate the measurable spin correlators, such as the equal time correlator $\langle S(0)S(R)\rangle$ and the spin susceptibility, which is measured by the NMR relaxation time. We find, that the correlation length which enters the equal time spin correlator is inversely proportional to the doping density and is weakly temperature dependent, whereas the copper NMR relaxation time decreases with temperature roughly as $1/\sqrt{T}$.

II. ANTIFERROMAGNETIC CORRELATIONS IN THE RVB STATE

To make these considerations more concrete we start by considering the properties of the t-J model which is probably the simplest model of holes in an antiferromagnet

$$H=\tfrac{1}{2}\sum_{i,j}[-t_{ij}c_{i\alpha}^{\dagger}c_{j\alpha}+J_{ij}S_iS_j] \tag{1}$$

subject to the single occupancy constraint $c_{i\alpha}^{\dagger}c_{i\alpha}\leq 1$ ($\alpha=1,2$).

In these systems, holes move fast (we assumes $t\geq J$, otherwise phase separation is likely to destroy the interesting physics[23]) and disorder the nearby spins surrounding them. The classical Néel state thereby becomes a very poor approximation to the true spin state. We do not have a quantitative theory describing the destruction of the Néel order by holes. Such a description may be based on the notion of the resonating-valance-bond (RVB) state introduced by Anderson.[14,24] In this state, the low-energy excitations are fermions with spin $\frac{1}{2}$ ("spinons"). At large doping the spinons acquire electrical charge and become undistinguishable from electrons. The RVB states become the exact ground states for the large-N generalizations of model (1), in which spin index α runs over N values instead of two.[16,17]

We consider here only RVB states where the spinons form a fermi sea bounded by a large Fermi surface. These states are stabilized by doping.[25] The total number of spinons is governed by Luttinger theorem which implies that it is equal to the number of sites occupied by spins in the magnetic system. As in Fermi-liquid theory, the spinon states close to the Fermi surface are similar to the states in a Fermi gas.

We cannot prove that such states are close to the ground state of (1) for $N=2$. However, assuming that it is true, one can derive the effective Lagrangian describing the low-energy physics of this state and study its magnetic properties. We will show that such a state always becomes unstable at large scales in the limit of zero doping and some magnetic long-range order is spontaneously formed. Finite doping stops this instability at large scales and stabilizes the RVB state. However, at shorter scales the AF correlations are left intact.

The derivation of the effective Lagrangian describing the low-energy physics can be found in a number of papers (see, e.g., Refs. 15 and 17). To derive it, one introduces auxiliary fields which describe the fluctuations of the bond variables $\Delta_{ij}\approx c_{i\alpha}^{\dagger}c_{j\alpha}$ to decouple the exchange term in (1). In the large N limit the amplitude fluctuations of these fields around a value $\Delta_{ij}=\Delta$ are small and short ranged. Neglecting them one finds the Lagrangian:

$$L=\frac{1}{2m_F}f_{\alpha}^{\dagger}[\nabla-i(A+a)]^2f_{\alpha}+\frac{1}{2m_b}b^{\dagger}[\nabla-ia]^2b+\phi(b^{\dagger}b+f_{\alpha}^{\dagger}f_{\alpha}-1)\,, \tag{2}$$

where a is the continuum limit of the phase of the bond variable $\Delta_{i,j}=\Delta\exp(ia_{ij})$, A is the external electromagnetic field, and ϕ is a Lagrange multiplier that plays the role of a longitudinal scalar potential. Its fluctuations are also short ranged. The effective Lagrangian (2) describes the subsystem of fermions ($f^{\dagger}$) and bosons ($b^{\dagger}$) interacting with a gauge field.[24] The number of bosons coincides with the number of holes introduced by doping. The in-

teraction with the gauge field is very important. The transverse gauge field (a) describes an overdamped collective mode which mediates a long range interaction between the Bose and Fermi particles.[17]

A. The undoped case

We consider the undoped case first. The Lagrangian (2) describes a dense liquid of interacting fermions. The interaction is mediated by the gauge field. The longitudinal part of the field is short ranged. This part of the interaction is repulsive, and small in the limit $N\to\infty$. We assume that it does not result in qualitative changes of the state even at $N=2$. We neglect this interaction below. The transverse part is long ranged and far more dangerous. The interaction mediated by it is attractive between fermions moving in the same direction, but repulsive between fermions moving in opposite directions. Thus, this interaction does not lead to a Cooper instability, but can result in an instability in the particle-hole channel. This instability corresponds to the formation of a SDW.

Here we estimate this interaction and show that it, indeed, results in an instability with wave vector $2p_F$, i.e., the formation of a magnetic state. To simplify the discussion, we make the approximation of a circular Fermi surface. The Green function of the transverse gauge field a was computed in Ref. 17. At small frequencies and momenta it has the general form

$$D_{\mu,\eta}(\omega,q)=\frac{\delta_{\mu,\eta}-q_\mu q_\eta/q^2}{\chi_F q^2+\Gamma(q)|\omega|}\ , \tag{3}$$

where χ_F is the orbital part of the susceptibility of the Fermi system with respect to the internal "magnetic" field and $\Gamma(q)$ is its damping coefficient. The "susceptibility" of the Fermi system can be approximated by its values in a Fermi gas:

$$\chi_F=\frac{N}{24\pi m_F}\ , \tag{4}$$

where N is the number of fermion species, $N=2$ for the physical systems. The damping coefficient depends strongly on q. The main part of the interaction is mediated by the low-frequency "photons" with $\omega\ll v_F q$. At these frequencies $\Gamma(q)$ describes Landau damping, which for a Fermi gas is

$$\Gamma(q)=\gamma/q,\quad \gamma=Np_F/(2\pi)\ . \tag{5}$$

Both estimates (4) and (5) neglect Fermi-liquid corrections.

In our estimate of the effect of the gauge field on the magnetic correlations we shall restrict ourselves to the simplest ladder approximation, neglecting the corrections to the particle Green function and renormalization of the particle-gauge field interaction. Within this approximation one should solve the problem of the spinons interacting with the holons via the gauge field. The interaction mediated by the gauge field is retarded. Thus, we must solve the Bethe-Salpeter equation for the particle-hole pair, or in other words find their scattering amplitude by solving the Dyson equation for the renormalized vertex function V_Q at zero external frequency:

$$V_Q(\varepsilon,p,\varepsilon',p')=V_0(\varepsilon,p,\varepsilon',p')+\sum_\omega\int(dq)W(\varepsilon,p,\omega,q)V_Q(\varepsilon+\omega,p+q,\varepsilon',p')\ , \tag{6}$$

$$W(\varepsilon,p,\omega,q)=\frac{1}{m^2}D_{\mu,\eta}(q,\omega)(p+q/2)_\mu(p+Q+q/2)_\eta\frac{1}{[i(\varepsilon+\omega)-\xi_{p+q}][i(\varepsilon+\omega)-\xi_{Q+p+q}]}\ , \tag{7}$$

$$V_0(\varepsilon,p)=\frac{1}{m^2}D_{\mu,\eta}(p-p',\varepsilon-\varepsilon')(p+p')_\mu(p+p'+Q)_\eta\ , \tag{8}$$

where $\xi_p=p^2/(2m)-\mu$ and V_0 is the scattering amplitude in the Born approximation.

The kernel of this integral equation becomes large if both ξ_p and ξ_p+Q are small. Thus, attraction between particles has maximal effect at $Q=2p_F$. The particles most strongly affected by this interaction are those close to the Fermi surface which have $p\approx -Q/2$. The instability appears first at these values of Q and p. For these Q,p the main contribution to the integral (6) comes from q, which are nearly parallel to the Fermi surface and transverse to Q,p. We neglect the dependence of $D(\omega,q)$ on the small longitudinal component of q, and perform the integral over this component:

$$W(\varepsilon,p,\omega,q)=\frac{v_f q\,\mathrm{sgn}(\varepsilon+\omega)}{2[\varepsilon+\omega+i(p+q)^2/(2m_f)](\gamma|\omega|+\chi q^3)}, \tag{9}$$

where p,q are now only the components of p,q transverse to Q. The vertex function increases at small ε,p. To make an estimate of the eigenvalue of the integral operator in the right-hand side of (6) we replace it by a constant that gives us a lower bound for this eigenvalue. At $\varepsilon=p=0$ the sum of (9) over ω,q diverges logarithmically. Evaluating this sum for finite ω,q with logarithmic accuracy we find:

$$\int(dq)\sum_\omega W(\varepsilon,p,\omega,q)=\frac{p_F\ln[\min\{p^{-1},\varepsilon^{-1/2},T^{-1/2}\}]}{4\pi\gamma}\ . \tag{10}$$

The estimate (10) shows that at low temperatures, this eigenvalue becomes large and the ground state becomes unstable with respect to the formation of a spin-density wave (SDW). Clearly, this instability is due to the singular behavior of the gauge Green function (3) and the ex-

istence of gapless fermion excitations.

This is, of course, a singularly awkward way to describe a magnetically ordered state. However, we will see that it allows us to deal simply with the moderately doped state; the singular behavior of the photon is removed at finite doping which destroys the long-range antiferromagnetic order.

B. The doped case

There are two scenarios we have considered for the destruction of the antiferromagnetic order.

(i) In the first, which is more likely relevant to reduced T_c materials such as oxygen deficient Y-Ba-Cu-O or $La_{2-x}Sr_xCuO_4$, the holes are quite dilute and are strongly scattered at all relevant temperatures above T_c. A hole gains a delocalization energy in the spin state where all spins are ferromagnetically aligned. Thus, the only effect of doping is to introduce some effective frustration into the spin system which causes pairing of the spinons and the formation of a state that could be described as a strongly fluctuating spinon superconductor or, in the strong coupling limit, as a lightly doped dimer liquid[26–29] (the dimer represents a Cooper pair of spinons). In this case, a pseudogap Δ_s should open in the spin excitation spectrum below a spinon pairing temperature[26] T_s. The superconducting $T_c < T_s$ in this scenario is associated with bose condensation of holons and hence should be linearly dependent on the concentration x. While the dimer model adequately describes properties of the system at energies below the pseudogap, it is wholly inadequate at distances less than hv_F/Δ_s. At short distances and high energies the system should look like a weakly frustrated antiferromagnet. We have not yet developed a formalism capable of treating this situation in a more detailed fashion. This is an interesting regime and warrants further study.

(ii) At moderate doping, the holons, being bosons, occupy predominantly states at the bottom of the band. In the absence of scattering by the gauge fields, the bosons will Bose condense at a very high temperature $T_B^{(0)} \sim 2\pi x m_b^{-1}$. The scattering by the gauge field fluctuations decreases the transition temperature significantly.[30–32] In the large range of temperatures below $T_B^{(0)}$ the bose occupation number of the lowest energy level remains finite, so that no bose condensate is formed. However, at small but finite frequencies the response function of the bosons is similar to that of the bose condensate, and hence the bose contribution to the propagator of the gauge field can be approximated by a mass term:

$$\delta D^{-1} = m_b^{-1} x \ . \tag{11}$$

This term makes the fluctuations of the gauge field short range. At low temperatures $T \ll T_B^{(0)}$, either a bose condensate or a completely new state in which time reversal symmetry is broken is formed. In any of these cases the fluctuations of the gauge field remain short ranged. Therefore, doping removes the singularity of the interaction, which was the reason for the SDW instability in the first place. At large q the form of the gauge field propagator remains unchanged. This means that holes change the antiferromagnetic correlations only at large scales and low energies. To estimate these scales we insert the mass term into the gauge field propagator in (7) and note that it influences the integrals (6)–(10) only at small p:

$$p \leq p_0 = m_F/(m_b \gamma) x \ . \tag{12}$$

At short scales (at $r \langle p_0^{-1}$) and high energies ($\varepsilon \rangle p_F p_0/m_F$), the effect of holes on the gauge field Green function is small. The direct interaction of spin excitations with holes is short ranged, its effect becomes small at low doping and/or at large N. We thus assume it is not essential. Thus, the effect of holes at these scales is small and all spin correlations are the same as in an undoped antiferromagnet with somewhat renormalized interaction constants. In the RVB theory these correlations are due to the spinons far from the Fermi surface, which interact strongly with each other at $N=2$. At these scales the description in terms of localized Heisenberg spins becomes much more convenient.

Thus, we will treat the spinons close and far from the Fermi surface separately. From now on, by spinons we will mean spinons close to the Fermi surface only. We construct "quasispins" from the spinons far from the Fermi surface:

$$\tilde{S}_i^{\alpha} = \frac{1}{2} \sum_{p_1,p_2}{}' c_{p_1}^{\dagger} \sigma^{\alpha} c_{p_2} e^{-i(p_i - p_2)r_i} \ , \tag{13}$$

where $\sum'$ means that the sum is performed only over momenta far from the Fermi surface: $|p - p_F| \geq p_0$. The properties of the quasispin operators (13) are close to those of usual spin operators if $p_0 \ll 1$:

$$[\tilde{S}_i^{\alpha}, \tilde{S}_j^{\beta}] = \frac{i}{2} \sum_{p_1,p_2}{}' f(r) e^{(p_2 - p_1)R} c_{p_1}^{\dagger} \{ \varepsilon_{\alpha\beta\gamma} \sigma^{\gamma} \cos[r(p_1 + p_2)] - \delta_{\alpha\beta} \sin[r(p_1 + p_2)] \} c_{p_2} \ ,$$

$$f(r) = \delta(r) - \frac{1}{2\pi} \{ (p_F + p_0) J_1((p_F + p_0)r) - (p_F - p_0) J_1((p_F - p_0)r) \}, \quad r = r_i - r_j, \quad R = (r_i + r_j)/2 \ . \tag{14}$$

At small $rp_0 \ll 1$ the function $f(r)$ can be approximated by a δ function and the commutator (14) coincides with the usual spin commutator. Each site is occupied either by a spinon, a quasispin, or a hole. The average density of spin carried by the spinons is small:

$$\langle S_i^2 \rangle_{sp} = \frac{3 p_0 p_F}{4\pi} \ll 1 \ . \tag{15}$$

Therefore, the expectation value of the quasispin squared is close to the usual $\frac{3}{4}$. The Hamiltonian, describing the interaction between spinons and quasispins follows from (1):

$$H=\frac{1}{2}\sum_{i,j}J_{ij}[\tilde{S}_i^{\alpha}\tilde{S}_j^{\alpha}+\tilde{S}_i^{\alpha}\sum_{p_1,p_2}c_{p_1}^{\dagger}\sigma^{\alpha}c_{p_2}e^{-i(p_1-p_2)r_i}] , \tag{16}$$

where the sum over momenta is performed only over the momenta close to the Fermi surface. The interaction between the spinons [omitted in (16)] is short ranged, small and can be neglected. At short scales $rp_0 \ll 1$ the quasispin correlations coincide with the spin correlations in the undoped antiferromagnet. At larger scales they decrease. We do not know the exact form of the correlation function in the intermediate region. For the following estimates we will assume the simplest possible form for it:

$$\langle \tilde{S}(0)\tilde{S}(r)\rangle = \langle S\rangle_f^2 \exp(i\mathbf{Q}\cdot\mathbf{r}-\kappa r) , \tag{17}$$

where $Q=2p_F$, and $\langle \cdots \rangle_f$ denotes the average over "fast" spin fluctuations that happen at short scales. We estimate: $\langle S\rangle_f^2 \approx 0.1$ and the inverse correlation length by $\kappa \approx p_0$. Since the average spin carried by the spinons is small, their contribution to the spin correlator is small. Therefore, the quasispin correlation function (17) also describes the equal time correlations of the total spin in the system:

$$\langle S(0)S(0)\rangle_q \approx \langle \tilde{S}\,\tilde{S}\rangle_q = \langle S\rangle_f^2 \frac{2\pi\kappa}{[\kappa^2+(Q+q)^2]^{3/2}} . \tag{18}$$

In the following we will only need the static quasispin correlation function. The dynamics of the excitations with large momenta involve ineractions at small length scales only. Therefore their spectrum should coincide with the spectrum of the spin waves: $\omega=ck$ with $c\sim J$ and hence should be important only at very high energies. For the estimates in the following Section we will assume the simplest form of the dynamic correlation function which satisfies this requirement and yields (18) after being integrated over frequency:

$$\langle \tilde{S}\,\tilde{S}\rangle_{\omega,q} = \langle S\rangle_f^2 \frac{8\pi\kappa c^3}{[c^2(\kappa^2+(Q+q)^2)+\omega^2]^2} . \tag{19}$$

At reasonable temperatures all characteristic frequencies of the quasispin correlators are large compared with the temperature: $\omega \sim c\max\{q,\kappa\} \approx \sqrt{2}J\max\{q,\kappa\} \gg T$; thus, the quasispin correlators depend only weakly on temperature. The low-energy spinon excitations are more sensitive to temperature variations. Their existence reveals itself in the imaginary part of the spin correlation functions only. These correlations are measured by NMR experiments.

Finally, in this limit, we expect the superconducting transition to occur when the spinons pair and the holons simultaneously bose condense. In lightly doped materials the transition temperature coincide with the bose condensation temperature, which is linear in doping density. At larger doping, the bose condensation temperature becomes larger than the temperature at which the spinon gap is formed and the superconducting transition temperature is limited by the latter. The mechanism of spinon pairing is not clear to us at the present moment, but almost any spinon pairing is suppressed by the preexisting bose condensate which converts spinons into ordinary electrons. Thus T_c should be a strong function of x: $T_c \sim x$ at small x, saturates at moderate concentrations $x \sim x_0$, and then drops again. The transition $x \geq x_0$, in contrast to that at lower dopant concentration discussed previously, should be qualitatively like a BCS transition in that pairing and condensation occur simultaneously.

III. RELAXATION TIME IN NMR EXPERIMENTS

The magnetic moment of the nucleus at site n interacts with the surrounding spins:

$$H_{\rm hf}=\sum_{j,\alpha}A_{nj}^{\alpha}I_n^{\alpha}S_j^{\alpha} . \tag{20}$$

The constants A_{nj}^{α} of the hyperfine interaction are small, so this interaction can be treated as a small perturbation. The nuclear relaxation rate is related to the spin susceptibility by the well-known formula:[33]

$$\frac{1}{T_1T}=\frac{1}{\omega}\int (dq)\,A_{\alpha n}^2(q)\,{\rm Im}\chi_{\alpha\alpha}(\omega,q) . \tag{21}$$

The frequency ω of the NMR experiment is always very low, so the relaxation time $1/T_1$ is a very effective probe of the low-energy spin excitations. In our approach, we neglect the anisotropy of the spin correlation functions, so all components of the susceptibility are equal: $\chi_{\alpha,\alpha}=\chi$. The hyperfine coupling constants $A_{\alpha n}$ depend on the overlaps of the electronic orbitals on different sites. We will use the simplest model in which the spin of the copper nucleus interacts only with the spin on the copper site, whereas the spin of the oxygen nucleus interacts with spins on adjacent copper sites. Thus, we simplify (21) to

$$\begin{aligned}&\frac{1}{T_1T}=\frac{1}{\omega}\int (dq)\,A_n^2(q)\,{\rm Im}\chi(\omega,q) ,\\ &A_{\rm Cu}(q)=A,\ A_O(q)=B(1+\cos q_x) .\end{aligned} \tag{22}$$

To evaluate the susceptibility $\chi(\omega,q)$ we express it in terms of the Matsubara Green function,

$$\chi(\omega,q)=\int dt\,\langle T_\tau S_q^z(t)S_q^z(0)\rangle \exp(i\omega t) . \tag{23}$$

We estimate the spin correlation function (23) using a perturbation expansion in the interaction between quasispins and spinons. We restrict ourselves to the ladder diagrams:

$$\langle T_\tau S^zS^z\rangle_{\omega,q}=\frac{D(\omega,q)+\Pi_\chi(\omega,q)+2D(\omega,q)\Pi_\chi(\omega,q)}{1-J^2(q)D(\omega,q)\Pi_\chi(\omega,q)} , \tag{24}$$

where $D(\omega,q)$ is the correlation function of the quasispins, $D(\omega,q)=\langle T_\tau \tilde{S}_{\omega,q}\tilde{S}_{\omega,q}\rangle$, and $\Pi_\chi(\omega,q)$ is the spinon susceptibility. Since magnetic long-range order is absent, the excitations of the quasispins have a gap. Thus, the

imaginary part of the quasispin susceptibility is zero at low frequencies:

$$\mathrm{Im}\chi(\omega,q)=\mathrm{Im}\Pi_\chi(\omega,q)D^2(\omega,q)\tilde{V}(\omega,q)\ ,$$
$$\tilde{V}(\omega,q)=\frac{J(q)+D^{-1}(\omega,q)}{1-J^2(q)D(\omega,q)\Pi_\chi(\omega,q)}\ . \tag{25}$$

At $q\approx Q$, $D(\omega,q)$ becomes large, $D(\omega,q)J(q)\gg 1$. In this range we can neglect the second term in the nominator of (25). The denominator in (25) describes the renormalization of the quasispin correlation function due to the interaction with spinons. We will neglect this effect and estimate $\mathrm{Im}\chi(\omega,q)$ at $q\approx Q$ by

$$\mathrm{Im}\chi(\omega,q)=\mathrm{Im}\Pi_\chi(\omega,q)D^2(\omega,q)J^2(q)\ . \tag{26}$$

We see that the contribution to the relaxation rate coming from the vicinity of the point $q\approx Q$ is strongly enhanced by the factor $D(\omega,q)$. Far from this point $D(\omega,q)$ becomes small and the susceptibility can be estimated by the fermion polarization loop itself. Computing the fermion polarization loop we neglect small effects of the spinon relaxation rate and expand over $\omega\ll T$:

$$\mathrm{Im}\chi(\omega,q)\approx\mathrm{Im}\Pi_\chi(\omega,q)$$
$$=2N\omega\int\delta(\xi_{p+q}-\xi_p)\frac{\partial n(\xi_p)}{\partial\xi_p}(dp)\ , \tag{27}$$

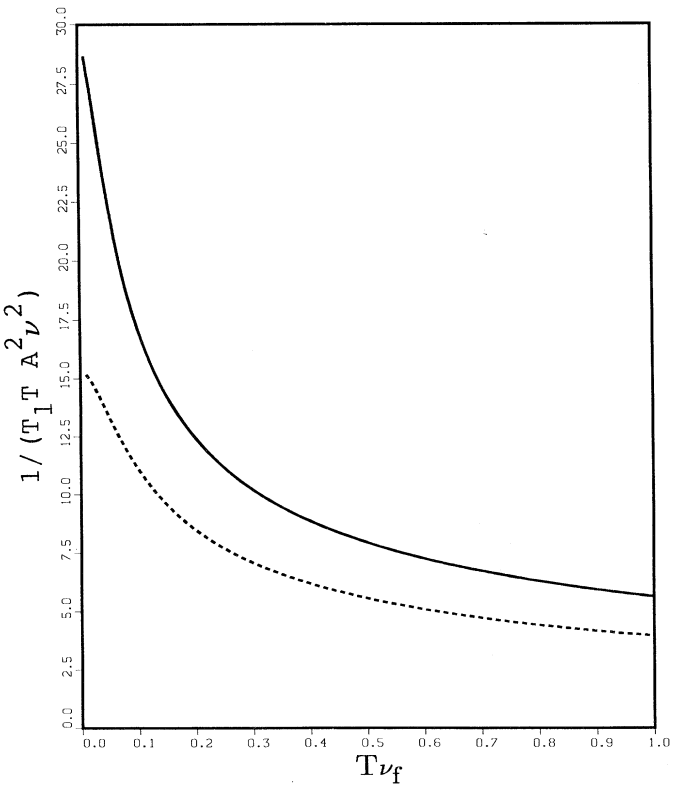

FIG. 1. NMR relaxation rate $1/(T_1TA^2\nu^2)$ for two AF correlation lengths: solid curve $L=2.5$, dashed curve $L=1.5$ (the latter was multiplied by factor 5).

where $n(\xi)$ is Fermi distribution function. The factor $(1+\cos q_x)$ suppresses the contribution from the vicinity of the line $q_x=\pi$. For reasonable doping concentrations and coupling constants J_{ij}, the vector $\mathbf{Q}$ lies close to this line, so the cntribution to the oxygen relaxation rate is substantially suppressed. Thus, we can use (27) to estimate the relaxation rate at the oxygen site:

$$\frac{1}{T_1T}\approx 2.0B^2\nu^2(\varepsilon_f)\ , \tag{28}$$

where $\nu(\varepsilon)$ is the spinon density of states and the numerical coefficient corresponds to a spherical Fermi surface ($\nu=m_F/(2\pi)$). This is the usual Korringa relation.

Now we consider the copper relaxation rate. The main contribution to it comes from the vicinity of the point $q=Q$. Therefore, in this case we use the estimate (26). The function $D(\omega,q)$ depends weakly on ω, so we can replace it by $D(q)=D(0,q)$. The function $D(q)$ is strongly peaked at $q=Q$ (19). If this peak is narrow compared with the temperature broadening of the Fermi surface ($v_F\kappa\ll T$), we can replace $D^2(q)$ in all integrals by a δ function: $D^2(q)=(2\pi)^3 8/3\langle S\rangle_f^4\kappa^{-4}c^{-2}\delta(q-Q)$. Using the estimate $J(Q)/c=2\sqrt{2}$, which holds for weakly frustrated Heisenberg antiferromagnets, we find

$$\frac{1}{T_1T}=0.16\kappa^{-4}A^2\frac{\nu^{3/2}}{T^{1/2}}\ . \tag{29}$$

At low temperatures the width of the peak becomes larger than the thermal broadening of the Fermi surface and the square root temperature growth (29) saturates. The behavior of the relaxation time in the whole range of temperatures is shown in Fig. 1.

The anisotropy of the Fermi surface, which happens near half filling enhances the temperature dependence of the Korringa ratio in the intermediate range of temperatures. In the limit of a nested (square) Fermi surface (29) becomes:

$$\frac{1}{T_1T}=0.25\kappa^{-4}A^2\frac{\nu}{T}\ . \tag{30}$$

IV. CONCLUSION

We have considered the magnetic properties of lightly doped Mott insulators. We assumed that the spin carrying quasiparticles in these systems are fermions with spin $\frac{1}{2}$, which form a Fermi sea with a large Fermi surface. We have shown that a small amount of doping stabilizes this phase against the spontaneous formation of long-range magnetic order, but the short-range antiferromagnetic order is preserved.

Here we consider briefly the other possible states of the doped materials and the applicability of our arguments to them. The first possibility is that the fermion subsystem has a small fermi surface, which shrinks to zero at zero doping. This state was discovered in Refs. 16 and 34. The gauge field fluctuations are larger in this state,[17] so that all instabilities are more pronounced. These instabilities are likely to result in long-range Neel order.[35] A finite concentration of holes again stabilizes this state,

leaving the short-range magnetic correlations intact. Thus, we expect that the spin correlations in this state are qualitatively similar to the correlations in the state with a large Fermi surface.

The second possibility is that the state is close to one of the dimerized states described for the undoped frustrated Heisenberg model.[26–29] In these states, spinons on adjacent sites form a singlet and spin excitations have a large gap. If the dimers crystalize into a state with long-range order, the motion of holes in this state is drastically impeded, so such a state can hardly exist at finite doping without phase separation. This is why we do not believe that this state can be used for the description of the doped materials. Otherwise, when the dimers form a liquid, the motion of a hole becomes easy and the phase separation is not inevitable.[28] It seems that this is the only phase in which the short-ranged AF correlations can be completely excluded self-consistently. However, it is hard to imagine a spin Hamiltonian that does not result in short-range AF fluctuations at all, so that these correlations should be probably included in this description as well. As discussed at the beginning of Sec. II, a version of this state may occur at smaller values of x.

In order to incorporate the short-range AF fluctuations into the spinon picture we separated the spinons close to the Fermi surface and constructed quasispins from the spinons far from it. We evaluated the physical spin correlators in the lowest-order approximation in the interaction between quasispins and spinons. At present it is not clear to us how to go to the next order in perturbation theory in which one should actually take into account the nontrivial commutation rules of the quasispins.

We have shown that such a scheme results in spin correlators whose behavior is similar to that observed experimentally. In particular, the oxygen relaxation time displays Korringa behavior, whereas the copper Korringa ratio falls with temperature as $T^{-1/2}$. The static spin correlation function displays no changes at reasonable temperatures, in agreement with neutron scattering experiments.[5–7] This point distinguishes the present scheme from the phenomenological model of Millis, Monien, and Pines[4] where the changes in the NMR relaxation rate were associated with the temperature dependence of the spin correlation length.

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