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Magnetotransport effects in paramagnetic gases

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A theory of magnetotransport phenomena in low-density paramagnetic systems at low temperature is proposed. The main contentions are based on considering a dissipation mechanism due to the inelastic scattering of paramagnetic particles with transverse collective spin fluctuations. In the presence of an external magnetic field and in the low-momentum limit the corresponding cross section may be very large. Theoretical estimates are in good agreement with recent indirect measurements of intrabeam scattering for cooled H atoms. The extra scattering channel results in a drastic reduction in the mean free path and, consequently, a decrease in the thermal conductivity and viscosity, in contrast to previous theories. In the case of degenerate Fermi systems, the calculations yield quite different temperature dependencies compared with those given by the Fermi-liquid approach. Application of the theory to gaseous H↓, ^{3}He↑, and dilute ^{3}He↑-^{4}He mixtures are briefly discussed, and it is shown that the thermal conductivity and viscosity of gaseous H↓ under common conditions are expected to be roughly 10^2 times less than the normal gas-kinetic values.

I. INTRODUCTION

Macroscopic properties of spin-polarized systems such as gaseous, liquid, and solid ^{3}He↑, dilute ^{3}He↑-^{4}He mixtures, atomic hydrogen H↓ and deuterium D↓, magnetically diluted ^{29}Si↑, etc., are intensively being investigated in laboratories throughout the world.[1] In view of these recent achievements, the question of how an external magnetic field or induced spin polarization can influence transport phenomena in gases and quantum fluids is becoming more important.

The first attempts to consider such an influence were based on rotational degrees of freedom. The magnetic moment of a rotating diatomic (or polyatomic) molecule interacts with an external magnetic field. This results in a field dependence of the viscosity and thermal conductivity in a molecular gas, known as the Senftleben-Beenakker effect. A general theoretical description of the effect was proposed by Gorter, Zernike, and Van Lier, Kagan and Maximov, etc. (see surveys[2]). However, the magnitude of the effect is usually rather small, and therefore it will not be discussed in this paper. The main efforts will be directed to studying transport properties of spin-polarized atomic gases or gaslike dilute systems of quasiparticles. If one deals with a gas of diatomic molecules, the temperature will be supposed to be less than the quantum of rotation.

In 1977, a different mechanism of magnetotransport phenomena was proposed.[3,4] This mechanism results from exchange effects which always exist and play a very important role in collisions of identical particles with spin. The necessity to symmetrize or antisymmetrize the wave function of two scattering atoms leads to a strong dependence of the cross section on their total spin. The scattering cross section can have quite different values depending on whether the total spin of a pair of colliding particles is even or odd.[5] Inasmuch as the occupation numbers for different spin states change upon polarizing the system, the ratio between the numbers of pairs with an even and odd total spin also changes. This considerably affects the collision term in the Boltzmann transport equation and hence the transport coefficients. Since the contribution of the exchange interaction falls off rapidly with increasing temperature, one can expect the system to exhibit magnetotransport properties connected with this mechanism only at low temperatures. It was demonstrated[3,4,1] that, in the case of a low-density degenerate Fermi system with a short-range interaction between the particles (for instance, dilute ^{3}He-^{4}He solutions), the magnetic polarization of a gas was accompanied by an increase of the viscosity and thermal conductivity. Later Lhullier and Laloe[6] and Meyerovich[7] constructed a similar theory for nondegenerate Boltzmann gases and obtained analogous results. There have been other papers on the enhancement of transport properties in polarized Fermi systems where different types of the interaction potential between particles have been used.[8] All these theories predict that the viscosity and thermal conductivity of a gas of fermions placed in an external magnetic field or polarized by means of some dynamical methods (optical pumping, injection of spins, etc.) are larger than in the unpolarized state.

Another interesting phenomenon in spin-polarized gases is connected with the existence of weakly damped transverse spin waves in the system. Describing spin modes in degenerate Fermi systems used to be done on the basis of Landau theory.[9–11,4] The theoretical prediction and experimental discovery of collective spin waves in nondegenerate paramagnetic Boltzmann gases were made during the period from 1981 to 1984.[12–16,6] Spin modes can be considered as some extra Bose branch of elementary excitations in the system. In general, it means that the interaction between paramagnetic particles and spin fluctuations, as well as the interaction be-

tween spin modes themselves, should be taken into account when discussing transport properties. In some cases the interaction of spin waves with each other plays an essential role even in thermodynamics.[17] On the other hand, the scattering of paramagnetic atoms with thermal spin fluctuations can contribute significantly to the transport processes.

The common way to describe quantitatively such scattering is to consider the interaction between the magnetic moment of a particle and the time-dependent and spatially varying magnetic field induced by the fluctuating macroscopic magnetization in spin mode. It used to be done on the basis of the relativistic Zeemann Hamiltonian and Maxwell equations. This is why the corresponding cross section normally turns out to be very small. However, it was shown[18] that some special conditions existed under which even weak relativistic coupling might result in a gigantic cross section. The effect was called the gigantic opalescence (because of some formal analogies with the mathematical description of critical phenomena) and the required conditions will be referred to hereafter as the conditions of gigantic opalescence. The conditions in question will come into effect if one creates a high enough degree of polarization, say, by optical pumping or in some other way, and simultaneously keeps the external magnetic field low. Equivalently, one may say that the available degree of polarization should be higher than the equilibrium magnetization that could be induced by the external field. Obviously such a state can exist only for time intervals less than the relaxation time τ_s of the longitudinal magnetization. For example, one can reach the conditions of gigantic opalescence by polarizing a gas in the external magnetic field and then by rapidly reducing the field during a time period less than τ_s.

A scattering mechanism of purely exchange origin that can influence transport coefficients was proposed in 1989.[19] This mechanism corresponds to the inelastic scattering of paramagnetic particles with thermal spin fluctuations. This kind of scattering is characterized by a cross section which is of the order of the gas-kinetic cross section even under normal conditions, where the system is polarized simply by applying an external magnetic field. Under conditions of gigantic opalescence, the corresponding cross section is also gigantic. Creating the appropriate experimental conditions in order to observe the effect becomes much easier. For example, gaseous ^{3}He$\uparrow$, atomic H$\downarrow$, and magnetically diluted ^{29}Si$\uparrow$ under common conditions are very good systems for experimentally detecting gigantic opalescence. To illustrate the enormous magnitude of the effect, let us give some numerical examples concerning a slow neutron beam propagating through a target with polarized nuclei. In the case of ^{29}Si$\uparrow$ and ^{3}He$\uparrow$ targets under normal experimental conditions[20,21] (temperature $T\sim 1-4$ K, an external magnetic field $H\sim 10$ G, and the degree of polarization $\alpha\sim 30-50\,\%$), the inelastic-scattering cross section mentioned above is expected to be of the order of 10^5-10^6 b.[19] This tremendous value is even more than the cross section of the absorption of neutrons by ^{3}He nuclei.

The large enough value of the scattering cross section for H atoms (of the order of 10^{-14} cm^2) has been recently measured indirectly in experiments with atomic hydrogen beams at $T=2$ K, $H=4.38$ T.[22] Perhaps it is the scattering mechanism that has been identified in the experiments.[22] Quantitative estimates made on the basis of the theory[19] may be in good agreement with the data obtained in Ref. 22. They will be discussed briefly in Sec. V of the present paper.

The appearance of an extra scattering channel which is characterized by such a large cross section must cause a dramatic reduction in the mean free path. This means that transport coefficients in spin-polarized gases will, under certain conditions, be considerably less than in the unpolarized state. This conclusion is quite opposite to all the previous theories. The scattering mechanism contributes significantly to transport properties even in the case of equilibrium polarization in an external magnetic field. The reduction in the mean free path is here predicted to be of the order of 74–82 % provided the external magnetic field is not too high. In strong fields the mean free path starts increasing once again, so the transport coefficients as a function of the magnetic field should possess a minimum. Under conditions of gigantic opalescence, the mechanism in question plays the most important role. The size of the "reducing effect" may be many orders of magnitude. In degenerate Fermi systems, the interaction between paramagnetic particles and transverse spin fluctuations is always predominant and the Fermi-liquid theory no longer describes the temperature dependence of transport coefficients. All transport processes are then determined by the spin-fluctuation-induced scattering considered above, which results in quite extraordinary temperature and field dependencies of kinetic coefficients. In order to obtain an appreciable cross section, one always needs to apply some not too small external magnetic field. Polarizing the system can be created by many different methods, but a magnetic field has to be switched on. That is why the phenomena in question are being considered here as magnetotransport effects in spin-polarized systems.

A theory of magnetotransport phenomena in paramagnetic gases polarized by the same magnetic field or in some other way is proposed in this paper. The viscosity and thermal conductivity of a gas are calculated on the basis of the linearized Boltzmann transport equation. All the collision terms connected with the emission or absorption of thermal spin excitations by paramagnetic atoms are expressed through the exact two-particle scattering amplitude. It is fortunate that, in the most interesting cases, we can solve the Boltzmann equation exactly, with no use of any approximations or model representations, and obtain the final results in a simple, analytic form. The applications of the theory to the cases of gaseous ^{3}H$\uparrow$, dilute ^{3}H$\uparrow$-^{4}He mixtures, atomic H$\downarrow$ are briefly discussed.

II. PHYSICAL MOTIVATION: SIMPLE GAS-KINETIC ESTIMATIONS

At the beginning we shall recollect some results concerning the inelastic scattering of paramagnetic particles

with collective spin fluctuations[19] and give a short physical back-ground. The exchange interaction between a particle with spin and the spin modes has the same origin as any collective phenomenon in quantum liquids and gases (including spin waves themselves). It results from the fact that the energy of a particle is a functional of the distribution function (or, in general, the density matrix). If the distribution function fluctuates, the self-energy will contain some time- and space-dependent terms which can be considered as the Hamiltonian of interaction with the macroscopic fluctuation field. A low-density system of particles with spin $\frac{1}{2}$ and a short-range interaction will be studied here. We will assume that a natural small parameter $Nr_0^3 \ll 1$ exists in the system. N is the atomic density and r_0 the range of interaction. Rarefied gases can exhibit macroscopic quantum-mechanical behavior most efficiently if the temperature T is not too high:[16,18]

$$\max\{\varepsilon_F, T\} \ll \frac{\hbar^2}{mr_0^2} , \tag{2.1}$$

where m is the mass of a particle, and ε_F is the quantum degeneracy temperature which is related to the density by means of the usual expression

$$\varepsilon_F = \frac{mv_F^2}{2} = \frac{(3\pi^2 N)^{2/3}\hbar^2}{2m} . \tag{2.2}$$

If the criterion (2.1) is fulfilled, the mean de Broglie wavelength will considerably exceed the size of a particle r_0 and the ultraquantum-mechanical case will occur. In this case, all momenta of colliding particles are small, $pr_0 \ll \hbar$, and the main contribution to the scattering amplitude is given by the s-wave scattering length which depends neither on the energy nor on the angle. The low-energy scattering and small transferred momenta provide one more advantage. The reason is that finite momenta would result in some structural factor like $e^{i\mathbf{p}\cdot\mathbf{r}}$ in the scattering probability. It would essentially reduce the total cross section of the particle-spin mode interaction in the final result.

Under the conditions discussed above, the corrections in the self-energy of a particle associated with the quantum-mechanical refraction are determined by the quite simple diagrams shown in Fig. 1. The analytical expression corresponding to these diagrams has the form

$$\delta H = gN(\mathbf{r},t) - \frac{1}{\beta} g\boldsymbol{\sigma}\cdot\mathbf{M}(\mathbf{r},t), \quad g \equiv \frac{2\pi\hbar^2}{m} a , \tag{2.3}$$

where β is the magnetic moment of a particle, $\boldsymbol{\sigma}$ Pauli matrices, g the coupling constant expressed through the s-wave scattering length a, $|a| \sim r_0$, and $\mathbf{M}$ the macroscopic magnetization related very simply to the degree of polarization α,

$$\mathbf{M} = \beta N \alpha \mathcal{M}, \quad \alpha = \frac{N_\uparrow - N_\downarrow}{N}, \quad N_\uparrow + N_\downarrow = N . \tag{2.4}$$

Here $\mathcal{M}$ is the unit vector in the direction of spin polarization, and $N_\uparrow$ and $N_\downarrow$ the number densities of particles with spin up and down, respectively. The imaginary part of δH, describing the attenuation of single-particle excitations, contains higher orders in the small momentum $\mathbf{p}$.

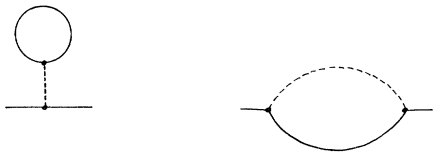

FIG. 1. The diagrammatic representation for the corrections in the self-energy of a particle, connected with quantum-mechanical refraction in the low-momentum limit $pr_0 \ll \hbar$.

The first term in (2.3) corresponds to the interaction of a particle in a gas with the density fluctuation field. In the long-wavelength limit it describes the inelastic scattering of slow atoms with phonons. It is completely similar to the problem of slow neutron scattering caused by density fluctuations in liquids and solids.[23] In this case, the differential cross section is determined by the dynamic structure factor, i.e., the Fourier transform of the binary correlation function. When a degenerate Fermi gas is considered, the first term in (2.3) and the related calculations are somewhat analogous to the problem of electron-phonon interaction in conducting media. Since we are going to study the influence of transverse with respect to $\mathbf{M}$ spin modes, which are not coupled with density fluctuations, we consider just the second term in the Hamiltonian (2.3).

The process under consideration can be treated as the inelastic scattering of a paramagnetic atom, accompanied by the emission or absorption of a thermal spin wave. Since the Hamiltonian in question has a purely exchange structure, the total magnetic moment has to be conserved. This automatically implies and it is also confirmed by quantum-mechanical calculations of matrix elements that only the two types of transitions shown in Fig. 2 are permitted by the Hamiltonian. In both cases the particle experiences the spin-flip transition and the change of the atomic spin is compensated by the magnetic moment of the emitted or absorbed transverse spin mode. One can interpret it as redistributing the magnetic moment between fermion spins and magnons, which may be treated as a system of delocalized, inversely directed, spins.

So, using the second term in (2.3) as the Hamiltonian of a particle-magnon interaction and calculating the probabilities of inelastic processes shown in Fig. 2, one can express the corresponding differential cross section through

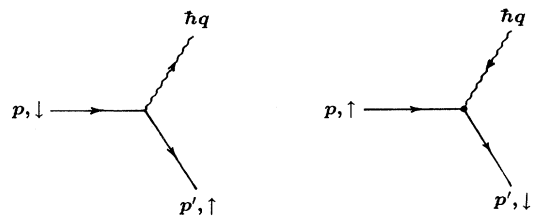

FIG. 2. Two types of spin-flip inelastic transitions permitted by the exchange Hamiltonian (2.3). In both cases the change of the fermionic spin is compensated by the magnetic moment of the emitted or absorbed thermal magnon.

the dynamic magnetic form factor $S_{ik}(\mathbf{k},\omega)$ (Ref. 19)

$$d\sigma_{\beta\to\alpha} = \frac{2m}{pN}\left[\frac{g}{\beta\hbar}\right]^2 [\sigma^x_{\alpha\beta}S_{xx}(\mathbf{q},\omega)-\sigma^y_{\alpha\beta}S_{xy}(\mathbf{q},\omega)]d\Gamma' ,$$

$$\hbar\omega=\frac{p^2-p'^2}{2m}-\beta H(\sigma^z_{\beta\beta}-\sigma^z_{\alpha\alpha}),\quad \mathbf{p}-\mathbf{p}'=\hbar\mathbf{q} , \tag{2.5}$$

$$d\Gamma=\frac{d^3p}{(2\pi\hbar)^3} ,$$

where $\alpha=\uparrow,\downarrow$, $\beta=\uparrow,\downarrow$ are spinor indices, and the external magnetic field and z axis are assumed to be directed along the vector $\mathbf{M}$. The transverse component of the magnetic structure factor in (2.5), which determines the transverse magnetization correlation function, is related to the dynamic susceptibility $\chi_{ik}(\mathbf{q},\omega)$, $i,k=x,y$. The relationship is tensorial even in a gas because the magnetic symmetry is broken and the related properties of a system are no longer isotropic. Substituting $\chi_{ik}(\mathbf{q},\omega)$ into Eq. (2.5), we get

$$\begin{aligned} &d\sigma_{\downarrow\to\uparrow}\equiv d\sigma_1 = A(\omega)\mathrm{Im}\chi_+(\mathbf{q},\omega)d\Gamma' ,\\ &d\sigma_{\uparrow\to\downarrow}\equiv d\sigma_2 = -A(\omega)\mathrm{Im}\chi_+(-\mathbf{q},-\omega)d\Gamma' ,\\ &A(\omega)=4\hbar\frac{m}{pN}\left[\frac{g}{\beta\hbar}\right]^2\left[1-\exp\left[-\frac{\hbar\omega}{T}\right]\right]^{-1} ,\\ &\chi_+(\mathbf{q},\omega)=\chi_{xx}(\mathbf{q},\omega)+i\chi_{yx}(\mathbf{q},\omega) . \end{aligned} \tag{2.6}$$

If spin oscillations are not too strongly damped the magnetic susceptibility, $\chi_+(\mathbf{q},\omega)$ is determined by the magnon dispersion law and has the following form:

$$\mathrm{Im}\chi_+(\mathbf{q},\omega)=\frac{2}{\hbar}\beta^2N\alpha\frac{|\omega''_q|}{(\omega-\Omega_H-\omega'_q)^2+\omega''^2_q} , \tag{2.7}$$

where the Larmor frequency of the uniform precession in an external magnetic field $\Omega_H=2\beta H/\hbar$ is introduced, ω''_q corresponds to the imaginary part of the magnon energy spectrum and describes the attenuation of spin waves, and $(\omega'_q+\Omega_H)$ gives the real part of the spectrum. In the case where external magnetic field is not too small,

$$\Omega_H \gg \max\{|\omega'_q|,|\omega''_q|\} , \tag{2.8}$$

and by making use of the known relationship

$$\lim_{\nu\to 0}\frac{1}{\pi}\frac{\nu}{x^2+\nu^2}=\delta(x) , \tag{2.9}$$

one can easily obtain

$$\mathrm{Im}\chi_+(\mathbf{q},\omega)=\frac{2\pi}{\hbar}\beta^2N\alpha\delta(\omega-\Omega_H) . \tag{2.10}$$

After this, integrating the expressions (2.6) for $d\sigma_1$, $d\sigma_2$ becomes trivial and yields with good accuracy the total inelastic-scattering cross sections

$$\begin{aligned} \sigma_1&=16\pi a^2\alpha(1-e^{-\hbar\Omega_H/T})^{-1}\\ &=16\pi a^2\alpha(\mathcal{N}_0+1) ,\\ \sigma_2&=16\pi a^2\alpha(e^{\hbar\Omega_H/T}-1)^{-1}=16\pi a^2\alpha\mathcal{N}_0 . \end{aligned} \tag{2.11}$$

Here $\mathcal{N}_0$ is the occupation number for magnons

$$\mathcal{N}_0=(e^{\hbar\Omega_H/T}-1)^{-1} \tag{2.12}$$

and only the Larmor energy gap in the spectrum of spin waves is taken into account. In this sense, one may treat $\mathcal{N}_0$ from (2.12) as the number of uniform precession quanta. Formulas (2.11) imply that the Einstein relationship for the induced emission and absorption of spin waves are fulfilled automatically.

In the hydrodynamic range $ql\ll 1$, where l is the mean free path, and within the exchange approximation, both ω'_q and ω''_q are quadratic in the wave vector $\mathbf{q}$ and can be represented in the form[10,6,16]

$$\begin{aligned} &\omega_q=\Omega_H+\omega'_q+i\omega''_q ,\\ &\omega'_q=bq^2\frac{\gamma^2}{1+\gamma^2},\quad \omega''_q=-\frac{\omega'_q}{\gamma} ,\\ &\gamma=\frac{D_0}{b}=\Omega_{\mathrm{int}}\tau,\quad \Omega_{\mathrm{int}}=-\frac{2gN\alpha}{\hbar} ,\\ &b=-\frac{\hbar}{2m}\frac{E_\uparrow-E_\downarrow}{g(N\alpha)^2} . \end{aligned} \tag{2.13}$$

Here D_0 is the spin-diffusion coefficient in the unpolarized gas, and τ is the gas-kinetic relaxation time. $E_\uparrow$ and $E_\downarrow$ are the total energies of the components consisting of particles with spins up and down

$$E_\alpha=\int\frac{p^2}{2m}n^{(0)}_\alpha d\Gamma,\quad \alpha=\uparrow,\downarrow , \tag{2.14}$$

where the equilibrium distribution functions $n^{(0)}_\alpha(\mathbf{p})$ for both species are introduced. The terms ω'_q and ω''_q determine the spatial dispersion in the frequency and attenuation of spin fluctuations. Keeping only the Larmor frequency Ω_H in the energy spectrum means neglecting both the absorption and q-dependent corrections in the frequency of spin waves. If the parameter $|\gamma|$ is large enough, $|\gamma|\gg 1$, the damping $|\omega''_q|$ is much less than $|\omega'_q|$ and nonuniform spin oscillations can propagate in the gas, but the contribution of q-dependent terms to the spectrum is much less than Ω_H for all accessable values of q. In the case $|\Omega_{\mathrm{int}}|\tau\gg 1$, collective spin modes can also propagate even in the nonhydrodynamic regime,[12,16] $ql\gg 1$, if their wavelengths are not too short $qv^*\ll|\Omega_{\mathrm{int}}|$. Here v^* is some average velocity. In a degenerate Fermi gas, v^* is the Fermi velocity and in the classical high-temperature limit, $v^*\sim(T/m)^{1/2}$. For the case in question, formulas (2.13) for ω'_q are still valid and one can easily convince oneself that, for all permitted q, the magnitude of $|\omega'_q|$ cannot exceed $|\Omega_{\mathrm{int}}|$. At higher wave numbers when $qv^*\gg|\Omega_{\mathrm{int}}|$, Landau collisionless damping makes the propagation of spin waves impossible.[24] In the opposite limiting case, $|\gamma|\ll 1$, the imaginary part plays the dominant role $|\omega''_q|\gg|\omega'_q|$. The nonuniform motions of macroscopic magnetization (diffusive spreading) may attenuate rapidly but the uniform Larmor precession lives for a much longer time (in the exchange approximation the lifetime is equal to infinity). The value of q in this case is cut off by the re-

ciprocal mean free path so the magnitude of $|\omega_q''|$ is not more than $D_0 l^{-2} \sim v^* l^{-1} \sim \tau^{-1}$. In general, the imaginary part of the frequency contains also some terms of the order of τ_s^{-1}, which is of the relativistic origin (say, the dipole-dipole interaction). Normally the value of τ_s^{-1} is much less than $|\omega_q''|$ and $|\omega_q'|$ estimated above. Anyway, the inequality $\Omega_H \tau_s \gg 1$ will also be assumed to be fulfilled.

To avoid misunderstanding, let us emphasize that neglecting all q-dependent terms in the spectrum and magnetic susceptibility does not mean that the moving particle emits or absorbs just the uniform-precession quantum and experiences the zero-angle scattering only. In the process in question, we deal with the isotropic scattering in all directions [differential cross sections (2.6) have been integrated over all the scattering angles] in the limiting case of small momenta $p \to 0$ and $q \to 0$. It is somewhat analogous to the case of the elastic s-wave scattering when, in the zero-momentum limit, the cross section remains finite and goes to some constant value. Cross sections (2.11) correspond to emitting and absorbing spin fluctuations with all possible $\mathbf{q}$. However, since the momentum p is very small, and, consequently, the value of q is small too, in the main approximation $p \to 0$, the spatial dispersion terms do not contribute essentially to the total cross sections which turn out to be finite because of the existence of the Larmor energy gap in the magnon spectrum.

It should be pointed out that, in some cases, especially in degenerate Fermi liquids, the contribution of ω_q' to the spectrum may be commensurate with that of the Larmor gap. It results in rather interesting effects.[17] However, such a situation will not be discussed here.

Using the estimations mentioned above, it is easy to figure out the range of magnetic fields where the theory developed can be applied and to evaluate the contribution of the scattering mechanism to transport processes. Criterion (2.8) reduces to

$$\Omega_H \gg \max\{|\Omega_{\text{int}}|, \tau^{-1}\} , \tag{2.15}$$

and being applied to the case of a nondegenerate Boltzmann-Maxwell gas it gives

$$\beta H \gg \begin{cases} \Delta N |a|^3 \alpha, & \text{if } \alpha \geq \alpha_c, \quad \alpha_c = \dfrac{|a|}{\Lambda} \\ \hbar N a^2 v_T & \text{if } \alpha \leq \alpha_c \end{cases} , \tag{2.16}$$

where the thermal velocity v_T, characteristic energy Δ, and average de Broglie wavelength are determined by the expressions

$$v_T^2 = \frac{T}{m}, \quad \Delta = \frac{\hbar^2}{ma^2}, \quad \Lambda = \frac{\hbar}{mv_T} . \tag{2.17}$$

Formulas (2.16) yield the lower limit for the external magnetic field below which the theory no longer holds. Combining common gas-kinetic relationships and expressions (2.11) and (2.12) for cross sections, one can easily estimate the thermal conductivity

$$\kappa \sim N_\uparrow v_T l_\uparrow + N_\downarrow v_T l_\downarrow ,$$
$$\kappa \sim \frac{v_T}{a^2 \alpha} \left[\frac{1-\alpha}{\mathcal{N}_0 + 1} + \frac{1+\alpha}{\mathcal{N}_0} \right] , \tag{2.18}$$

and viscosity

$$\eta \sim m N_\uparrow v_T l_\uparrow + m N_\downarrow v_T l_\downarrow \sim m\kappa . \tag{2.19}$$

If the external magnetic field is not too strong, $\beta H \ll T$, Eqs. (2.18) and (2.19) can be simplified to

$$\kappa \sim \frac{v_T}{a^2 \alpha} \frac{\beta H}{T} \sim \kappa_0 \frac{1}{\alpha} \frac{\beta H}{T} ,$$
$$\eta \sim \frac{m v_T}{a^2 \alpha} \frac{\beta H}{T} \sim \eta_0 \frac{1}{\alpha} \frac{\beta H}{T} , \tag{2.20}$$

where κ_0 and η_0 correspond to the thermal conductivity and viscosity in an unpolarized gas at $H = 0$. One can convince oneself that, under the conditions of gigantic opalescence, $\alpha \gg (\beta H / T)$, the transport coefficients (2.20) turn out to be much less than their values κ_0, η_0 in the unpolarized state. Therefore, in the case of nonequilibrium polarization, the predominant contribution to dissipative processes is given by the scattering mechanism and direct elastic fermion-fermion collisions need not be taken into account. In the case of thermodynamically equilibrium polarization, when

$$\alpha = \tanh \frac{\beta H}{T} \approx \frac{\beta H}{T} , \tag{2.21}$$

we find from (2.20) that quantities κ, η have the same order of magnitude as κ_0, η_0 and, in the first approximation, do not depend on H at all. It means that we have to take into account both the spin-fluctuation-induced mechanism and direct particle-particle scattering simultaneously. After all, the final reduction in the transport coefficients is not so drastic, but it is large enough to be measured experimentally (74–82 %). One must not forget, however, that these estimations cannot be extended to the zero-field limit $H = 0$ because of the existence of the lower boundary for H given by Eq. (2.16).

In the case of a degenerate Fermi gas, the relaxation time τ does not depend on the density

$$\tau \sim \frac{\hbar^2}{ma^2 T}$$

and, instead of (2.16) we get

$$\beta H \gg \begin{cases} \Delta N |a|^3 \alpha & \text{if } \alpha \geq \alpha_c \\ T(T/\Delta) & \text{if } \alpha \leq \alpha_c \end{cases} , \quad \alpha_c = \frac{p_F |a|}{\hbar} \left(\frac{T}{\varepsilon_F} \right)^2 . \tag{2.22}$$

In low magnetic fields, when $\beta H \ll T \ll \varepsilon_F$, all magnon energies are small compared with the thermal disturbance of the Fermi step function. This allows us to approximately treat the emission or absorption of a spin mode as a sort of elastic fermion scattering. (It is absolutely the same as in the case of electron-phonon interaction in metals at high temperatures.[2]) This is why no

small multiples such as $(T/\varepsilon_F)^2$ will appear in expressions for the thermal conductivity and viscosity as a result of Fermi statistics. Simple estimates yield

$$\begin{aligned}
&\kappa \sim \sum_\gamma N_\gamma \frac{T}{\varepsilon_{F_\gamma}} v_{F\gamma} l_\gamma \ , \\
&\eta \sim \sum_\gamma m N_\gamma v_{F\gamma} l_\gamma , \quad \gamma = \uparrow, \downarrow \ , \\
&\kappa \sim \frac{v_F}{a^2} \frac{\beta H}{\varepsilon_F} \frac{(1+\alpha)^{2/3}+(1-\alpha)^{2/3}}{\alpha}, \quad \varepsilon_F = \frac{m v_F^2}{2} \ , \\
&\eta \sim \frac{m v_F}{a^2} \frac{\beta H}{T} \frac{(1+\alpha)^{4/3}+(1-\alpha)^{4/3}}{\alpha} \ .
\end{aligned} \tag{2.23}$$

In the case of thermodynamical equilibrium, we have

$$\alpha = \frac{3\beta H}{2\varepsilon_F} \tag{2.24}$$

and formulas (2.23) reduce to

$$\kappa \sim \frac{v_F}{a^2} \quad \eta \sim \frac{m v_F}{a^2} \frac{\varepsilon_F}{T} \ , \tag{2.25}$$

which do not depend on the magnetic field in this approximation. We see that, in all cases (both the equilibrium and nonequilibrium polarization), the thermal conductivity and viscosity turn out to be much less than corresponding values in the unpolarized gas. The reduction factor is of the order of $(T/\varepsilon_F) \ll 1$ in the equilibrium case and $(T/\varepsilon_F)(\beta H/\varepsilon_F) \ll 1$ under conditions of gigantic opalescence when $\alpha \to 1$. The temperature dependencies of κ and η are quite different from those predicted by the theory of normal Fermi liquids. Thus, transport properties of spin-polarized degenerate Fermi gases placed into an external magnetic field are completely determined by scattering with transverse spin fluctuations and Fermi-liquid theory no longer holds provided criterion (2.22) is fulfilled and the temperature is not extremely small $\beta H \ll T \ll \varepsilon_F$. It should be mentioned that there is a conflict between the condition $\beta H \ll T$ on the one side and inequalities (2.16) and (2.22) on the other. A range of magnetic fields satisfying both conditions undoubtedly exists in all cases provided the density of a gas is not too high

$$1 \gg \frac{T}{\Delta} \gg N|a|^3 \ . \tag{2.26}$$

[Sometimes, e.g., in the case of Boltzmann-Maxwell statistics, this requirement may be weaker, $1 \gg (T/\Delta)^{1/2} \gg N|a|^3$.]

III. TRANSPORT EQUATIONS: COLLISION INTEGRALS

Transport coefficients should be found by solving Boltzmann transport equations for all kinds of elementary excitations in the system. In this case, we have two types of quasiparticles, namely, single-particle Fermi excitations (the particles themselves in the main approximation) and collective oscillations of magnetization. Thus, in general, one should deal with transport equations for both fermion and magnon components, taking into account all possible interactions between them. We, however, will consider the high-temperature limit, $T \gg \beta H$, when the number of magnons is large enough and the relaxation in the spin-fluctuation subsystem is very fast. In other words, the magnon distribution function may be set to equal to its equilibrium value in all transport equations. After this, one can obviously restrict oneself by considering the Boltzmann equation only for fermions but the collision term in the equation will certainly contain the particle-magnon interaction. In the case in question, the low-momentum inelastic particle-magnon scattering may be treated like the quasielastic one. This resembles very much the case of electron-phonon scattering in metals at high temperatures.

The particle-magnon collision term corresponding to the scattering of spin-up particles is given by the expression

$$\begin{aligned}
\mathrm{Coll}_1\{n_{p\uparrow}\} = \int [\, & W(\mathbf{p},\mathbf{q};\mathbf{p}')n_{p'\downarrow}(1-n_{p\uparrow})(\mathcal{N}_q+1) \\
& - W(\mathbf{p}';\mathbf{p},\mathbf{q})n_{p\uparrow}(1-n_{p'\downarrow})\mathcal{N}_q\,] \\
& \times \delta(\varepsilon+\hbar\omega_q-\varepsilon')\frac{d^3q}{(2\pi)^3} \ , \\
\varepsilon = \frac{p^2}{2m} - \beta H, \quad & \varepsilon' = \frac{p'^2}{2m} + \beta H, \quad \mathbf{p}-\mathbf{p}' = \hbar\mathbf{q} \ .
\end{aligned} \tag{3.1}$$

Here $n_{p\uparrow}$, $n_{p\downarrow}$, $\mathcal{N}_q$ are the spin-up particle, spin-down particle, and magnon distribution functions, respectively, $W(\mathbf{p}',\mathbf{q};\mathbf{p})$ and $W(\mathbf{p}';\mathbf{p},\mathbf{q})$ the probabilities of emission or absorption of a spin wave as illustrated in Fig. 2, and $\hbar\omega_q$ the energy of a collective spin excitation. In a similar way, one can formulate the collision term describing the change of the distribution function for spin-down particles

$$\begin{aligned}
\mathrm{Coll}_1\{n_{p\downarrow}\} = \int [\, & W(\mathbf{p},\mathbf{p}',\mathbf{q})n_{p'\uparrow}(1-n_{p\downarrow})\mathcal{N}_q \\
& - W(\mathbf{p}',\mathbf{q};\mathbf{p})n_{p\downarrow}(1-n_{p'\uparrow})(\mathcal{N}_q+1)\,] \\
& \times \delta(\varepsilon-\hbar\omega_q-\varepsilon')\frac{d^3q}{(2\pi)^3} \ , \\
\varepsilon = \frac{p^2}{2m} + \beta H, \quad & \varepsilon' = \frac{p'^2}{2m} - \beta H, \quad \mathbf{p}-\mathbf{p}' = \hbar\mathbf{q} \ .
\end{aligned} \tag{3.2}$$

As it follows from the result obtained in the previous section, probabilities $W(\mathbf{p}',\mathbf{q};\mathbf{p})$ and $W(\mathbf{p}';\mathbf{p},\mathbf{q})$ reduce to the same constant W_0 in the low-momentum limit $pr_0 \ll \hbar$,

$$\begin{aligned}
W(\mathbf{p}',\mathbf{q};\mathbf{p}) &= W(\mathbf{p}';\mathbf{p},\mathbf{q}) \\
&\equiv W_0 = 4(2\pi\hbar)^3 \left[\frac{a}{m}\right]^2 N\alpha \ .
\end{aligned} \tag{3.3}$$

On the other hand, when discussing the case of not too high magnetic fields $\beta H \ll T$, we may consider the magnon distribution function $\mathcal{N}_q$ as the thermal-equilibrium one. Besides, as mentioned above, if the external magnetic field is not too small, one can restrict oneself by taking into account only the Larmor gap in the spectrum $\hbar\omega_q \approx \hbar\Omega_H$ and distribution function $\mathcal{N}_q \approx \mathcal{N}_0$, where $\mathcal{N}_0$

is determined by Eq. (2.12). After this, collision integrals (3.1) and (3.2) are essentially simplified and reduce to the form

$$\begin{aligned}\mathrm{Coll}_1\{n_{p\uparrow}\}&=W_0\int[n_{p'\downarrow}(1-n_{p\uparrow})(\mathcal{N}_0+1)\\&\quad-n_{p\uparrow}(1-n_{p'\downarrow})\mathcal{N}_0]\delta\left(\frac{p^2}{2m}-\frac{p'^2}{2m}\right)d\Gamma',\\\mathrm{Coll}_1\{n_{p\downarrow}\}&=W_0\int[n_{p'\uparrow}(1-n_{p\downarrow})\mathcal{N}_0\\&\quad-n_{p\downarrow}(1-n_{p'\uparrow})(\mathcal{N}_0+1)]\\&\quad\times\delta\left(\frac{p^2}{2m}-\frac{p'^2}{2m}\right)d\Gamma',\\d\Gamma&=\frac{d^3p}{(2\pi\hbar)^3}=g_p d\varepsilon\frac{d0}{4\pi},\quad g_p=\frac{pm}{2\pi^2\hbar^3}.\end{aligned}\tag{3.4}$$

In most cases, collision terms (3.4) entirely determine the dissipative characteristics of spin-polarized quantum gases. The exception is the case of a Boltzmann-Maxwell gas which is in full thermal equilibrium being placed into an external magnetic field. In this case, one should take into account direct elastic interparticle collisions and add the appropriate collision terms which have the common form[4]

$$\begin{aligned}\mathrm{Coll}_2\{n_{p\uparrow}\}&=\int w(n_{p'\uparrow}n_{p_1'\downarrow}-n_{p\uparrow}n_{p_1\downarrow})d\Gamma' d\Gamma_1 d\Gamma_1',\\\mathrm{Coll}_2\{n_{p\downarrow}\}&=\int w(n_{p'\downarrow}n_{p_1'\uparrow}-n_{p\downarrow}n_{p_1\uparrow})d\Gamma' d\Gamma_1 d\Gamma_1',\\\mathbf{p}+\mathbf{p}_1&=\mathbf{p}'+\mathbf{p}_1',\quad\frac{p^2}{2m}+\frac{p_1^2}{2m}=\frac{p'^2}{2m}+\frac{p_1'^2}{2m}.\end{aligned}\tag{3.5}$$

In the general case, two linearized coupled kinetic equations

$$\begin{aligned}&\frac{\partial}{\partial t}n_{p\alpha}+\mathbf{v}\cdot\nabla n_{p\alpha}=\mathrm{Coll}_1\{n_{p\alpha}\}+\mathrm{Coll}_2\{n_{p\alpha}\}\equiv\mathrm{Coll}\{n_{p\alpha}\},\\&\mathbf{v}=\frac{\mathbf{p}}{m},\quad\alpha=\uparrow,\downarrow\end{aligned}\tag{3.6}$$

need to be solved in order to determine transport properties of a spin-polarized gas in an external magnetic field.

IV. THERMAL CONDUCTIVITY AND VISCOSITY

In order to find the thermal conductivity and viscosity, one has to calculate the heat and momentum fluxes induced by temperature and macroscopic flow velocity spatial gradients. To do this, we may substitute the local equilibrium distribution function

$$\begin{aligned}n_{p\alpha}&=\frac{1}{2}\left[1-\tanh\frac{\varepsilon-\mathbf{pV}-\mu_\alpha}{2T}\right],\quad\alpha=\uparrow,\downarrow,\\\varepsilon&=\frac{p^2}{2m},\quad\int n_{p\alpha}d\Gamma=N_\alpha,\end{aligned}\tag{4.1}$$

with slightly nonuniform temperature and velocity fields $T(\mathbf{r})$, $\mathbf{V}(\mathbf{r})$ into the left-hand side of transport Eq. (3.6). Since we are interested in the first viscosity, we may simply consider the case $\mathrm{div}\mathbf{V}=0$ when all time derivatives vanish according to hydrodynamic equations. Expanding the left-hand side of Eq. (3.6) in a power series of small gradients ∇T, $\partial V_i/\partial x_k$, and linearizing the collision terms, one can get the transport equations in the common form

$$\begin{aligned}&\frac{n_{p\alpha}(1-n_{p\alpha})}{T}G_{p\alpha}=\mathrm{Coll}\{\delta n_{p\alpha}\},\quad\alpha=\uparrow,\downarrow,\\&G_{p\alpha}=\left[\frac{\varepsilon-\mu_\alpha}{T}+\frac{\partial\mu_\alpha}{\partial T}\right]\mathbf{v}\cdot\nabla T+mF_{ik}V_{ik},\\&mF_{ik}=v_ip_k-\tfrac{2}{3}\varepsilon\delta_{ik},\quad V_{ik}=\frac{1}{2}\left[\frac{\partial V_i}{\partial x_k}+\frac{\partial V_k}{\partial x_i}\right].\end{aligned}\tag{4.2}$$

As the gas-kinetic estimations demonstrated, in the cases of gigantic opalescence and a degenerate Fermi gas as well, the main contribution to collision processes is determined by spin-flip inelastic scattering with transverse spin fluctuations. Thus, we may neglect $\mathrm{Coll}_2\{\delta n_{p\alpha}\}$ and keep only $\mathrm{Coll}_1\{\delta n_{p\alpha}\}$ in the collision term $\mathrm{Coll}\{\delta n_{p\alpha}\}$ of Eq. (4.2). [The case of a Maxwell-Boltzmann gas polarized by a brute-force technique where $\mathrm{Coll}_2\{\delta n_{p\alpha}\}$ cannot be neglected, will be examined separately in Sec. IV C.]

In a monatomic gas, any deviation of the distribution function caused by temperature and velocity gradients should have the form

$$\delta n_{p\alpha}=A_\alpha(\varepsilon)\mathbf{p}\cdot\nabla T+B_\alpha(\varepsilon)(p_ip_k-\tfrac{1}{3}p^2\delta_{ik})V_{ik}.\tag{4.3}$$

That is why all the terms in $\mathrm{St}_1(\delta n_{p\alpha})$ containing $\delta n_{p'\alpha}$ vanish upon integration and the collision integrals are drastically simplified:

$$\begin{aligned}\mathrm{Coll}_1\{\delta n_{p\uparrow}\}&=-W_0g_p(\mathcal{N}_0+n_{p\downarrow})\delta n_{p\uparrow},\\\mathrm{Coll}_1\{\delta n_{p\downarrow}\}&=-W_0g_p(\mathcal{N}_0+1-n_{p\uparrow})\delta n_{p\downarrow}.\end{aligned}\tag{4.4}$$

This allows us to solve exactly the linearized transport equations without use of any polynomial expansions or other approximations. Actually, the problem is reduced to calculating the heat and momentum fluxes caused by the small perturbations $\delta n_{p\alpha}$ which can be found from Eqs. (4.2) and (4.4)

$$\begin{aligned}\delta n_{p\uparrow}&=-\frac{n_{p\uparrow}(1-n_{p\uparrow})G_{p\uparrow}}{W_0g_p(\mathcal{N}_0+n_{p\downarrow})T},\\\delta n_{p\downarrow}&=-\frac{n_{p\downarrow}(1-n_{p\downarrow})G_{p\downarrow}}{W_0g_p(\mathcal{N}_0+1-n_{p\uparrow})T}.\end{aligned}\tag{4.5}$$

Of course, there exist many kinds of transport phenomena in multicomponent systems (spin-up and spin-down particles, magnons) connected with a temperature gradient, e.g., thermodiffusion, etc.[25] But now we shall restrict our attention to the case of pure thermal conductivity at a constant relative concentration of species.

A. Degenerate Fermi system

In order to calculate correctly the heat flow which determines the thermal conduction coefficient κ, one has to subtract the convective current associated with the motion of a Fermi sphere as a whole. The correct

definition of heat flux is well known from the theory of the electron thermal conductivity in metals with impurities[2] and, in this case, is given by the expression

$$\mathbf{Q}=\sum_{\alpha}\int(\varepsilon-\mu_{\alpha})\mathbf{v}\delta n_{p\alpha}d\Gamma=-\kappa\nabla T,\quad \alpha=\uparrow,\downarrow\ . \tag{4.6}$$

When dealing with strongly degenerate Fermi systems, $T\ll\varepsilon_F$, one can put $(\partial\mu_{\alpha}/\partial T)=0$ in Eqs. (4.5) as the basic approximation. Besides, in the low magnetic field limit $\beta H\ll T\ll\varepsilon_F$, the magnon occupation number is large

$$\mathcal{N}_0\approx\frac{T}{2\beta H}>1\ , \tag{4.7}$$

so both $n_{p\alpha}$ and $(1-n_{p\alpha})$ in Eqs. (4.5) are negligibly small compared with $\mathcal{N}_0$ and can be omitted. Then, substituting expressions (4.5) in (4.6) and taking into account the relationships

$$\begin{aligned} n_{p\alpha}(1-n_{p\alpha})&=-T\frac{\partial n_{p\alpha}}{\partial\varepsilon}\ ,\\ \frac{\partial n_{p\alpha}}{\partial\varepsilon}&=-\delta(\varepsilon-\mu_{\alpha})-\frac{\pi^2}{6}T^2\frac{\partial^2}{\partial\varepsilon^2}\delta(\varepsilon-\mu_{\alpha})\ , \end{aligned} \tag{4.8}$$

one can easily obtain

$$\kappa=\frac{\pi}{96a^2}v_F\frac{\beta H}{\varepsilon_F}\frac{(1+\alpha)^{2/3}+(1-\alpha)^{2/3}}{\alpha}\ . \tag{4.9}$$

Thus, in the main approximation the thermal conductivity (4.9) does not depend on the temperature at all in contrast to predictions of the traditional Fermi-liquid theory. At a fixed value of α (the nonequilibrium situation for the time less than τ_s), the magnitude of κ turns out to be proportional to an external magnetic field. In the case of equilibrium polarization (the brute-force technique), when the degree of polarization (2.24) is determined by an external field, the thermal conductivity is just a constant and does not depend on H,

$$\kappa=\frac{\pi}{72}\frac{v_F}{a^2}\ . \tag{4.10}$$

Following the same procedure, we may define the viscosity of a gas by means of the usual equation for the momentum flux tensor Π_{ik}

$$\begin{aligned} \Pi_{ik}&=\sum_{\alpha}\int p_i v_k n_{p\alpha}d\Gamma\\ &=P\delta_{ik}+\rho V_i V_k-\eta_{ik,lm}V_{lm}\ , \end{aligned} \tag{4.11}$$

where P and ρ are the total pressure and density and $\eta_{ik,lm}$ is the viscosity tensor. In an isotropic gas, the tensor $\eta_{ik,lm}$ must have the form

$$\eta_{ik,lm}=\eta(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}-\tfrac{2}{3}\delta_{ik}\delta_{lm}) \tag{4.12}$$

so that the scalar coefficient of viscosity has the simple form

$$\eta=\tfrac{1}{10}\eta_{ik,ik}\ . \tag{4.13}$$

Combining Eqs. (4.5), (4.11), and (4.13), we immediately find

$$\eta=\frac{1}{80\pi a^2}mv_F\frac{\beta H}{T}\frac{(1+\alpha)^{4/3}+(1-\alpha)^{4/3}}{\alpha}\ . \tag{4.14}$$

In the case of total thermodynamical equilibrium (2.24), the viscosity does not depend on an external magnetic field either,

$$\eta=\frac{1}{60\pi a^2}mv_F\frac{\varepsilon}{T}\ . \tag{4.15}$$

Thus, we see the coefficient of viscosity η also exhibits quite unusual, for degenerate Fermi systems, temperature and field dependencies. It should be emphasized once again that formulas (4.9), (4.10), (4.14), and (4.15) cannot at all be extrapolated to the zero-field limit $H=0$ since the theory developed here holds only when $\beta H_c\ll\beta H\ll T\ll\varepsilon_F$, where H_c is determined by Eqs. (2.22).

There is now a temptation to apply the results obtained to the case of ^{3}He↑-^{4}He degenerate solutions. Unfortunately, the experimental fact is that the s-wave scattering description which has been used here does not work sufficiently well even at low enough concentrations of a mixture. One could probably expect the approach to work better at concentrations of the order of 10^{-2}–10^{-1} at. %. In order to get at least a qualitative picture, let us make numerical estimations for the 0.1 at. % dilute ^{3}He↑-^{4}He mixture polarized by an external magnetic field at $T=1$ mK. In this case the theory is expected to be valid within the magnetic field range 10 kG $\gg H\gg$ 4 G. Using formulas (2.11), (4.10), and (4.15), one can easily estimate the mean free path $l\sim10^{-4}$ cm, viscosity $\eta\sim5\times10^6$ cgs, and thermal conductivity $\kappa\sim10^{17}$ cgs, all of which turn out to be considerably less than those calculated from the Fermi-liquid approach. Thus, qualitatively, the effects in question could lead to nonmonotonic dependencies of the viscosity and thermal conductivity on an external magnetic field and to creating a deep and wide enough minimum in the curves $\eta(H)$ and $\kappa(H)$ which might be located within the field range shown above. The experimental identification of such a minimum might require accurate low-field measurements inasmuch as the lower boundary of the field range is comparable with the background magnetic field.

B. Boltzmann statistics: Gigantic opalescence

When the temperature of a gas is high enough and all particles obey the Maxwell-Boltzmann statistics, we can restrict ourselves by only considering the term $\mathrm{St}_1(\delta n_{p\alpha})$ in the collision integrals in the case of a nonequilibrium polarization. In this case one can find the viscosity and thermal conductivity with better accuracy than for degenerate systems discussed earlier. In the classical limit, particle occupation numbers $n_{p\alpha}$ are always small $n_{p\alpha}\ll1$. It enables us to omit $n_{p\alpha}$ and keep not only $\mathcal{N}_0$ but also 1 in the denominators of Eqs. (4.5).

The chemical potentials μ_{α} can be represented in the form

$$\mu_{\alpha}=-T\ln T^{5/2}+T\times\mathrm{const}\ . \tag{4.16}$$

Then it is easy to verify that

$$\mu_\alpha - T\left[\frac{\partial\mu_\alpha}{\partial T}\right]_{P,c} = \tfrac{5}{2}T \ , \tag{4.17}$$

where the total pressure P and relative concentration c are defined as

$$N_\uparrow = cN = c\frac{P}{T}, \quad N_\downarrow = (1-c)N = (1-c)\frac{P}{T} \ . \tag{4.18}$$

Finally, calculating the heat flux

$$\mathbf{Q} = \sum_\alpha \int \varepsilon \mathbf{v}\, \delta n_{p\alpha} d\Gamma = -\kappa \nabla T \tag{4.19}$$

leads to the final result

$$\kappa = \frac{1}{24\pi a^2}\left[\frac{2T}{\pi m}\right]^{1/2}\left[\sinh\frac{2\beta H}{T} + \alpha\left[\cosh\frac{2\beta H}{T} - 1\right]\right] . \tag{4.20}$$

Similarly, using Eqs. (4.5) and (4.11)–(4.13), one can find the viscosity of a gas

$$\eta = \frac{m}{30\pi a^2}\left[\frac{2T}{\pi m}\right]^{1/2}\left[\sinh\frac{2\beta H}{T} + \alpha\left[\cosh\frac{2\beta H}{T} - 1\right]\right] . \tag{4.21}$$

Formulas (4.20), and (4.21), which contain the same combination

$$\frac{N_\uparrow}{\mathcal{N}_0} + \frac{N_\downarrow}{\mathcal{N}_0+1} = 2\sinh\frac{\hbar\Omega_H}{T} + 2\alpha\left[\cosh\frac{\hbar\Omega_H}{T} - 1\right] , \tag{4.22}$$

actually hold when $\beta H < T$.

Gaseous ^{3}He↑, which is normally polarized by optical pumping[1,21] and can be placed in an arbitrary magnetic field, would be an ideal object for experimental measurements if it could be cooled down to low enough temperatures. Regrettably, the saturated vapor pressure is exponentially small at such temperatures and, under the common conditions $T \sim 1$ K, $N \sim 10^{17}$ cm^{-3}, $\alpha \sim 30\%$, the s-wave scattering description works even worse than in the case of dilute ^{3}He↑-^{4}He mixtures. Nevertheless, the evaluations within the approach developed might be useful in order to figure out when one could hope to detect some anomalies in the transport properties of gaseous ^{3}He↑. The estimations carried out under the conditions mentioned above result in the following magnetic field range $H \gg 50$ G (the criterion $\beta H < T$ is automatically fulfilled for all available magnetic fields). In the case $\alpha \gg \tanh(\beta H/T)$, the viscosity and thermal conductivity are expected to be essentially less than the common gas-kinetic values.

C. Boltzmann statistics: Equilibrium polarization

Now we shall discuss the case of a Boltzmann paramagnetic gas polarized by the brute-force method. Thus, complete thermodynamical equilibrium is assumed to take place and the degree of polarization is determined by an external magnetic field as shown by Eq. (2.21). In this case, as it was demonstrated above, calculating transport coefficients requires taking into account the direct particle-particle collisions, described in terms of $\mathrm{Coll}_2\{n_{p\alpha}\}$ from Eq. (3.5), which were omitted in all previous computations. It makes the problem more complicated since the exact solution cannot be found in this case and one has to use some polynomial approximations. It is very convenient to use Sonine's polynomials when dealing with collision integrals of the type (3.5).[2,25] Of course, when expanding both $\mathrm{Coll}_1\{n_{p\alpha}\}$ and $\mathrm{Coll}_2\{n_{p\alpha}\}$ in a series of Sonine's polynomials, we lose some accuracy. The convergence of the Sonine expansion for $\mathrm{Coll}_2\{n_{p\alpha}\}$ is pretty good. However, considering only the leading term in the Sonine expansion for $\mathrm{Coll}_1\{n_{p\alpha}\}$ indeed results in a worse precision than in the case of gigantic opalescence where practically the exact solution of a linearized Boltzmann equation was found. Nonetheless, in this paper we will restrict ourselves to discussing the main term in a series of Sonine's polynomials for the total collision integrals $\mathrm{Coll}\{n_{p\alpha}\}$. Let us introduce the notation

$$\delta n_{p\alpha} = -\frac{\partial n_{p\alpha}}{\partial\varepsilon}\chi_{p\alpha} = \frac{n_{p\alpha}}{T}\chi_{p\alpha} \ . \tag{4.23}$$

In the first approximation of the Chapman-Enskog method applied to the problem of thermal conductivity on a Maxwell-Boltzmann gas, we can seek the solutions of transport equations (4.2) with collision integrals (3.5) and (4.4) in the form

$$\chi_{p\alpha} = A_\alpha \frac{\gamma}{N_\alpha} S^1_{3/2}\left[\frac{\varepsilon}{T}\right]\mathbf{v}\cdot\nabla T, \quad \gamma = \frac{m}{2T}, \quad \alpha = \uparrow,\downarrow, \tag{4.24}$$

where $S^1_{3/2}(\varepsilon/T)$ is the Sonine polynomial determined in the usual manner,[2,25] and coefficients A_α have to be found. According to the definition (4.19), the coefficient of thermal conductivity κ can be calculated as $\kappa = \frac{5}{4}(A_\downarrow + A_\downarrow)$. Substituting Eqs. (4.23) and (4.24) into the transport equations and following the traditional Chapman-Enskog procedure, one can reduce the problem to a system of linear equations for coefficients A_α (see the Appendix). After some algebra we finally get

$$\kappa = \frac{\kappa(0)}{172}\frac{1}{1+\Phi(\alpha)}\left[59\left[\frac{N_\uparrow}{N_\downarrow} + \frac{N_\downarrow}{N_\uparrow}\right] + 54 + F(\alpha)\right] ,$$

$$F(\alpha) = 104\sqrt{2}\left[\frac{\mathcal{N}_0}{N_\uparrow} + \frac{\mathcal{N}_0+1}{N_\downarrow}\right]\alpha N \ ,$$

$$\kappa(0) = \frac{75}{256a^2}\left[\frac{T}{\pi m}\right]^{1/2} , \tag{4.25}$$

$$\Phi(\alpha) = \frac{767\sqrt{2}}{344}\left[\frac{\mathcal{N}_0}{N_\downarrow} + \frac{\mathcal{N}_0+1}{N_\uparrow}\right]\alpha N + \frac{338}{43}\mathcal{N}_0(\mathcal{N}_0+1)\frac{(\alpha N)^2}{N_\uparrow N_\downarrow} \ .$$

Here $\kappa(0)$ corresponds to the thermal conductivity of an

unpolarized gas in the absence of an external magnetic field, and functions F and Φ depend on the degree of polarization and magnetic field only but do not depend on the total density N. Equations (4.25) take into account both scattering mechanisms and provide the description valid for any polarized state. But we are particularly interested in the case of equilibrium polarization. Then, using definitions (2.12) and (2.21), one can easily prove the identity

$$\alpha N\left[\frac{\mathcal{N}_0+1}{N_\downarrow}+\frac{\mathcal{N}_0}{N_\uparrow}\right]=\frac{N_\uparrow}{N_\downarrow}+\frac{N_\downarrow}{N_\uparrow} \tag{4.26}$$

and obtain the field dependence of the thermal conductivity:

$$\frac{\kappa(H)}{\kappa(0)}=\frac{2(59+104\sqrt{2})\cosh(2\beta H/T)+54}{1524+767\sqrt{2}} . \tag{4.27}$$

When considering the viscosity in the same approximation, we should seek the deviation of the distribution function in the form $\chi_{p\alpha}=g_{ik}^{(\alpha)}V_{ik}$, where

$$g_{ik}^{(\alpha)}=-\frac{\gamma^2}{N_\alpha}F_{ik}B_\alpha,\quad \alpha=\uparrow,\downarrow . \tag{4.28}$$

Here F_{ik} is the same as in Eqs. (4.2), and it is taken into account that $S^0_{5/2}(\varepsilon/T)=1$. As it follows from the definitions (4.11)–(4.13), the viscosity of a gas is determined by the expression $\eta=(m/4T)(B_\uparrow+B_\downarrow)$. Then, following the standard method (see the Appendix), we find

$$\eta=\frac{\eta(0)}{5}\frac{1}{1+\Xi(\alpha)}\left[2\left[\frac{N_\uparrow}{N_\downarrow}+\frac{N_\downarrow}{N_\uparrow}\right]+1+R(\alpha)\right],$$
$$R(\alpha)=3\sqrt{2}\left[\frac{\mathcal{N}_0}{N_\uparrow}+\frac{\mathcal{N}_0+1}{N_\downarrow}\right]\alpha N ,$$
$$\eta(0)=\frac{5}{64a^2}\left[\frac{mT}{\pi}\right]^{1/2} , \tag{4.29}$$
$$\Xi(\alpha)=\frac{8\sqrt{2}}{5}\left[\frac{\mathcal{N}_0}{N_\downarrow}+\frac{\mathcal{N}_0+1}{N_\uparrow}\right]\alpha N + \tfrac{24}{5}\mathcal{N}_0(\mathcal{N}_0+1)\frac{(\alpha N)^2}{N_\uparrow N_\downarrow} ,$$

where $\eta(0)$ is the viscosity of a gas in the unpolarized state. In the case of equilibrium polarization in an external magnetic field, the calculations similar to those carried out above for the thermal conductivity yield

$$\frac{\eta(H)}{\eta(0)}=\frac{2(2+3\sqrt{2})\cosh(2\beta H/T)+1}{29+16\sqrt{2}} . \tag{4.30}$$

Thus, at high enough magnetic field $H\gg H_c$, where H_c is determined by Eqs. (2.16) and (2.21), the thermal conductivity and viscosity are linear functions of $\cosh(\hbar\Omega_H/T)$. It is the same H dependence that was given by previous theories considering the elastic particle-particle scattering only. But the numerical coefficients in Eqs. (4.27) and (4.30) make $\kappa(H)$ and $\eta(H)$ at $H\gg H_c$ less than even $\kappa(0)$ and $\eta(0)$ (see Fig. 3). The dotted curves in Fig. 3 result from calculations neglecting the scattering mechanism and are given by the expressions

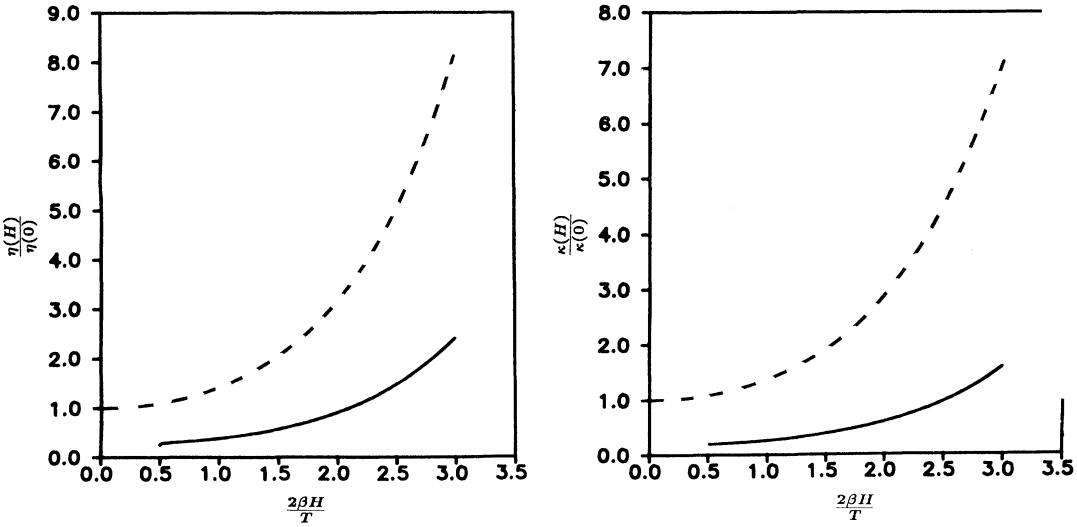

FIG. 3. The magnetic field dependencies of the viscosity η and the thermal conductivity κ. The dotted curves correspond to the old calculations [see Eqs. (4.31)] which did not take into account the inelastic scattering of paramagnetic particles with transverse spin fluctuations. The solid curves result from the present theory [see Eqs. (4.27) and (4.30)]. They cannot be extrapolated to the zero-field limit at $H<H_c$. This implies that there should be at least one minimum in the functions $\eta(H)$ and $\kappa(H)$ within the field range $0<H<H_c$.

$$\left[\frac{\kappa(H)}{\kappa(0)}\right]_{\rm dir}=\frac{59\cosh(2\beta H/T)+27}{86}\ ,$$
$$\left[\frac{\eta(H)}{\eta(0)}\right]_{\rm dir}=\frac{4\cosh(2\beta H/T)+1}{5}\ . \tag{4.31}$$

The solid curves include both scattering mechanisms and are given by Eqs. (4.27) and (4.30). Thus, one can expect some nonmonotonic behavior of the thermal conductivity and viscosity and the appearance of at least one minimum in the curves $\eta(H)$ and $\kappa(H)$ within the field range $0<H<H_c$. The relative depth of the minima can be easily evaluated as

$$\lim_{H\to 0}\frac{\kappa(H)}{\kappa(0)}=0.18,\quad \lim_{H\to 0}\frac{\eta(H)}{\eta(0)}=0.26\ . \tag{4.32}$$

In the case of a very diluted nondegenerate ^{3}He↑-^{4}He solution with concentration $x\leq 0.1$ at. % at $T\approx 30$ mK, we may estimate that $H_c\approx 300$ G. Under these conditions the contribution of rotons and phonons is certainly negligible and cannot mask the effect in question. However, one should keep in mind all the restrictions and stipulations mentioned previously when applying the theory to the case of ^{3}He↑-^{4}He mixtures.

V. SPIN-POLARIZED ATOMIC HYDROGEN H↓

We have just discussed the magnetotransport effects in Fermi gases but practically all of the above statements are also valid in the case of paramagnetic Bose gases. In view of this, atomic hydrogen H↓ is probably the ideal object for experimentally studying the phenomenon under consideration. First, gaseous H↓ is not liquified and possesses a high enough density at the required temperatures (2.1). Second, the quantum-mechanical refraction mechanism at low-momentum scattering holds perfectly in this case. It was given experimental evidence after detecting the collective spin waves in H↓.[14,1,16] And, last, when considering the nuclear magnetization one can conclude that gaseous H↓ is always under conditions of gigantic opalescence since the degree of nuclear polarization is normally much higher than that which could be induced by the available magnetic field. Thus, one can expect spin-polarized atomic hydrogen H↓ to exhibit anomalous transport properties affected by inelastic scattering with coherent spin fluctuations.

When discussing the exchange interaction between the nuclear spin of a hydrogen atom and the long-wavelength fluctuations of transverse nuclear magnetization, we have to define the degree of polarization as

$$\alpha=\frac{|N_a-N_b|}{N},\quad N=N_a+N_b\ , \tag{5.1}$$

where N_a and N_b are the number densities of atoms in the states $|a\rangle=|\Downarrow\downarrow\rangle-\xi|\Uparrow\uparrow\rangle$ and $|b\rangle=|\Downarrow\uparrow\rangle$, respectively. Here the arrows $\Downarrow$ and ↑ denote the electronic and proton spin, ξ is the small hyperfine structure parameter. Transport equations (4.2) are just slightly transformed, that is to say, we must replace the factor $(1-n_{p\alpha})$ by the factor $(1+n_{p\alpha})$, where $n_{p\alpha}$ now looks like the Bose-distribution function and the index α enumerates the $|a\rangle$ and $|b\rangle$ states, i.e., in the leading approximation it indicates the projection of a protonic spin. Regretfully, the Bose-Einstein condensation in gaseous H↓ has not yet been reached and the temperature available is normally much higher than the degeneracy temperature. That is why, in this case, we may directly use all the formulas for the viscosity and thermal conductivity obtained in Sec. IV B in the limit of Maxwell-Boltzmann statistics.

Under typical experimental conditions,[1,14,26,27] $T=220$ mK, $H=8$ T, and $\alpha=0.8$, one can easily calculate η and κ by means of Eqs. (4.20) and (4.21) which yield

$$\frac{\kappa}{\kappa_0}=0.13\frac{1}{\alpha}\frac{\beta H}{T}\approx 4.7\times 10^{-3}\ ,$$
$$\frac{\eta}{\eta_0}=0.38\frac{1}{\alpha}\frac{\beta H}{T}\approx 1.4\times 10^{-2}\ . \tag{5.2}$$

Thus, we have obtained a dramatic reduction in the viscosity and thermal conductivity of gaseous H↓. This result provides the basic statement of the theory.

Such a drastic reduction is certainly due to the anomalously large cross sections (2.11) describing the inelastic scattering of paramagnetic H atoms with macroscopic spin fluctuations. The first experimental evidence for the scattering mechanism was probably found by Hershcovitch[22] in the measurements of intrabeam scattering of H atoms at $T=2$ K, $H=4.38$ T. He observed an intrabeam scattering process which prevented the focusing of H atoms in the beam. He also managed to infer the scattering cross section per single H atom which turned out to be unexpectedly large, $\sigma_t\approx 10^{-14}$ cm^2. It is difficult to ascribe such an enormous cross section to real H-H collisions which do not seem to play any important role (the corresponding mean free path is larger than the size of a beam). But, the origin of the scattering mechanism in question, as well as the quantum-mechanical refraction, in general, does not at all require that the distances should be more than the mean free path calculated for interparticle collisions. It is created at smaller distances and exhibits itself even in the collisionless regime[16,18,19] (as seen in neutron optics). There are no data concerning the nuclear magnetization of atomic H in Ref. 22. But, if one assumed the effective degree of polarization α to be equal to just 0.01, then the measured value of the total cross section is obtained

$$\sigma_t\simeq\sigma_1\simeq\sigma_2\simeq 8\pi a^2\alpha\frac{T}{\beta H}\simeq 10^{-14}\ \text{cm}^2\ . \tag{5.3}$$

Of course, such an explanation of the effect must be verified in separate experiments. In any case, it looks plausible and has a number of advantages. Probably the best way to carry out the crucial experiment is to directly measure the transport coefficients or some other dissipative function (such as sound absorption, etc.) in gaseous H↓, which are expected to be considerably less than in the case of a common unpolarized gas under the same thermodynamical conditions. Beam-target-like experiments with atomic H↓ may also provide a good opportunity to observe the phenomenon. Spin-polarized atom-

ic deuterium D↓ might be a good system for an experimental study too.[28]

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APPENDIX

Let quantities X_α and Y_α be defined as

$$X_\alpha=\left[\frac{2\pi\hbar^2}{mT}\right]^{3/2} \times\int we^{-\varepsilon/T}(\chi_{p'\alpha}-\chi_{p\alpha})d\Gamma' d\Gamma_1 d\Gamma_1' ,$$
$$Y_\alpha=\left[\frac{2\pi\hbar^2}{mT}\right]^{3/2} \times\int we^{-\varepsilon/T}(\chi_{p_1'\alpha}-\chi_{p_1\alpha})d\Gamma' d\Gamma_1 d\Gamma_1' , \quad \alpha=\uparrow,\downarrow . \tag{A1}$$

Then, taking into account Eq. (4.23), one can express linearized collision integrals (3.5) in terms of X_α and Y_α as

$$\mathrm{Coll}_2\{n_{p\uparrow}\}=\frac{n_{p\uparrow}}{T}N_\downarrow(X_\uparrow+Y_\downarrow) ,$$
$$\mathrm{Coll}_2\{n_{p\downarrow}\}=\frac{n_{p\downarrow}}{T}N_\uparrow(X_\downarrow+Y_\uparrow) . \tag{A2}$$

Substituting Eqs. (4.23), (4.24), (A1), and (A2) into transport equations (4.2) with both $\mathrm{Coll}_1\{n_{pa}\}$ and $\mathrm{Coll}_2\{n_{pa}\}$, and multiplying them by the factor $\mathbf{v}n_{p\alpha}S^1_{3/2}(\varepsilon/T)$, and then integrating over the momentum space $d\Gamma$, we get the system of linear equations

$$\left[c_\uparrow\frac{N_\downarrow}{N_\uparrow}+\frac{\mu_0}{N_\uparrow}\right]A_\uparrow+d_\downarrow A_\downarrow=\frac{15}{2}\frac{T}{m} ,$$
$$d_\uparrow A_\uparrow+\left[c_\downarrow\frac{N_\uparrow}{N_\downarrow}+\frac{\nu_0}{N_\downarrow}\right]A_\downarrow=\frac{15}{2}\frac{T}{m} , \tag{A3}$$

where the quantities c_α, d_α, μ_0, and ν_0 are defined by the following:

$$c_\alpha=-\left[\frac{2\pi\hbar^2}{mT}\right]^{3/2}\gamma \times\int e^{-\varepsilon/T}\mathbf{v}S^1_{3/2}\left[\frac{\varepsilon}{T}\right]X_\alpha(\mathbf{v}S^1_{3/2})d\Gamma ,$$
$$d_\alpha=-\left[\frac{2\pi\hbar^2}{mT}\right]^{3/2}\gamma \times\int e^{-\varepsilon/T}\mathbf{v}S^1_{3/2}\left[\frac{\varepsilon}{T}\right]Y_\alpha(\mathbf{v}S^1_{3/2})d\Gamma , \tag{A4}$$
$$\nu_0=208\left[\frac{2\pi T}{m}\right]^{1/2}Na^2\alpha(\mathcal{N}_0+1) ,$$
$$\mu_0=208\left[\frac{2\pi T}{m}\right]^{1/2}Na^2\alpha\mathcal{N}_0 .$$

The way to calculate the parameters c_α and d_α was actually developed in Ref. 25 for the case of a binary gas mixture. Comparing Eqs. (A4) with the results of Ref. 25 and considering the s-wave scattering only, one can easily obtain

$$c_\downarrow=c_\uparrow=118\left[\frac{\pi T}{m}\right]^{1/2}a^2 ,$$
$$d_\uparrow=d_\downarrow=-54\left[\frac{\pi T}{m}\right]^{1/2}a^2 . \tag{A5}$$

Combining Eqs. (A3)–(A5) we obtain the result (4.25).

In order to calculate the viscosity we should multiply Eqs. (4.2) by $n_{p\alpha}F_{ik}d\Gamma$ and integrate them taking into account the relationships (4.23), (4.28), (A1), and (A2). This procedure leads to the following linear equations:

$$10\frac{T^2}{m}=b_\uparrow B_\uparrow+\left[a_\downarrow\frac{N_\uparrow}{N_\downarrow}+\frac{8\nu_0}{13N_\downarrow}\right]B_\downarrow ,$$
$$10\frac{T^2}{m}=\left[a_\uparrow\frac{N_\downarrow}{N_\uparrow}+\frac{8\mu_0}{13N_\uparrow}\right]B_\uparrow+b_\downarrow B_\downarrow . \tag{A6}$$

Here the quantities a_α, b_α are determined by the formulas

$$a_\alpha=-\left[\frac{2\pi\hbar^2}{mT}\right]^{3/2}\gamma^2\int e^{-\varepsilon/T}F_{ik}X_\alpha(F_{ik})d\Gamma, \quad \alpha=\uparrow,\downarrow , \tag{A7}$$
$$b_\alpha=-\left[\frac{2\pi\hbar^2}{mT}\right]^{3/2}\gamma^2\int e^{-\varepsilon/T}F_{ik}Y_\alpha(F_{ik})d\Gamma ,$$

and, in the s-wave scattering limit, they are equal to

$$a_\uparrow=a_\downarrow=\frac{256}{3}\left[\frac{\pi T}{m}\right]^{1/2}a^2, \quad b_\uparrow=b_\downarrow=-\tfrac{1}{4}a_\uparrow . \tag{A8}$$

Then, solving Eqs. (A6) with coefficients (A4) and (A7), we find the result (4.29).

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