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Transient regimes of flux creep in high-T_c superconductors

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Initial nonlogarithmic ($t<\tau$) stages of flux creep in superconductors have been studied both theoretically and experimentally. Measurements of the time constant τ in grain-oriented $YBa_2Cu_3O_{7-x}$ at $T=20$ K and $B=6$ T and various sweep rates dB_e/dt are presented. The experimental data as well as an analysis of nonlinear flux diffusion for different flux-creep models indicate that the constant τ is a macroscopic quantity essentially depending upon initial conditions and sample geometry. The value τ is shown to be inversely proportional to the sweep rate with τ reaching 5×10^3 s at $dB_e/dt=5\times10^{-6}$ T/s.

Thermal fluctuations are known to be a very important factor affecting the critical current density j_c and pinning of vortex structures in high-T_c superconductors for which the observed rapid relaxation of magnetization (flux creep) leads to essential dependence of irreversible magnetic properties on induced electric fields. Initial stages of the flux creep from a subcritical state ($j<j_c$) well below the irreversibility line[1,2] are usually described phenomenologically as follows,[3,4]

$$M(t)=M_0-M_1\ln(1+t/\tau)\ , \tag{1}$$

where $M(t)$ is the magnetic moment of a sample, M_0 is the initial value of $M(t)$, M_1 determines the flux-creep rate $S=-dM/d\ln t$, and τ is a time constant. At $t\gg\tau$ formula (1) takes the conventional form

$$M(t)=M_c-M_1\ln(t/t_0)\ , \tag{2}$$

where the constant t_0 generally differs from τ in Eq. (1) and depends on the choice of M_c, as only the combination $M_c+M_1\ln t_0$ has physical relevance. Usually t_0 is fixed by the condition that the value M_c should be given by the Bean model.[5,6] Formula (2) is a known result of the Anderson-Kim model,[5,6] and also describes the initial stages of the flux creep in a vortex glass[7,8] and collective creep[9,10] models.

As for the time constants τ and t_0 their meaning is discussed in the literature. In early works[5,6] the values τ and t_0 had been related to a microscopic frequency of oscillations of pinned vortices and put to be of order 10^{-10}–10^{-13} s. Then the dissipation in vortex cores upon the oscillations was taken into account, which enabled one to express τ and t_0 via macroscopic quantities such as the flux-flow resistivity ρ_f, sample sizes, etc.[4,10,11-15] Nevertheless, the formulas for τ and t_0 obtained in Refs. 10, 13, and 15 still contain uncertain microscopic parameters such as an attempt frequency or vortex bundle size, which allows some arbitrariness upon their estimations.

In this paper we show that both τ and t_0 are macroscopic quantities determined by nonlinear flux diffusion, which enables one to express them via directly measured parameters. Furthermore, it will be shown that usually $\tau\gg t_0$ with t_0 being an intrinsic time constant unlike τ, which essentially depends on initial conditions, in particular, on the sweep rate $\dot B_e=dB_e/dt$. Here characteristic values of t_0 range within 10^{-3}–10^{-6} s, whereas τ varies from 1 to 10^3 s upon the change of $\dot B_e$ from 10^{-2} to 10^{-5} T/s. The time τ determines a transient period after which the logarithmic relaxation of $M(t)$ begins. These statements will be confirmed below by exact solutions of nonlinear equations describing the flux diffusion and by flux-creep experiments done on oriented-grain $YBa_2Cu_3O_{7-x}$.

We consider first a slab of thickness $2a$ along the x axis and infinite in the yz plane with the magnetic field $\mathbf{H}_e$ parallel to the z axis (Fig. 1). Let the external field $B_e(t)$ increase with a constant sweep rate $\dot B_e$ until $t=0$, and then remain fixed. This induces an initial electric field $E(x)=\dot B_e x$ which then decays at $t>0$ due to the flux diffusion through the sample. Such a process has been considered by Beasley, Labusch, and Webb[6] in terms of the Maxwell equations for magnetic induction B proportional to the vortex density. For our aims it is more convenient to present the Maxwell equations $\dot{\mathbf{B}}=-\nabla\times\mathbf{E}$ and $\nabla\times\mathbf{H}=\mathbf{j}$ as a single equation for the y component of $\mathbf{E}=\hat{\mathbf{y}}E(x,t)$:

$$E''=\mu_0(\partial j/\partial E)\dot E\ , \tag{3}$$

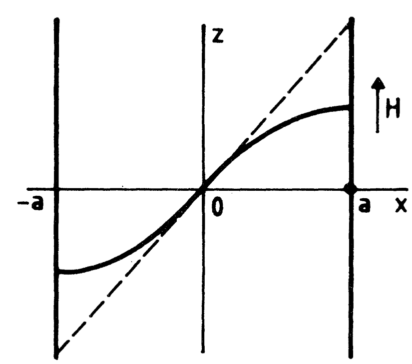

FIG. 1. The electric-field profile $E(x,t)$ described by Eq. (8). The dashed line shows the initial distribution $E(x,t)=\dot B_e x$.

where the prime and overdot denote the differentiation with respect to x and t, respectively, and we also put $B=\mu_0 H$ assuming that $H_{c1} \ll H \ll H_{c2}$. We are interested in solutions of Eq. (3) obeying the boundary conditions $E(0,t)=0$, $E'(a,t)=\dot{B}_e(a,t)=0$, and the initial condition $E(x,0)=\dot{B}_e x$.

Qualitative features of the flux creep entirely determined by the nonlinear dependence of the differential conductivity $\partial j/\partial E$ on E can be obtained from Eq. (3) by assuming $E'' \sim E/a^2$ and $\dot{E} \sim E/t$, which yields

$$t \sim \mu_0 (\partial j/\partial E) a^2 . \tag{4}$$

This diffusion-type relation enables one to get long-time asymptotes of $E(t)$ for various flux-creep models. For instance, for the exponential I-V curve $E=E_c \exp[-(j_c-j)^m/j_1^m]$, $m>1$, one finds $E \propto 1/t$ and $j(t)=j_c-j_1 \times \ln^{1/m}(t/t_0)$, with an accuracy to slowly varying logarithmic terms $\ln t$ as compared to t at $m \neq 1$. Likewise for the power I-V curve $E=E_c(j/j_c)^n$, one has $E \propto 1/t^{n/(n-1)}$ and $j(t) \propto 1/t^{(n-1)}$, whereas for the vortex-glass model with $E=E_0 \exp[-(j_0/j)^\beta]$ (Refs. 7–10) one gets $E(t) \propto 1/t$ and $j(t) \sim j_0/\ln^{1/\beta}(t/t_0)$, where we again neglect $\ln t$ as compared with t at $t \gg t_0$. Here E_c plays the role of a crossover field between the flux-flow and flux-creep regimes.

These general results based in essence only on a dimensional analysis show the independence of the long-time behavior of $E(t)$ from initial conditions. The latter occurs at $t \gg \tau$, where the time constant τ can be estimated by substituting a characteristic initial electric field $E=\dot{B}_e a$ into Eq. (4), which gives a time needed for the beginning of the above steady-state regime after a diffusion redistribution of magnetic flux due to the abrupt change of $\dot{B}_e(t)$ at $t=0$. Hence it follows that

$$\tau = a\mu_0 j_1/m\dot{B}_e \ln^{1-1/m}(E_c/a\dot{B}_e) , \tag{5}$$

$$\tau = (a^2\mu_0 j_c/nE_c)(E_c/a\dot{B}_e)^{1-1/n} , \tag{6}$$

$$\tau = a\mu_0 j_0/\beta\dot{B}_e \ln^{1+1/\beta}(E_0/a\dot{B}_e) , \tag{7}$$

where Eqs. (5)–(7) correspond to the exponential, power, and vortex-glass I-V curves, respectively. Notice that the time τ essentially depends on $\dot{B}_e$ (see also Refs. 4, 10, 14, and 15), this dependence being close to an universal law $\tau \propto 1/\dot{B}_e$ for all of the above-mentioned I-V curves if one neglects slowly varying logarithmic terms $\ln\dot{B}_e$ or takes into account that usually $n \gg 1$.[16] This is due to the fact that the difference between different flux-creep models manifests itself only at long times $t \gg \tau$,[17–19] whereas at $t \sim \tau$ the results are close to that given by the exponential I-V curve. Indeed, the field $E(j)$ at $j<j_c$ can be present in the form $E=E_c \exp[-U(j)/k_B T]$, where $U(j)$ is a flux-creep potential barrier which is, in general, a nonlinear function of j vanishing at $j=j_c$ (see Refs. 6–10). At initial stages of the decay of the critical state $[j(0) \simeq j_c]$ we can expand $U(j)$ in a power series of j_c-j keeping only the first term, i.e., $U/k_B T \simeq (j_c-j)/j_1$ with j_1 the observed flux-creep rate: $j_1=-dj/d\ln t$ (in the Anderson-Kim model $j_1=j_c k_B T/U_0$, where U_0 is a flux-creep activation energy[5,6]).

Since we are interested in the transient stages of the flux creep ($t \sim \tau$), we consider the case of the exponential I-V curve in more detail, assuming for simplicity $m=1$. By substituting $\partial j/\partial E = j_1/E$ into Eq. (3) we find an exact solution obeying the boundary conditions $E(0,t)=E'(a,t)=0$ and $E'(0,0)=\dot{B}_e$:

$$E(x,t)=\mu_0 j_1(ax-x^2/2)/(t+\tau), \quad x>0 , \tag{8}$$

where $\tau = a\mu_0 j_1/\dot{B}_e$ is given by Eq. (5) with $m=1$. Using Eq. (8) we can calculate the magnetic moment per unit lateral area of the slab, $M=\frac{1}{2}\int xj\,dx$, where $j=j_c-j_1\ln[E_c/E(x,t)]$. The result is given by Eq. (1) with

$$M_0=\tfrac{1}{2}a^2[j_c+j_1\ln(8a\dot{B}_e/E_c)-3j_1], \quad M_1=\tfrac{1}{2}a^2 j_1 . \tag{9}$$

Notice that the parabolic profile (8) forms from the initial distribution $E=\dot{B}_e x$ during a time $t \sim \tau$ needed for the diffusion of electric-field perturbations from the lateral surface toward the slab center (Fig. 1).[20] At $t \gg \tau$ formula (1) reduces to Eq. (2) with

$$t_0 = 8a^2\mu_0 j_1/e^3 E_c . \tag{10}$$

To estimate the crossover field E_c we consider a flux-flow part of $E(j)$ at $j>j_c$, where $E=(j-j_c)\rho_f$. This formula is valid if $j-j_c \gg j_1$ as j_1 determines the smearing of the I-V curve due to the flux creep. Hence $E_c \sim \rho_f j_1$ and Eq. (10) yields $t_0 \sim a^2\mu_0/\rho_f$, which correlates with estimations of t_0 obtained for various flux-creep models.[11–15]

Therefore the transient nonlogarithmic stage of magnetic relaxation can take considerable time, which may even exceed a time window in flux-creep experiments. For instance, if $a=1$ mm, $j_c=10^6$ A/cm^2, $j_1=2\times10^4$ A/cm^2 (Ref. 11), we get $\tau = a\mu_0 j_1/\dot{B}_e = 25$ s at $\dot{B}_e=10^{-2}$ T/s and $\tau=2.5\times10^4$ s at $\dot{B}_e=10^{-5}$ T/s. Taking $\rho_f=\rho_n(B/B_{c2}) \simeq 10$ $\mu\Omega$ cm with $B/B_{c2} \simeq 0.1$ and the normal-state

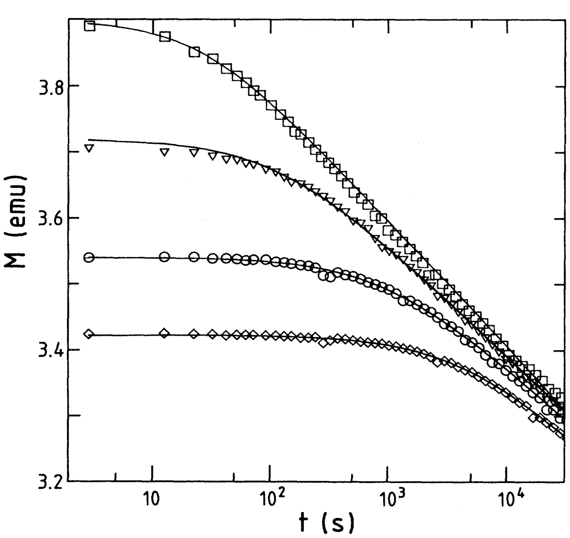

FIG. 2. Examples of the relaxation curves $\mathbf{M}=\mathbf{M}(\ln t)$ at $T=20$ K, $B=6$ T, and various $\dot{B}_e$: 10^{-3} T/s (□), 1.3×10^{-4} T/s (▽), 2×10^{-5} T/s (○), 5×10^{-6} T/s (◇). The solid curves correspond to Eq. (1).

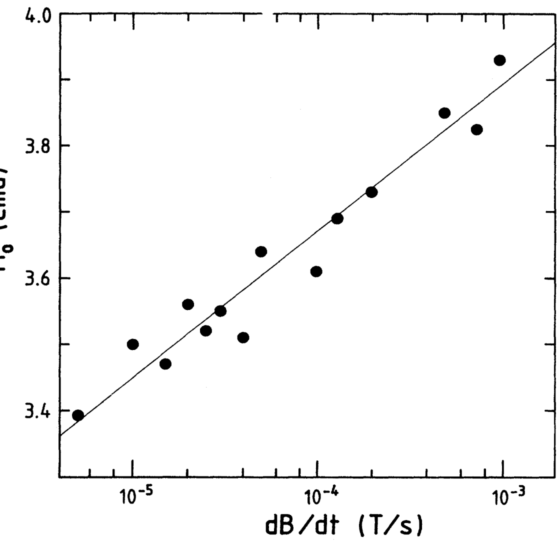

FIG. 3. Dependence of M_0 on $\dot{B}_e$. The line gives the best fit.

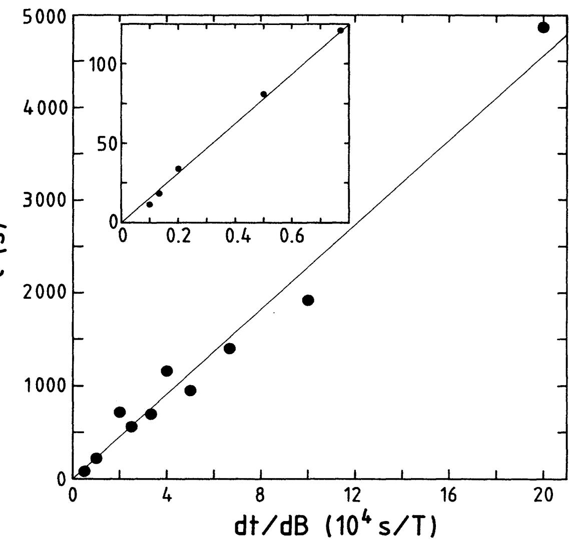

FIG. 4. Dependence of τ on $1/\dot{B}_e$. Inset: data for higher $\dot{B}_e$. The lines give the best fits; their slopes in the main figure and the inset differ by $\sim 20\%$.

resistivity $\rho_n \simeq 10^2$ $\mu\Omega$ cm, one finds the time constant $t_0 \sim 10^{-5}$ s is much less than τ.

In order to observe the transient stages of the flux creep we have studied the magnetic relaxation in grain-oriented $YBa_2Cu_3O_{7-x}$ at various sweep rates $5\times10^{-6} < \dot{B}_e < 1.2\times10^{-2}$ T/s. We used a sample of rectangular shape $0.47\times3.5\times2.5$ mm^3 with the sides parallel to the *a*, *b*, and *c* axes, respectively. The magnetic field $B=6$ T was parallel to the *c* axis, which ensured complete flux penetration into the sample and stable current configuration in the isotropic *ab* plane.[21] The relaxation of $M(t)$ was measured at low temperatures ($T=20$ K), where the flux creep has been observed to be nearly logarithmic with the maximum value of $j_1(T)$ in Eq. (5). The experimental setup has been described elsewhere.[11]

Shown in Fig. 2 are typical relaxation curves $M(t)$ measured at different $\dot{B}_e$. Being plotted as a function of $\ln t$, the curves $M(\ln t)$ clearly display a plateau corresponding to the transient nonlogarithmic stage, the plateau increasing as $\dot{B}_e$ decreases. As follows from Eq. (1) the value $\ln\tau$ is the ordinate of the intersection point of the straight line $M=M_0-M_1\ln(t/\tau)$ being the long-time asymptotes of $M(t)$ and the horizontal line $M=M_0$. We have found a good fit of Eq. (1) with the experimental curves $M(t)$ when treating M_1 and τ as fit parameters (Fig. 2).

The dependences of the measured quantities M_0 and τ upon $\dot{B}_e$ have been found in good agreement with those given by Eqs. (5)–(7) and (9). For example, Fig. 3 shows the logarithmic dependence of the initial magnetic moment M_0 on $\dot{B}_e$, which reflects the effect of the induced electric fields on j_c. The slope $dM_0/d\ln\dot{B}_e^{-1}$ coincides with the slope $M_1=-dM/d\ln t$, in accordance with Eq. (9). Furthermore, we have observed the inverse dependence of the time constant $\tau(B_e)$ upon the sweep rate $\dot{B}_e$, the value $\tau(\dot{B}_e)$ reaching $\sim 5\times10^3$ s at $\dot{B}_e=5\times10^{-6}$ T/s (Fig. 4). Notice, however, that the slopes $d\tau/d\dot{B}_e^{-1}$ prove to be slightly different at $5\times10^{-6} < \dot{B}_e < 10^{-4}$ T/s and $10^{-4} < \dot{B}_e < 1.2\times10^{-3}$ T/s, which may be due to slow logarithmic factors or a nonzero value of $1/n$ in Eqs. (5)–(7) resulting from a nonexponential form of $E(j)$.

In the above experiment we have specially used reduced sweep rates in order to reveal the transient stage. Usually the value $\dot{B}_e$ is taken by several orders of magnitude larger than our lowest $\dot{B}_e$, which shifts this stage into the ms region.[17] Notice here the recent work by Gao *et al.*,[18] who have measured the flux creep beginning with $t\sim0.1$ ms with the use of a pulse technique giving $\dot{B}_e \sim 10^2$–10^3 T/s. Under these conditions the above estimations indicate that the transient stage cannot be observed since $\tau < 0.1$ ms with $E(0)\sim 1$ mW/cm $\sim E_c$. However, the decrease of $\dot{B}_e$ (or E in resistive experiments) leads to the essential growth of τ imposing a lower limit for the minimum time window needed for measurements of steady-state flux-creep or resistive characteristics. For instance, in a sample with $a\sim1$ mm the time constant $\tau\sim a^2\mu_0 j_1/E$ exceeds 1 h at $E < 7\times10^{-10}$ V/cm.

In conclusion, we have studied both theoretically and experimentally initial nonlogarithmic stages of the flux creep and have shown their essential dependence on the sweep rate. Systematic measurements of the time constant $\tau(\dot{B}_e)$ are presented, and the inverse dependence $\tau \propto 1/\dot{B}_e$ has been observed. The results are shown to be similar for various flux-creep models over a wide region of the parameters.

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[20] This leads to a correction $\Delta M(t)$ to $M(t)$ at $t<\tau$; its maximum value $M(0)$ can be calculated in the same way as Eq. (9) but with $E=\dot{B}_e x$. As a result we get a numerically small value $\Delta M(0)=(2.5-\ln 8)M_1 \simeq 0.4M_1$, which does not change qualitatively the results obtained. Here Eqs. (5)–(7) are valid with an accuracy to numerical factors of the order of unity, which could be found by the computer simulation of Eq. (3).

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