Josephson-type effect in resonant-tunneling heterostructures

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We consider resonant-tunneling penetration through double-well semiconductor heterostructures, which may reveal some quantum-mechanical phenomena at the macroscopic level. The Josephson-type effects in these systems are investigated. We obtain simple expressions for the resonant current under a constant and an alternating bias voltage. It is shown that ac oscillations in the resonant current appearing in the case of a constant bias are exponentially damped. Nevertheless, a dc resonant current behaves similarly to the Josephson supercurrent: it rapidly decreases with bias voltage, but it peaks at the points corresponding to Shapiro steps when an alternating voltage is superimposed. Yet the heights of the peaks show a different dependence on the bias voltage than in the case of a Josephson junction. The possibility of observing the Josephson-type effect at room temperatures is discussed.

I. INTRODUCTION

The substantial improvement in epitaxial crystal growth during the past years has given a powerful tool to engineer semiconductor heterostructures of parallel layers. It is quite natural that the carriers in such systems exhibit properties of a two-dimensional electron gas in the plane parallel to the layers (y,z). However, along the direction normal to the layers (x) the system looks like a one-dimensional superlattice of potential barriers and quantum wells, each of them confining only one or few isolated quantum states.

The research in this field has mainly concentrated on the problem of penetration through double-barrier potentials (resonant-tunneling diode),[1] Fig. 1. Such a process is usually described by a multiple-scattering theory where the incident free wave impinging on the system is trapped in the quantum well between the barriers. The trapped wave undergoes multiple reflections off the two barriers. As a result, the total transmission probability peaks at the certain energy (E_0), which corresponds to the quasistationary state in the double-barrier potential (the analog phenomenon in optics is the Fabry-Pérot effect).

The resonance penetration through double-well barriers has been extensively studied both theoretically and experimentally.[1] It has been observed in different experiments and even at room-temperature regime.[2] However, a more interesting case of resonant tunneling, namely, the penetration of carriers through double-well potential structures with close levels (Fig. 2), has not received adequate attention.[3] In this case the carriers penetrating through the system are accumulated in each of the wells at the quasistationary levels E_1 (E_2). Thus, inside the wells the carriers would belong to the same quantum state $\Phi_1(x)$ [$\Phi_2(x)$], which is an eigenstate in the first (second) quantum well when the couplings with the neighboring well and the states E_L^j, E_R^k at the left- and the right-hand sides of the structure are neglected. [Note that the motion of carriers is free in the plane parallel to the layers, so that the wave functions are factorized: $\Psi(x,y,z) \propto \exp(ik_y y + ik_z z)\Phi(x)$.] As a result, there arises a resonant current across this system due to coherent tunneling of carriers through the barrier separating the wells. Such a phenomenon resembles the Josephson effect in two superconductors connected by an insulating barrier, where a dc supercurrent appears due to coherent tunneling of the electron pairs through the barrier.[4] This current drops down when a potential difference V is maintained across the barrier, and then an ac supercurrent of frequency $qV/\hbar$ flows between the two superconductors ($q=2e$ is the charge of the electron pair).

Although the resonant-tunneling heterostructures and superconductors separated by a weak link are quite different systems, their dynamics may nevertheless look similar, since both of them are governed by a coherent tunneling process. Hence, it is rather natural to look for quantum coherence effects (for instance, of the Josephson type) in the resonant-tunneling heterostructures. Note that the coherence in the resonant-tunneling process is implied by the existence of discrete (quasistationary) levels $E_{1,2}$ inside the wells, which makes it possible to find such effects even at room temperatures. In particular, this last possibility seems to be very attractive. It would therefore be desirable to perform a detailed study of tunneling through double-well potential barriers. In this paper we consider such an investigation. We use for our method an (time-dependent) approach, which is most appropriate for treating resonant-tunneling problems.[5–7] In this approach the resonance state is built up from bound-state wave functions, rather than from free waves propagating back and forth, as in the time-independent multiple-scattering theory (thus avoiding the explicit treatment of complicate multiple-reflection terms). The approach is briefly described in Sec. II, using the decay of a quasistationary state to continuum as an example.

A general treatment of resonant tunneling through double-well potentials is given in Sec. III. We obtain simple expressions for the resonant current across the structure under constant voltage bias. These expressions allow

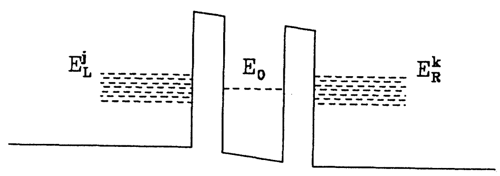

FIG. 1. Resonant tunneling through a double-barrier potential. E_L^j and E_R^k denote the energy levels at the left- and at the right-hand sides of the structure.

for a comparison of the dynamics of the resonant current in semiconductor heterostructures with the Josephson supercurrent in superconductor systems. Although the calculation does not address elastic and inelastic scattering and Coulomb interaction, the analysis does establish similarities and distinctions in the behavior of the two systems.

There are many interesting Josephson-type effects, which can be investigated in the resonant-tunneling heterostructures. We choose one, which is very important for applications and can be realized in present-time experiments, namely, the influence of alternating voltage (microwave radiation) on the system. It is known that if a Josephson junction maintained under a constant voltage V is irradiated with microwave radiation of frequency $qV/n\hbar$, a dc supercurrent reappears across the junction.[4] (This effect has many applications, and, in particular, it has been used for the most precise measurements of the ratio h/e.) It is thus necessary to establish whether this effect appears in the semiconductor heterostructures too. This question is investigated in Sec. IV, where we study the resonant tunneling through double-well potentials under an oscillating external voltage. The last section is the summary.

II. TIME-DEPENDENT APPROACH TO THE DECAY OF THE QUASI-STATIONARY STATE

Consider a standard tunneling problem of decay to continuum through the barrier penetration, Fig. 3(a). The initial state is prepared in the narrow well, while the continuum is represented by dense states in the right (large) well ($L_R \to \infty$). We take the initial state $\Phi_0(x)$ as the eigenstate of the Hamiltonian

$$H_0 = -(1/2m)(d^2/dx^2) + U_0(x)$$

with energy E_0, where the potential $U_0(x)$ is obtained from the original potential $V(x)$ by replacing the second well by a constant potential, Fig. 3(b). (Here and below we have adopted units with $\hbar = 1$.) As soon as the distorting potential $V(x) - U_0(x)$ is switched on (at $t=0$), the state $|\Phi_0>$ is no longer an eigenstate of the total Hamiltonian

$$H = -(1/2m)(d^2/dx^2) + V(x)$$

but rather is a wave packet spreading in time,

$$\Psi_0(x,t) = b_0(t)e^{-iE_0 t}\Phi_0(x) + \int b_k(t)e^{-iE_k t}\Phi_k(x)\frac{dk}{2\pi} , \tag{2.1}$$

where $\Phi_k(x)$ are the eigenfunctions of the Hamiltonian H_0, which belongs to the continuum spectrum [without loss of generality, we assumed that the potential $U_0(x)$ contains only one bound state]. Equation (2.1) is supplemented with the initial condition $b_0(0)=1$ and $b_k(0)=0$.

The spreading of the wave packet is determined by the

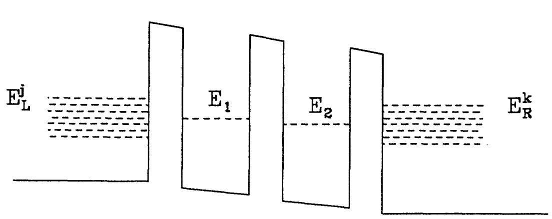

FIG. 2. Resonant tunneling through a double-well potential. E_L^j and E_R^k denote the energy levels at the left- and at the right-hand sides of the structure. The energy level E_1 (E_2) corresponds to the bound state Φ_1 (Φ_2) in the first (second) well where the couplings with the neighboring well and the states E_L^j, E_R^k are neglected.

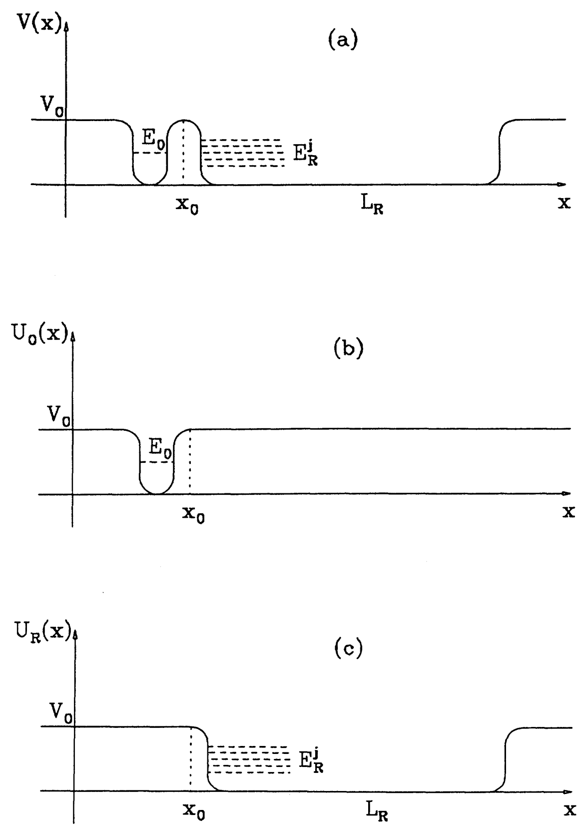

FIG. 3. Potential $V(x)$ and the auxiliary potentials $U_0(x)$ and $U_R(x)$ used in the time-dependent approach.

probability amplitudes $b_0(t)$ and $b_k(t)$, which can be found from the standard equations of time-dependent perturbation theory. One can show[5] that the exponential and oscillatory components of the probability amplitudes are defined by the poles of the total Green's function $G(E)=1/(E-H)$, and therefore their behavior does not depend on the particular choice of the initial wave packet [or on the particular decomposition of V into U_0 and $(V-U_0)$]. In a case of resonance states the oscillatory and exponential components dominate in the probability amplitudes $b_0(t), b_k(t)$. Taking only these components into consideration, the equations describing time behavior of $b_0(t)$, $b_k(t)$ are simplified considerably. However, the main simplification is achieved by reexpanding the second term in Eq. (2.1) into the eigenfunctions $\chi_R^j(x)$ of the Hamiltonian

$$H_R=-(1/2m)(d^2/dx^2)+U_R(x) ,$$

where the potential $U_R(x)$ is obtained from the potential $V(x)$ by replacing the narrow well by a constant potential, Fig. 3(c). Then, the time dependence of the wave packet is given by the following effective Schrödinger equation[5–7]

$$i\frac{d}{dt}\psi(t)=H^{\text{eff}}\psi(t) , \tag{2.2}$$

where $\psi(t)$ and the effective Hamiltonian are

$$\psi(t)=\begin{bmatrix} b_0(t)\\ b_R^1(t)\\ b_R^2(t)\\ \vdots\\ b_R^n(t)\end{bmatrix}$$

and (2.3)

$$H^{\text{eff}}=\begin{bmatrix} E_0 & \Omega_R^1 & \Omega_R^2 & \cdots & \Omega_R^n\\ \Omega_R^1 & E_R^1 & 0 & \cdots & 0\\ \Omega_R^2 & 0 & E_R^2 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \Omega_R^n & 0 & 0 & \cdots & E_R^n\end{bmatrix} .$$

Here $|b_R^j(t)|^2$ is the probability to find the system in the large (right) well at the level E_R^j, Fig. 3(c) and $\Omega_R^j\equiv\Omega_R(E_R^j)$ is the effective coupling between the level E_0 in the narrow and the level E_R^j in the right well. For $E_R^j=E_0$ it is given by product[5,8]

$$\Omega_R(E_0)=\frac{p(x_0)}{m}\Phi_0(x_0)\chi_R^j(x_0) , \tag{2.4}$$

where x_0 is some point inside the barrier (Ω_R is not sensitive to the choice of x_0), and

$$p(x_0)=\sqrt{2m[V(x_0)-E_0]} .$$

If $E_R^j\neq E_0$, there is an additional correction term in Eq. (2.4) proportional to the integral over the overlap of Φ_0 and χ_R^j, which is of order $(E_R^j-E_0)\Omega_R^j/V_0$. If $(E_R^j-E_0)\sim\Omega_R^j$ this term can be neglected.

The differential equation (2.2), describing the propagation of exponential and oscillatory components of the wave packets in terms of probability amplitudes, is the central point of the time-dependent approach to tunneling. It can be shown that all other tunneling problems can be reduced to equation of the same type,[6,7] resembling the Weisskopf-Wigner equation for decay of quasistationary states.[9] The easiest way to solve such equations is to perform the Laplace transform:

$$\tilde{\psi}(E)=\int_0^\infty \exp(iEt)\psi(t)dt .$$

Then Eq. (2.2), supplemented with the initial condition $b_0(0)=1$ and $b_R^j(0)=0$, turns over the system of algebraic equations

$$(E-E_0)\tilde{b}_0-\sum_j \Omega_R^j\tilde{b}_R^j=i , \tag{2.5a}$$

$$(E-E_R^j)\tilde{b}_R^j-\Omega_R^j\tilde{b}_0=0 , \tag{2.5b}$$

where the amplitudes $\tilde{b}_0(E), \tilde{b}_R^j(E)$ are, respectively, the Laplace transform of the amplitudes $b_0(t), b_R^j(t)$. Substituting $\tilde{b}_R^j$ from Eq. (2.5b) into Eq. (2.5a) we get

$$\left[E-E_0-\sum_j\frac{\Omega_R^2(E_R^j)}{E-E_R^j}\right]\tilde{b}_0(E)=i , \tag{2.6}$$

where $\Omega_R(E_R^j)\equiv\Omega_R^j$. In the limit of very dense states in the right well (continuum) one can replace the sum in Eq. (2.6) by the integral, $\sum_j\to\int\rho_R(E_R)dE_R$, where $\rho_R(E_R)$ is the density of states in the right well. Then Eq. (2.6) can be rewritten as

$$\tilde{b}_0(E)=\frac{i}{E-E_0-\Delta(E)+i[\Gamma(E)/2]} . \tag{2.7}$$

Here Δ is the principal part of the integral, and

$$\Gamma(E)=2\pi\Omega_R^2(E)\rho_R(E) . \tag{2.8}$$

The probability amplitude $b_0(t)$ to find the system in the narrow well is the inverse Laplace transform of $\tilde{b}_0(E)$, which is performed by closing the integration contour in the lower half plane of E and taking into account the singularities of $\tilde{b}_0(E)$. For small Ω_R the leading contribution will arise from the complex pole $E=E_0+\Delta_0-i(\Gamma_0/2)$, Eq. (2.7), where $\Delta_0=\Delta(E_0)$ and $\Gamma_0=\Gamma(E_0)$, and therefore one gets the usual exponential decay: $|b_0(t)|^2=\exp(-\Gamma_0 t)$.

As another illustration of the time-dependent approach, we consider the "reverse" decay problem (which will be useful in subsequent discussions). This consists of tunneling from the large (right) well to the narrow one when all the levels in the large well are occupied, and the narrow well is initially empty, Fig. 3(a). The solution is equally given by Eq. (2.2), supplemented now with the initial conditions $b_0(0)=0$ and $b_R^j(0)=\delta_{jl}$. Hence, instead of Eqs. (2.5) one obtains

$$(E-E_0)\tilde{b}_0-\sum_j\Omega_R^j\tilde{b}_R^j=0 , \tag{2.9a}$$

$$(E-E_R^j)\tilde{b}_R^j-\Omega_R^j\tilde{b}_0=i\delta_{jl} . \tag{2.9b}$$

Therefore,

$$\tilde{b}_0(E)=\frac{i\Omega_R(E_R^l)}{(E-E_0+i\gamma_0)(E-E_R^l)}\ , \tag{2.10}$$

where $\gamma_0=\Gamma_0/2$, and Δ_0 has been included in the definition of E_0. Using the inverse Laplace transform one finds

$$b_0(t)=\frac{\Omega_R(E_R^l)}{E_R^l-E_0+i\gamma_0}(e^{-iE_R^l t}-e^{-iE_0 t-\gamma_0 t}) \tag{2.11}$$

and the correspondent probability

$$N_0^{(l)}(t)=|b_0(t)|^2=\frac{\Omega_R^2(E_R^l)}{(E_R^l-E_0)^2+\gamma_0^2}[1-2\cos(E_0-E_R^l)t\ e^{-\gamma_0 t}+e^{-2\gamma_0 t}]\ . \tag{2.12}$$

To obtain the full probability of finding the system in the narrow well one needs to sum over all states (l) in the right well. Using $\sum_l \to \int \rho_R(E_R)dE_R$ and Eq. (2.8) one easily obtains

$$\sum_l N_R^{(l)}(t)=1-\exp(-\Gamma_0 t)\ .$$

Therefore, in the limit $t\to\infty$ the initially empty level E_0 in the narrow well is occupied.

III. RESONANT TUNNELING THROUGH DOUBLE-WELL BARRIER

Consider a double-well barrier shown schematically in Fig. 2. Using the approach presented in the preceding section we find that the system is described by Eq. (2.2) with

$$\psi(t)=(\mathbf{b}_L(t),b_1(t),b_2(t),\mathbf{b}_R(t))\ ,$$

where the vector

$$\mathbf{b}_{L(R)}=(b_{L(R)}^1,\ldots,b_{L(R)}^n)$$

represents the probability amplitudes of finding a carrier at the left-hand (right-hand) side of the structure, and $b_{1(2)}(t)$ is the probability amplitude to find it in the first (second) narrow well at the energy level $E_{1(2)}$. Then Eq. (2.2) can be written explicitly as

$$i\frac{d}{dt}b_L^j(t)=E_L^j b_L^j(t)+\Omega_L^j b_1(t)\ , \tag{3.1a}$$

$$i\frac{d}{dt}b_1(t)=E_1 b_1(t)+\sum_j \Omega_L^j b_L^j(t)+\Omega_0 b_2(t)\ , \tag{3.1b}$$

$$i\frac{d}{dt}b_2(t)=E_2 b_2(t)+\sum_k \Omega_R^k b_R^k(t)+\Omega_0 b_1(t)\ , \tag{3.1c}$$

$$i\frac{d}{dt}b_R^k(t)=E_R^k b_R^k(t)+\Omega_R^k b_2(t)\ , \tag{3.1d}$$

where $\Omega_L^j\equiv\Omega_L(E_L^j)$ is the coupling between the levels E_L^j and E_1, $\Omega_R^k\equiv\Omega_R(E_R^k)$ is the coupling between the levels E_R^k and E_2, and Ω_0 is the coupling between the levels E_1 and E_2.

We investigate the time development of the system, which is initially localized at the left-hand side of the structure. We assume that at $t=0$ all the levels E_L^j are occupied up to the Fermi level E_F, where $E_F\gg E_{1,2}\gg 0$. Therefore one has to solve Eq. (3.1) for the initial conditions $b_L^j(0)=\delta_{jl}$ and $b_1(0)=b_2(0)=b_R^k(0)=0$. Using the Laplace transform, $b(t)\to\tilde{b}(E)$, one can rewrite Eqs. (3.1) as

$$(E-E_L^j)\tilde{b}_L^j(E)-\Omega_L^j\tilde{b}_1(E)=i\delta_{jl}\ , \tag{3.2a}$$

$$(E-E_1)\tilde{b}_1(E)-\sum_j \Omega_L^j\tilde{b}_L^j(E)-\Omega_0\tilde{b}_2(E)=0\ , \tag{3.2b}$$

$$(E-E_2)\tilde{b}_2(E)-\sum_k \Omega_R^k\tilde{b}_R^k(E)-\Omega_0\tilde{b}_1(E)=0\ , \tag{3.2c}$$

$$(E-E_R^k)\tilde{b}_R^k(E)-\Omega_R^k\tilde{b}_2(E)=0\ . \tag{3.2d}$$

Substituting $\tilde{b}_L^j$ from Eq. (3.2a) into Eq. (3.2b), and $\tilde{b}_R^k$ from Eq. (3.2d) into Eq. (3.2c) and replacing the sums on j and k by the integrals, we obtain

$$\left[E-E_1+i\frac{\Gamma_L}{2}\right]\tilde{b}_1(E)-\Omega_0\tilde{b}_2(E)=i\frac{\Omega_L(E_L^l)}{E-E_L^l}\ , \tag{3.3a}$$

$$\left[E-E_2+i\frac{\Gamma_R}{2}\right]\tilde{b}_2(E)-\Omega_0\tilde{b}_1(E)=0\ , \tag{3.3b}$$

where

$$\Gamma_L=2\pi\Omega_L^2(E_1)\rho_L(E_1)\ ,\quad \Gamma_R=2\pi\Omega_R^2(E_2)\rho_R(E_2) \tag{3.4}$$

are, respectively, the partial width of the levels E_1 and E_2 due to tunneling to continuum, and the corresponding tunneling energy shifts $\Delta_{L,R}$ were included in E_1 and E_2.

In order to calculate the total current in this structure, one first needs to find the charge accumulated at the right-hand side of the structure, $Q(t)$. This can be easily written as

$$Q(t)=\frac{qm}{\pi}\sum_{l,k}(E_F-E_L^l)|b_R^k(t)|^2\ . \tag{3.5}$$

The factor $m(E_F-E_L^l)/\pi$ comes from the integration over the transverse electron momenta. Replacing sums over l,k by the integrals and using $\tilde{b}_R^k(E)=\Omega_R^k\tilde{b}_2(E)/(E-E_R^k)$, which follows from Eq. (3.2d), one can rewrite Eq. (3.5) as

$$Q(t)=\frac{qm}{4\pi^3}\int_0^{E_F}dE_L\rho_L(E_L)(E_F-E_L)\int dE\,dE'dE_R\frac{\rho_R(E_R)\Omega_R^2(E_R)\tilde{b}_2(E)\tilde{b}_2^*(E')}{(E-E_R)(E'-E_R)}e^{i(E'-E)t}\ . \tag{3.6}$$

It is useful to introduce the amplitudes $\hat{b}_{1,2}(E)$

$$\tilde{b}_1(E)=\hat{b}_1(E)\frac{\Omega_L(E_L^l)}{E-E_L^l}\ ,\quad \tilde{b}_2(E)=\hat{b}_2(E)\frac{\Omega_L(E_L^l)}{E-E_L^l}\ , \tag{3.7}$$

which satisfy the equations

$$\left[E-E_1+i\frac{\Gamma_L}{2}\right]\hat{b}_1(E)-\Omega_0\hat{b}_2(E)=i\ , \tag{3.8a}$$

$$\left[E-E_2+i\frac{\Gamma_R}{2}\right]\hat{b}_2(E)-\Omega_0\hat{b}_1(E)=0\ . \tag{3.8b}$$

(Notice that $\hat{b}_1$ and $\hat{b}_2$ describe the time dependence of the electron initially localized in the first well.) Substituting $\tilde{b}_2(E)$ from Eq. (3.7) into Eq. (3.6), one can make an approximate integration over E_L and E_R by extending the integration limits over the whole real axis and closing the integration contour in the complex plane. Since the poles $E_{L,(R)}=E,E'$ are on different sides of the real axis we get

$$Q(t)=-\frac{qm}{\pi}\int dE\, dE'\frac{(E_F-E)\rho_L(E)\rho_R(E)\Omega_L^2(E)\Omega_R^2(E)}{(E'-E)^2}\times\hat{b}_2(E)\hat{b}_2^*(E')e^{i(E'-E)t}\ . \tag{3.9}$$

The (resonant) current across the structure, $I(t)=dQ(t)/dt$, is therefore

$$I(t)=\frac{qm}{4\pi^3}\int dE\, dE' i(E_F-E)\frac{\Gamma_L\Gamma_R}{E-E'}\times\hat{b}_2(E)\hat{b}_2^*(E')e^{i(E'-E)t}\ . \tag{3.10}$$

Here we used Eq. (3.4). Solving Eqs. (3.8) one obtains

$$\hat{b}_2(E)=\frac{i\Omega_0}{(E-E_+)(E-E_-)}\ , \tag{3.11}$$

where

$$E_\pm=\bar{E}-i\gamma\pm\left[\Omega_0^2+\left[\epsilon-i\frac{\gamma_L-\gamma_R}{2}\right]^2\right]^{1/2}\ . \tag{3.12}$$

Here we used $\gamma_L=\Gamma_L/2$, $\gamma_R=\Gamma_R/2$, $\gamma=(\gamma_L+\gamma_R)/2$, $\bar{E}=(E_1+E_2)/2$, and $\epsilon=(E_1-E_2)/2$.

Consider first the symmetric case, $\gamma_L=\gamma_R=\gamma$. Then $E_\pm=\bar{E}-i\gamma\pm\omega$, where $\omega=(\Omega_0^2+\epsilon^2)^{1/2}$. Substituting Eq. (3.11) into Eq. (3.10) and performing integration over E and E' by closing the integration contour into the upper or the lower complex half plane we find

$$I(t)=\frac{qm(E_F-\bar{E})}{\pi}\gamma\frac{\Omega_0^2}{\omega^2+\gamma^2}\times\left[1-e^{-2\gamma t}\left[1+\frac{\gamma}{\omega}\sin 2\omega t+\frac{2\gamma^2}{\omega^2}\sin^2\omega t\right]\right]\ . \tag{3.13}$$

It follows from Eq. (3.13) that the ac component of $I(t)$ disappears when $t\to\infty$, so that the stationary current I across the structure is

$$I=\frac{qm(E_F-\bar{E})}{\pi}\gamma\frac{\Omega_0^2}{\Omega_0^2+\epsilon^2+\gamma^2}\ . \tag{3.14}$$

The resonant current $I(t)$ has been calculated for the initial conditions $b_L^j(0)=\delta_{jl}$ and $b_1(0)=b_2(0)=b_R^k(0)=0$, which correspond to the carriers being initially localized on the levels E_L^j at the left-hand side of the structure, Fig. 2. More realistic initial conditions, however, would consider the case where the level E_1 in the first narrow well is occupied too. Indeed, usually the levels E_1,E_2 in the wells are not aligned. As a result the carriers do not penetrate to the second well, but the level E_1 in the narrow well will be occupied [see the reverse decay problem, Eq. (2.12)]. By applying an additional external bias voltage, one can align the levels E_1 and E_2, so that the resonant current $I(t)$ appears across the structure. Now as about a calculation of $Q(t)$, Eq. (3.5), and $I(t)$, Eq. (3.10), one needs to add the contribution for the initial conditions, where the carriers start from the level E_1 in the first well [$b_1(0)=1$ and $b_L^j(0)=b_2(0)=b_R^k(0)=0$]. We finally obtain

$$I(t)=\frac{qm(E_F-\bar{E})}{\pi}\times\gamma\frac{\Omega_0^2}{\omega^2+\gamma^2}\left[1-e^{-2\gamma t}\left[\cos 2\omega t+\frac{\gamma}{\omega}\sin 2\omega t\right]\right]\ . \tag{3.15}$$

The transient component of the resonant current $I(t)$ does depend on the initial condition. As a result, Eq (3.15) is different from Eq (3.13). Nevertheless, the stationary (dc) component of the resonant current $I=I(t\to\infty)$ is independent of initial conditions, and therefore it is again given by Eq. (3.14).

In the general case $\gamma_L\neq\gamma_R$ the expression for the resonant current $I(t)$ is more complicated than Eqs. (3.13) and (3.15), which we obtained for the symmetric case. However, the dc component I has a rather simple form. It can be easily obtained from Eq. (3.10) if we take into account that the poles of $\hat{b}_2(E)$ [$\hat{b}_2^*(E')$], Eq. (3.12), are always below (above) the real axis, and therefore their contributions to the current $I(t)$ decrease exponentially at $t\to\infty$. Hence, only the pole $E=E'$ contributes to the stationary current I. As a result, we get

$$I=\frac{qm}{2\pi^2}\int dE(E_F-E)\Gamma_L\Gamma_R|\hat{b}_2(E)|^2=\frac{qm}{2\pi^2}\int\frac{(E_F-E)\Omega_0^2\Gamma_L\Gamma_R dE}{(E-E_+)(E-E_-)(E-E_+^*)(E-E_-^*)}\ . \tag{3.16}$$

Using Eq. (3.12), one finally obtains

$$I=\frac{qm(E_F-\bar{E})}{\pi}\frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}\frac{\Omega_0^2}{\Omega_0^2+\frac{\Gamma_L\Gamma_R}{4}+4\epsilon^2\frac{\Gamma_L\Gamma_R}{(\Gamma_L+\Gamma_R)^2}}. \tag{3.17}$$

If all the barriers have approximately the same width, and therefore $\Omega_0 \sim \Omega_{L(R)}$, one finds from Eq. (3.4) that $\Omega_0 \gg \Gamma_L, \Gamma_R$, and Eq. (3.17) can be rewritten as

$$I=\frac{qm(E_F-\bar{E})}{\pi}\frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}\frac{1}{1+\left[\frac{E_1-E_2}{\Omega_0}\right]^2\frac{\Gamma_L\Gamma_R}{(\Gamma_L+\Gamma_R)^2}}. \tag{3.18}$$

Let us compare the resonant current I across the double-well potential structure, Fig. 2, with the resonant current $I_{\rm DB}$, flowing through the double-barrier tunneling diode, Fig. 1. The latter is given by[7,10]

$$I_{\rm DB}=\frac{qm(E_F-E_0)}{\pi}\frac{\Gamma_L\Gamma_R}{\Gamma_L+\Gamma_R}, \tag{3.19}$$

where Γ_L, Γ_R are the partial widths of the quasistationary level E_0 due to tunneling to continuum through the left or the right barriers, Eq. (3.4). Surprisingly, the same current will flow across the double-well potential structure, Eq. (3.18), if only the levels are aligned ($E_1=E_2$). It looks like as the barrier in the middle has no influence on the resonant current. However, when an additional external voltage V is applied ($E_1-E_2=qV$), the resonant current I drops down as $\sim(\Omega_0/qV)^2$, Eq. (3.18), whereas $I_{\rm DB}$ remains approximately the same (as far as the level E_0 is inside the Fermi sea of the occupied states E_L^j).

Such strong falloff of the resonant dc current I across the double-well structure resembles the falloff of the Josephson dc supercurrent when a bias voltage V is applied. However, in the case of the Josephson junction, there appear oscillations of frequency qV in the supercurrent. Similar oscillations with frequency $2\omega \cong qV$, Eq. (3.15), appear in the resonant tunneling current too, but only for the transient component of $I(t)$. Indeed, for $t \ll \gamma^{-1}$, one obtains from Eq. (3.15),

$$I(t)=\frac{qm(E_F-\bar{E})}{\pi}\gamma\frac{4\Omega_0^2}{q^2V^2}[1-\cos(qVt)]. \tag{3.20}$$

Yet, these oscillations do disappear in the limit $t\rightarrow\infty$, Eq. (3.15), which is a result of the finite width (2γ) of the levels $E_{1,2}$ due to coupling with continuum states of carriers reservoir.

The main manifestation of the oscillations in the case of a Josephson junction is considered to be an appearance of the dc component in the supercurrent, when the external alternative voltage of the frequency qV/n is superimposed (Shapiro steps).[4] The question is whether a similar effect would exist in the resonant tunneling heterostructures too, in spite of the exponential damping of the oscillations in the steady state current. This problem is investigated in the next section.

IV. RESONANT TUNNELING UNDER ALTERNATING BIAS

Let us consider resonant tunneling through the double-well potential structure, Fig. 2, when an alternating voltage, $V(t)=V_0+V_1\cos\omega_0 t$ is superimposed. In this case we can replace

$$E_{1,2}=\pm(u_0+u_1\cos\omega_0 t)$$

in Eq. (3.1), where $u_{0,1}=qV_{0,1}/2$ and q is the electron charge. [Without loss of generality we choose $\bar{E}=(E_1+E_2)/2=0$.] It is useful to eliminate $V(t)$ by means of the canonical transformation of the amplitudes $b_{1,2}(t)$:

$$b_{1,2}(t)=b'_{1,2}(t)e^{\pm i(u_0t+\xi\sin\omega_0t)}=b'_{1,2}(t)\sum_{n=-\infty}^{\infty}J_n(\xi)e^{\pm i(u_0+n\omega_0)t}, \tag{4.1}$$

where $\xi=u_1/\omega_0$. Then Eqs. (3.1) can be rewritten as

$$i\frac{d}{dt}b_L^j(t)=E_L^jb_L^j(t)+\Omega_L^j\sum_n J_n(\xi)e^{-iu_{0n}t}b'_1(t), \tag{4.2a}$$

$$i\frac{d}{dt}b'_1(t)=\sum_j\Omega_L^j\sum_n J_n(\xi)e^{iu_{0n}t}b_L^j(t)+\Omega_0\sum_{n'}J_{n'}(2\xi)e^{i(u_{0n'}+u_0)t}b'_2(t), \tag{4.2b}$$

$$i\frac{d}{dt}b'_2(t)=\sum_k\Omega_R^k\sum_n J_n(\xi)e^{-iu_{0n}t}b_R^k(t)+\Omega_0\sum_{n'}J_{n'}(2\xi)e^{-i(u_{0n'}+u_0)t}b'_1(t), \tag{4.2c}$$

$$i\frac{d}{dt}b_R^k(t)=E_R^kb_R^k(t)+\Omega_R^k\sum_n J_n(\xi)e^{iu_{0n}t}b'_2(t). \tag{4.2d}$$

Here we denoted $u_{0n}=u_0+n\omega_0$. Time dependence of the continuum states E_L^j and E_R^k has not been taken into account in Eqs. (4.2), since all these states are finally summed over, and, as a result, their time dependence has no influence on the total resonant current.

Performing the Laplace transform $[b(t)\rightarrow\tilde{b}(E)]$ one obtains

$$(E-E_L^j)\tilde{b}_L^j(E)-\Omega_L^j\sum_n J_n(\xi)\tilde{b}'_1(E-u_{0n})=i\delta_{jl}, \tag{4.3a}$$

$$E\tilde{b}'_1(E)-\sum_j\Omega_L^j\sum_n J_n(\xi)\tilde{b}_L^j(E+u_{0n})-\Omega_0\sum_n J_n(2\xi)\tilde{b}'_2(E+u_0+u_{0n})=0, \tag{4.3b}$$

$$E\tilde{b}'_2(E)-\sum_k\Omega_R^k\sum_n J_n(\xi)\tilde{b}_R^k(E-u_{0n})-\Omega_0\sum_n J_n(2\xi)\tilde{b}'_1(E-u_0-u_{0n})=0, \tag{4.3c}$$

$$(E-E_R^k)\tilde{b}_R^k(E)-\Omega_R^k\sum_n J_n(\xi)\tilde{b}'_2(E+u_{0n})=0. \tag{4.3d}$$

The charge $Q(t)$, accumulated on the right well, can be written as [cf. Eqs. (3.5) and (3.6)]:

$$Q(t)=\frac{qm}{4\pi^3}\int_0^{E_F}dE_L\rho_L(E_L)(E_F-E_L)\int dE\,dE'dE_R\frac{\rho_R(E_R)\Omega_R^2(E_R)\tilde{B}_2(E)\tilde{B}_2^*(E')}{(E-E_R)(E'-E_R)}e^{i(E'-E)t}\,, \tag{4.4}$$

where

$$\tilde{B}_2(E)=\sum_n J_n(\xi)\tilde{b}_2'(E+u_{0n})\,. \tag{4.5}$$

Correspondingly, the resonant current across the structure is $I(t)=dQ(t)/dt$.

Substituting the amplitudes $\tilde{b}_L^j,\tilde{b}_R^k$ from Eqs. (4.3a) and (4.3d) into Eqs. (4.3b) and (4.3c), we find that the amplitudes $\tilde{b}_{1,2}'(E)$ satisfy the system of finite difference equations:

$$E\tilde{b}_1'(E)-\sum_{j,n,n'}\frac{(\Omega_L^j)^2J_n(\xi)J_{n+n'}(\xi)}{E+u_{0n}-E_L^j}\tilde{b}_1'(E-n'\omega_0)-\Omega_0\sum_n J_n(2\xi)\tilde{b}_2'(E+u_0+u_{0n})=\sum_n\frac{i\Omega_L^lJ_n(\xi)}{E+u_{0n}-E_L^l}\,, \tag{4.6a}$$

$$E\tilde{b}_2'(E)-\sum_{k,n,n'}\frac{(\Omega_R^k)^2J_n(\xi)J_{n+n'}(\xi)}{E-u_{0n}-E_R^k}\tilde{b}_2'(E+n'\omega_0)-\Omega_0\sum_n J_n(2\xi)\tilde{b}_1'(E-u_0-u_{0n})=0\,. \tag{4.6b}$$

Similarly to previous treatment, we replace the sums over the states E_L^j,E_R^k in Eqs. (4.6) by the corresponding integrals, so that the zeroes in the energy denominators generate the energy widths. Assuming that the widths vary slowly with the energy, one can sum over n and n' in the second terms of Eqs. (4.6a) and (4.6b) using $\sum_n J_n(\xi)J_{n+n'}(\xi)=J_{n'}(0)$. As a result, Eqs. (4.6) can be rewritten as

$$\left[E+i\frac{\Gamma_L}{2}\right]\tilde{b}_1'(E)-\Omega_0\sum_n J_n(2\xi)\tilde{b}_2'(E+2u_0+n\omega_0)=\sum_{n'}\frac{i\Omega_L^lJ_{n'}(\xi)}{E+u_{0n'}-E_L^l}\,, \tag{4.7a}$$

$$\left[E+i\frac{\Gamma_R}{2}\right]\tilde{b}_2'(E)-\Omega_0\sum_n J_n(2\xi)\tilde{b}_1'(E-2u_0-n\omega_0)=0\,, \tag{4.7b}$$

where $\Gamma_{L,R}$ are given by Eq. (3.4).

Let us compare Eqs. (4.7) with a constant bias voltage case analyzed in the previous section. Taking $u_1=0$ (or $\xi=0$) in Eqs. (4.7) we get

$$\left[E+i\frac{\Gamma_L}{2}\right]\tilde{b}_1'(E)-\Omega_0\tilde{b}_2'(E+2u_0)=i\frac{\Omega_L^l}{E+u_0-E_L^l}\,, \tag{4.8a}$$

$$\left[E+i\frac{\Gamma_R}{2}\right]\tilde{b}_2'(E)-\Omega_0\tilde{b}_1'(E-2u_0)=0\,. \tag{4.8b}$$

These equations coincide with Eqs. (3.3) for $\tilde{b}_{1,2}(E)=\tilde{b}_{1,2}'(E\mp u_0)$, and $E_{1,2}=\pm u_0$. Consider $u_0\gg\Omega_0$ and $\omega_0\gg\Omega_0$. Then, as follows from Eq. (3.18), the dc resonant current for I for the constant bias voltage is strongly suppressed $\sim(\Omega_0/u_0)^2$ in comparison with the case of aligned levels ($V_0=0$). On the other hand, the bias potential $2u_0$ is replaced by $2u_0+n\omega_0$ for the case of an alternating voltage, Eqs. (4.7). Therefore, for the values of V_0 such that $qV_0-N\omega_0\sim\Omega_0$, the contribution from $n=-N$ terms in Eqs. (4.7) will generate a dc resonant current $I=I_N$ of the same order as in the case of aligned levels.

Consider the resonant current $I=I_N$ in regions where $qV_0-N\omega_0\sim\Omega_0$. We retain only the terms $n=-N$ in Eqs. (4.7), since the others are of the order (Ω_0/ω_0) (as can be easily shown by iteration). Then, substituting

$$\tilde{b}_{1,2}'\left[E\mp u_0\pm\frac{N\omega_0}{2}\right]=\hat{b}_{1,2}(E)\sum_{n'}\frac{i\Omega_L^lJ_{n'}(\xi)}{E+(N\omega_0/2)+n'\omega_0-E_L^l} \tag{4.9}$$

into Eqs. (4.7), we find that the amplitudes $\hat{b}_{1,2}(E)$ satisfy Eqs. (3.8), where $E_{1,2}=(\pm qV_0\mp N\omega_0)/2$, and the coupling Ω_0 is replaced by $(-1)^NJ_N(2\xi)\Omega_0$. The amplitude $\tilde{B}_2(E)$, Eq. (4.5), can now be written as

$$\tilde{B}_2(E)=i\Omega_L^l\sum_{n,n'}\frac{J_n(\xi)J_{n'-n-N}(\xi)}{E+n'\omega_0-E_L^l}\hat{b}_2\left[E+n\omega_0+\frac{N\omega_0}{2}\right]. \tag{4.10}$$

Substituting Eq. (4.10) into Eq. (4.4) one can perform the E_L,E_R integrations just in the same way as it was done in the preceding section. After the summation over n', one finally obtains for the stationary component of the resonant current $I=I_N$,

$$I_N=\frac{qm}{2\pi^2}\sum_n J_n^2(\xi)\int dE(E_F-E)\Gamma_L\Gamma_R\left|\hat{b}_2\left[E+n\omega_0+\frac{N\omega_0}{2}\right]\right|^2. \tag{4.11}$$

The last integration over E is done by closing the integration contour over the singularities of $\hat{b}_2$ [see Eq. (3.17)]. Assuming again a weak energy dependence of the widths, we finally obtain

$$I_N = \frac{qmE_F}{\pi} \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \frac{\Omega_0^2 J_N^2(qV_1/\omega_0)}{\Omega_0^2 J_N^2(qV_1/\omega_0) + \Gamma_L \Gamma_R/4 + (qV_0 - N\omega_0)^2 [\Gamma_L \Gamma_R/(\Gamma_L + \Gamma_R)^2]} . \quad (4.12)$$

Let us compare the resonant current I_N, Eq. (4.12), with the Josephson supercurrent across the junction irradiated with an external frequency ω_0. Both currents, taken as a function of the bias voltage V_0, peak at the same points: $qV_0 = N\omega_0$, corresponding to tunneling with emission (or absorption) on N photons. However, the values of the currents at these points exhibit different N dependences. For instance, if $\Omega_0 J_N(qV_1/\omega_0) \gg \Gamma_{L,R}$, the resonant current $I = I_N$ shows no N dependence, whereas the Josephson supercurrent[4] is proportional to $J_N(qV_1/\omega_0)$. The N dependence of the resonant current appears only in widths of the peaks, which are proportional to $J_N(qV_1/\omega_0)$ [see Eq. (4.12)].

As an example of such behavior, the resonant current $I = I_N$ is given in Fig. 4 as a function of the bias voltage V_0, Eq. (4.12), for $\Gamma_R = 4\Gamma_L$, $qV_1/\omega_0 = 3$, and $\omega_0/\Omega_0 = 10$. We consider two cases: (a) $\Omega_0 J_N(qV_1/\omega_0) \gg \Gamma_L$, so that the second term in the denominator of Eq. (4.12) is neglected, and (b), where $\Gamma_L/\Omega_0 = 0.2$. Also dimensionless quantities qV_0/ω_0 and I/I_0 have been used, where I_0 is the resonant current for zero voltage, $V_0 = V_1 = 0$ in Eq. (4.12).

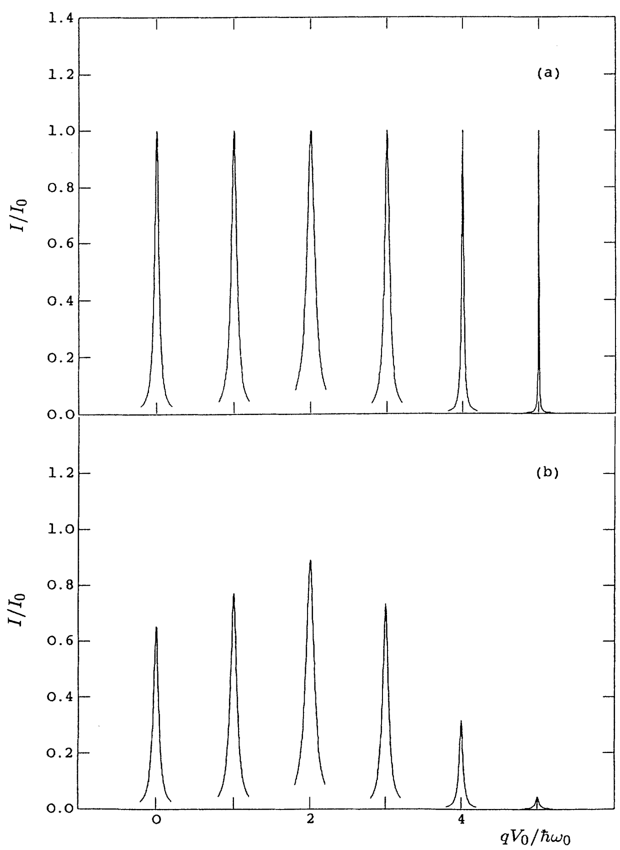

FIG. 4. Resonant current I across the double-well potential structure as a function of the bias voltage V_0, when the alternating voltage $V_1 \cos\omega_0 t$ is superimposed. Dimensionless units $qV_0/\hbar\omega_0$ and I/I_0 are used, where I_0 is the resonant current for zero voltage ($V_0 = V_1 = 0$). The other parameters are $\Gamma_R = 4\Gamma_L$, $qV_1/\hbar\omega_0 = 3$, $\omega_0/\Omega_0 = 10$, and (a) $\Omega_0 J_N(qV_1/\omega_0) \gg \Gamma_L$ and (b) $\Gamma_L/\Omega_0 = 0.2$.

The insensitivity of the resonant current amplitude to the number of emitted or absorbed photons N (for the case of $\Omega_0 \gg \Gamma_{L,R}$) is not so surprising. It can be understood in the following way. We found in the preceding section that for aligned levels in a double-well structure, Fig. 2, the middle barrier does not play a role. As a result, the resonant current is the same as that flowing across the double-barrier diode, Fig. 1. On the other hand, it has been shown earlier,[10] that the presence of phonons (phonon assisted tunneling) does not influence the total resonant current across a double-barrier diode. (The same could be expected for a photon case.) Therefore, in so far as the levels in the double-well structure are effectively aligned, i.e., $E_1 - E_2 = N\omega_0$, the resonant current will reach the same value as in the case of the double-barrier diode.

The N dependence of the linewidths, Fig. 4, has also a simple interpretation. Consider the limit $\Omega_0 \gg \Gamma_{L,R}$. Then, it is the coupling between the two wells that gives the main contribution in the line broading. In the case of photon assisted tunneling, the coupling is a product of the amplitude of tunneling, Ω_0, and the amplitude of the emission (or absorption) of N photons, $J_N(qV_1/\omega_0)$ (cf. the ionization amplitude in a strong electromagnetic field[11]). As a result, the resonant current has such a peculiar N dependence in the resonances' width.

V. SUMMARY

In this paper we studied the resonant current through a double-well potential heterostructure of semiconductor layers under a constant and an oscillating bias voltage. In both cases simple analytical expressions for the resonant current have been obtained. We found that a dc resonant current will appear at zero bias voltage (where the levels in two wells are aligned), but it decreases very rapidly when the bias voltage is applied. Such a phenomenon resembles the Josephson supercurrent in superconductors separated by a weak link. The difference, however, is that the latter would exhibit oscillations with a frequency proportional to the external voltage, whereas the resonant current displays such oscillations only in its transient component.

On a first sight, the absence of oscillations in a stationary component of the resonant current would limit an analogy with the Josephson supercurrent, so that the most important ac Josephson effects (for instance, Shapiro steps) could not be revealed in semiconductors heterostructures. We demonstrated that this is not the case by analyzing the behavior of the resonant currents under an oscillating voltage. It has been shown that a dc resonant current should reappear in the double-well heterostructure under a constant bias voltage V_0, when an alternating voltage (microwave radiation) is superimposed. It is remarkable that both the Josephson and the resonant currents display a dc component on the same condition: $qV_0=N\hbar\omega_0$, where q is the charge of the tunneling carriers, and ω_0 is the frequency of an external oscillating voltage. A difference between these currents appears in the N dependence of the Josephson and the resonant current amplitudes. While the height of peaks of the Josephson supercurrent is governed by the corresponding Bessel function, the same quantity for the resonant current is given by a more complicate expression, which involves the quasistationary level widths. It is interesting that in the case where the widths are much smaller than the level splitting due to tunneling between the wells, the height of peaks is independent of N. Only the widths of the peaks display the N dependence.

One of the most attractive possibilities that can be realized in semiconductors heterostructures is finding quantum coherence effects at room temperatures. Indeed, the coherence in such systems is achieved by the existence of isolated quasistationary quantum levels between the barriers. By a proper engineering of the semiconductor heterostructure one can make the potential barriers high enough in order to diminish the thermal excitation of carriers above the barriers. This would guarantee the widths of the quasistationary levels inside the wells to be small enough, so that the desirable effects (Josephson type) could be observed at room temperatures. Note that the usual resonant-tunneling current across the double-barrier resonant diode has been already observed at room-temperature regime.

We hope that this paper can stimulate further research on resonant tunneling through double-well potential heterostructures, focusing on different effects, which were not addressed here. For instance, we have not considered elastic and inelastic scattering of the carriers, which sometimes may modify the resonant current in an unexpected manner.[7] However, the most important phenomenon not addressed here is the Coulomb interaction of the carriers accumulated in the same quantum states between the wells. Since the double-well structure represents a capacitor, the accumulated charges can influence the alignment of the quantum levels (if the capacity of the structure is small enough). As a result, the resonant current (which is very sensitive to the levels alignment) can display very interesting quantum coherence effects on the macroscopic level, similar to those in small Josephson junctions. We even suggest that the study of the Coulomb interaction effects in the semiconductor heterostructures can help us reach a better understanding of quantum coherence phenomena in small Josephson junctions, for which contradictory theoretical descriptions still exist.

We also did not discuss the influence of magnetic field and a possible analogy with the Josephson effects in a SQUID. Unfortunately, the experimental technique in semiconductor heterostructures is not developed enough for engineering appropriate samples where a phenomena similar to Josephson effects in a SQUID can be observed. We believe, however, that this will be possible in the near future, and the theoretical investigation of magnetic field effects in double-well heterostructures will be very important.

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