Quantum-statistical theory of high-field transport phenomena

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On the basis of Tani's formalism for nonlinear response, a closed-form expression of the field-dependent dc conductivity is introduced. The field-dependent dc conductivity of electron-background (impurity and phonon) systems is evaluated by using the Mori-type projection technique. We also obtain the field-dependent self-energy operators, which show that the collisional broadening appears in both weak and high fields, while the intracollisional field effect is observed only in high field, as predicted by Barker. The results are compared with the work of some other authors.

I. INTRODUCTION

Recently we have seen a remarkable advance in the techniques of crystal growth and device processing.[1-4] This affords a realm of physics on the submicrometer and subpicosecond dimensional scale, and makes the study of high-field transport active. One of the subjects of numerous theoretical and experimental investigations that have received considerable attention in recent years is the transport of electrons in very high electric and/or magnetic fields.[1-44]

Early theoretical studies on high-electric-field transport were mainly based on semiclassical techniques including the Boltzmann transport theory.[5-7] The approaches, however, led to unsatisfactory results, the reason being that high fields alter the quantum states and the energy spectra of the carriers.[5,8] Some quantum theories appeared in efforts to improve the results.

Barker,[9] while studying a steady-state Boltzmann equation for high fields, predicted the existence of two quantum effects as a consequence of the level broadening due to the relaxation and the accelerating effects of the field in the collision event: the "collisional broadening" (CB) and the "intracollisional field effect" (ICFE), respectively. We see that these effects can be described by proper[9,10,27] or complete[11] treatment of quantum transport. Many other attempts for high-field transport, such as Feynmann's path-integral approach,[12-15] the force-balance approach,[16-18] the Green's-function approach,[19-23] the generalized-quantum-Langevin-equation approach,[24-26] the Monte Carlo method,[27,34] the projection-operator method,[35] the resolvent-superoperator approach[11] based on Tani's nonlinear-response formalism,[42-44] the Stark-ladder-representation approach,[36] and the Wigner-representation approach[37-41] would be capable of including the CB and ICFE effects in submicrometer structures.

In this paper, we will present a high-field quantum transport theory based on Tani's nonlinear-response formalism and the Mori-type projection-operator technique,[45] which could account for a wide variety of high-field effects, magnetophonon effects, and intervalley scatterings. A closed-form expression for the steady-state current with field-dependent dc conductivity shall be derived. We will also obtain a general expression of the temperature- and field-dependent self-energy, which are closely related to the CB and ICFE effects and show a detailed method of practical calculation of the field-dependent conductivity. Finally comparison with some other theories shall also be made.

II. NONLINEAR STATIC CONDUCTIVITY

Consider a system of many electrons subject to an external electric field $\mathbf{E}(t)$ given by

$$\mathbf{E}(t)=\mathbf{E}\exp(\epsilon t), \quad 0<\epsilon\ll 1 \ . \tag{2.1}$$

Then, the total Hamiltonian of the system is

$$H_T(t)=H+H_E(t) \ . \tag{2.2}$$

Here the time-independent part is given by

$$H=H_e+\eta V+H_B=\sum_l h^{(l)}+H_B \ , \tag{2.3}$$

$$h=h_e+\eta v \ , \tag{2.4}$$

where h_e is the unperturbed part of the single-electron Hamiltonian and v is the scattering potential which is the impurity part plus the phonon part, i.e.,

$$v=v_{e-i}+v_{e-\text{ph}} \ , \tag{2.5}$$

$$v_{e-i}=\sum_{\mathbf{q}}\tilde{v}(\mathbf{q})\exp(i\mathbf{q}\cdot\mathbf{r}) \ , \tag{2.6}$$

$$v_{e-\text{ph}}=\sum_{\mathbf{q}}(\gamma_{\mathbf{q}}b_{\mathbf{q}}+\gamma_{\mathbf{q}}^{\dagger}b_{\mathbf{q}}^{\dagger}) \ , \tag{2.7}$$

and H_B is the background Hamiltonian which contains the phonon part only, i.e.,

$$H_B=\sum_{\mathbf{q}}(b_{\mathbf{q}}^{\dagger}b_{\mathbf{q}}+\tfrac{1}{2})\hbar\omega_{\mathbf{q}} \ . \tag{2.8}$$

The time-dependent part is given by

$$H_E(t)=\sum_l h_E^{(l)}\exp(\epsilon t) , \tag{2.9}$$

$$h_E=-e\mathbf{r}\cdot\mathbf{E} . \tag{2.10}$$

The l in Eqs. (2.3) and (2.9) denotes the single-electron index, $\mathbf{r}$ is the position of the conduction electron with effective mass m, $\tilde{v}(\mathbf{q})$ means the Fourier transform of the impurity potential, $b_\mathbf{q}^\dagger(b_\mathbf{q})$ is the creation (annihilation) operator of the phonon with momentum $\hbar\mathbf{q}$ and energy $\hbar\omega_\mathbf{q}$, $\gamma_\mathbf{q}[\equiv C_\mathbf{q}\exp(i\mathbf{q}\cdot\mathbf{r})]$ describes the interaction of the electron and phonon, $C_\mathbf{q}$ depends on the type of interaction, and η is a dimensionless expansion parameter which shall be set equal to unity at the final stage. It should be noted that electron-electron interactions can be partly included in $\gamma_\mathbf{q}$ through the screened interaction potential.[11]

We suppose that the external field is initially absent and the system is in thermodynamic equilibrium with temperature T. The initial state can be described in terms of the grand-canonical density operator

$$\begin{aligned}\rho_{\text{eq}}(H)&=\exp(\alpha\bar{N}-\beta H)/Z\\ &=\exp[\alpha\bar{N}-\beta(H_e+H_B)](S_{\text{even}}-S_{\text{odd}})/Z\\ &\equiv\rho_{eB}-\rho_V\end{aligned} \tag{2.11}$$

with

$$\rho_{eB}=\frac{\exp[\alpha\bar{N}-\beta(H_e+H_B)]}{\text{Tr}\{\exp[\alpha\bar{N}-\beta(H_e+H_B)]\}} , \tag{2.12a}$$

$$\rho_V=\exp[\alpha\bar{N}-\beta(H_e+H_B)][\text{Tr}(S_{\text{even}})-S_{\text{even}}+S_{\text{odd}}]/Z , \tag{2.12b}$$

$$\begin{aligned}Z&=\text{Tr}[\exp(\alpha\bar{N}-\beta H)]\\ &=\text{Tr}\{\exp[\alpha\bar{N}-\beta(H_e+H_B)]S_{\text{even}}\} ,\end{aligned} \tag{2.13}$$

$$S_{\text{even}}\equiv 1+\sum_{k=1}^{\infty}\int_0^\beta d\beta_1\int_0^{\beta_1}d\beta_2\cdots\int_0^{\beta_{2k-1}}d\beta_{2k}V(\beta_1)V(\beta_2)\cdots V(\beta_{2k}) , \tag{2.14a}$$

$$S_{\text{odd}}\equiv \sum_{k=1}^{\infty}\int_0^\beta d\beta_1\int_0^{\beta_1}d\beta_2\cdots\int_0^{\beta_{2k-2}}d\beta_{2k-1}V(\beta_1)V(\beta_2)\cdots V(\beta_{2k-1}) . \tag{2.14b}$$

Here $\bar{N}$ is the total number of electrons in the system, $\alpha=\beta\zeta$, $\beta=(k_BT)^{-1}$ and

$$V(\beta)=\exp[\beta(H_e+H_\beta)]V\exp[-\beta(H_e+H_B)] ,$$

where k_B and ζ, respectively, are the Boltzmann constant and the chemical potential, and Tr denotes the many-body trace. Note that we have used $\text{Tr}(S_{\text{odd}}=0)$ in Eq. (2.13) by the orthogonality of the background state.

We also assume that as the time-dependent external field is applied to the system, the statistical density changes as

$$\rho(t)=\rho_{\text{eq}}+\rho_1(t) , \tag{2.15}$$

in which $\rho_1(t)$ represents the field-dependent density operator and is not necessarily small. The Liouville equation $i\hbar\partial\rho/\partial t=[H_T(t),\rho(t)]$ can be written as

$$\begin{aligned}\frac{i\hbar\partial\rho_1(t)}{\partial t}&=[[H+H_E(t)],\rho_1(t)]\\ &\quad+[H_E(t),\rho_{eB}]-[H_E(t),\rho_V] ,\end{aligned} \tag{2.16}$$

where we have used the fact that $[H,\rho_{\text{eq}}]=0$ and $\partial\rho_{\text{eq}}/\partial t=0$. The second and third terms on the right-hand side of (2.16) represent the rate of change of ρ_{eq} due to the external electric field alone. Then the last term depends on the rate of change of the electron-background interaction due to the external field. When an external electric field is applied, energy is supplied to the electron system. This energy, which increases rapidly with increasing field, is dissipated to the background. Thus, the power supplied by the external field to the electron system is transferred to the background via the interaction between electrons and the background. This transfer energy which is due to the electron-background interaction generally brings about the heating of the background. For a system which has a low density of electrons, such as in semiconductors, we now assume that the background is part of the heat reservoir for the electron system and is in contact with another huge heat reservoir with temperature T. The system we consider here is nearly in thermal equilibrium with the outer world, so the background temperature can be kept constant. This means that the amount of energy which is transferred to the background from the conduction electrons is quite small, so the last term of Eq. (2.16) which includes the effect of the heating of the background can be neglected. However, this assumption is certainly invalid for metals and for the extremely high-field regime where the effects of nonequilibrium heating of the background occur.[11] Studies of the effects will be left for future investigation. We kept the term $[H_E(t),\rho_1(t)]$ since the electric field is not necessarily weak.

In order to obtain $\rho_1(t)$, we define the density operator in the Dirac picture as

$$\rho_{1D}(t)=\exp[iH_T(t)t/\hbar]\rho_1(t)\exp[-iH_T(t)t/\hbar] . \tag{2.17}$$

Differentiating Eq. (2.17) and considering Eqs. (2.16), we obtain

$$\frac{i\hbar\partial\rho_{1D}(t)}{\partial t}=[H_{ED}(t),\rho_{eB}]\ , \tag{2.18}$$

where

$$H_{ED}(t)=\exp[iH_T(t)t/\hbar]H_E(t)\exp[-iH_T(t)t/\hbar]\ . \tag{2.19}$$

Integrating Eq. (2.18) and taking into account Eq. (2.17) we have ρ_s, the steady-state value of $\rho_1(t)$, as

$$\rho_s=(1/i\hbar)\lim_{\epsilon\to 0^+}\int_{-\infty}^{0}dt\exp(\epsilon^+t)\exp[iH_T(t)t/\hbar]\times[H_E,\rho_{eB}]\exp[-iH_T(t)t/\hbar] \tag{2.20}$$

with $\epsilon^+=\epsilon+s\ (s\to 0^+)$, which amounts to the adiabatic switching on the perturbation given by Eq. (2.9).

On the other hand, the average steady-state current density can be calculated in terms of the density operator as

$$\langle\Delta\mathbf{J}\rangle=\mathrm{Tr}\{\mathbf{J}\rho_s\}=(1/i\hbar)\lim_{\epsilon\to 0^+}\int_0^\infty dt\exp(-\epsilon^+t)\mathrm{Tr}\left[\left[\rho_{eB},-\sum_l e r^{(l)}\right]\mathbf{J}(t|H_T(-t))\right]\cdot\mathbf{E}\ , \tag{2.21}$$

where we have used Eqs. (2.9), (2.10), and (2.20) and $\mathbf{J}(t|H)$ is a total current operator in the Heisenberg picture given by

$$\mathbf{J}(t|H)\equiv\exp(iHt/\hbar)\mathbf{J}\exp(-iHt/\hbar)\ , \tag{2.22}$$

$$\mathbf{J}=-\sum_l e(d\mathbf{r}^{(l)}/dt)=\sum_l j^{(l)}\ . \tag{2.23}$$

Here j is the single-electron operator in the one-band approximation, and $\Delta\mathbf{J}$ is the total current minus the initial value.

The k component of Eq. (2.21) becomes

$$\langle\Delta J_k\rangle=\sum_l\lim_{\epsilon\to 0^+}\int_0^\infty dt\exp(-\epsilon^+t)\phi_{kl}[t|H_T(-t)]E_l\ , \tag{2.24}$$

where $k,l=x,y,z$ and $\phi_{kl}[t|H_T(-t)]$ is the field-dependent response function given by

$$\phi_{kl}[t|H_T(-t)]=(1/i\hbar)\mathrm{Tr}\left[\left[\rho_{eB},-\sum_l e r_l^{(l)}\right]J_k[t|H_T(-t)]\right] \tag{2.25a}$$

$$=\int_0^\beta\mathrm{Tr}\{\rho_{eB}J_l(-i\hbar\beta_1|H_e+H_B)J_k[t|H_T(-t)]\}d\beta_1 \tag{2.25b}$$

$$=\lim_{u_l\to 0}\frac{\partial}{\partial u_l}\mathrm{Tr}\{\rho_{eB}(\tilde{H})J_k[t|H_T(-t)]\}\ . \tag{2.25c}$$

Here we have used the Kubo identity[46] and the following identity:[9,47]

$$\int_0^\beta d\beta_1\rho_{eB}\mathbf{J}(-i\hbar\beta_1|H_e+H_B)=\lim_{u_l\to 0}\frac{\partial}{\partial u_1}\rho_{\mathrm{eq}}(\tilde{H})\ . \tag{2.26}$$

In Eqs. (2.25c) and (2.26) $\tilde{H}\equiv H_e+H_B-\mathbf{u}\cdot\mathbf{J}$, $\mathbf{u}$ being a c-number vector, is the modified Hamiltonian[47] that gives a more compact response function. Considering Eqs. (2.24), (2.25b), and (2.25c) we obtain the formal expression for the generalized field-dependent conductivity tensor $\sigma_{kl}(E_l)$ as

$$\sigma_{kl}(E_l)=\Omega^{-1}\int_0^\infty dt\exp(-st)\int_0^\beta d\beta_1\mathrm{Tr}[\rho_{eB}J_l(-i\hbar\beta_1|H_e+H_B)J_k(t|H+H_E)] \tag{2.27a}$$

$$=\lim_{u_l\to 0}\frac{\partial}{\partial u_l}\Omega^{-1}\int_0^\infty dt\exp(-st)\mathrm{Tr}[\rho_{eB}(\tilde{H})J_k(t|H+H_E)] \tag{2.27b}$$

since $\langle\Delta J_k\rangle/\Omega=\sum_l\sigma_{kl}(E_l)E_l$,[11,42] where Ω represents the volume of the system. We see that Eq. (2.27) is a function of the external field $\mathbf{E}$ and includes nonlinear terms. If the Ohmic condition $\mathbf{E}\to 0$ is taken in Eqs. (2.27a) and (2.27b), Eqs. (2.27a) and (2.27b) are reduced to the Kubo formula[46] for the linear static conductivity.

In order to express Eq. (2.27b) in the single-electron representation for an electron-background system we assume that the statistical operator $\rho_{eB}(\tilde{H})$ in Eq. (2.27b) is factorized as[6,9,11,28]

$$\rho_{\mathrm{eq}}(\tilde{H})\approx\rho_B(H_B)\rho\left[\sum_l h_e^{(l)}-\mathbf{u}\cdot\mathbf{J}\right] \tag{2.28}$$

and the many-body trace Tr is reduced to $\mathrm{tr}\,\mathrm{Tr}^{(B)}$.[9,11] Here, $\rho_B(H_B)\equiv\exp(-\beta H_B)/\mathrm{Tr}^{(B)}[\exp(-\beta H_B)]$ and the symbols tr and $\mathrm{Tr}^{(B)}$ mean the single-electron trace and the many-body trace over background coordinates, respectively. Then, the exact field-dependent conductivity formula, Eq. (2.27b), can be expressed in terms of the single-electron trace as

$$\sigma_{kl}(E_l)=\Omega^{-1}\int_0^\infty dt\,\exp(-st)\left\langle \mathrm{tr}\left[\lim_{u_l\to 0}\left[\frac{\partial f}{\partial u_l}\right]j_k(t|h_{eE}+\eta v+H_B)\right]\right\rangle_B , \tag{2.29}$$

where $h_{eE}\equiv h_e+h_E$, $\langle\ \rangle_B$ denotes the average over the background (impurity and/or phonon) scatterings, and f is the modified Fermi-Dirac operator given by

$$f\equiv\{\exp[\beta'(h_e+\mathbf{u}\cdot\mathbf{j}-\zeta)]+1\}^{-1} \tag{2.30}$$

with $\beta'=(k_BT_e)^{-1}$.

In order to rewrite Eq. (2.29) in a more convenient form, we represent the interaction term in the background average as

$$\mathrm{tr}\left[\lim_{u_l\to 0}\left[\frac{\partial f}{\partial u_l}\right]j_k(t|h_{eE}+\eta v+H_B)\right] =\sum_{\lambda_1,\lambda_2}\frac{1}{2\pi i}\oint_c dz\, f(z)\langle\lambda_1|R_zj_lR_z|\lambda_2\rangle \times\langle\lambda_2|j_k(t|h_{eE}+\eta v+H_B)|\lambda_1\rangle , \tag{2.31}$$

where we have used

$$\lim_{u_l\to 0}\frac{\partial}{\partial u_l}(z-h_e-\mathbf{u}\cdot\mathbf{j})^{-1}=R_zj_lR_z \tag{2.32}$$

with $R_z=(z-h_e)^{-1}$ and $f(z)$ is defined by

$$f(z)=\{\exp[\beta'(z-\zeta)]+1\}^{-1} . \tag{2.33}$$

The λ in Eq. (2.31) denotes the single-electron state corresponding to the energy eigenvalue E_λ:

$$h_{eE}|\lambda\rangle=E_\lambda|\lambda\rangle . \tag{2.34}$$

It should be noted that for our choice of the electron state we shall adopt a representation in which h_{eE} is diagonal.[11]

Then, by considering Eqs. (2.29) and (2.31) the field-dependent conductivity is reduced to

$$\sigma_{kl}(E_l)=\Omega^{-1}\sum_{\lambda_1,\lambda_2}\langle\langle\lambda_1|Y_z|\lambda_2\rangle\langle\lambda_2|\tilde{j}_k(E_l)|\lambda_1\rangle\rangle_B , \tag{2.35}$$

where $\tilde{j}_k(E_l)$ is the Laplace transform of $j_k(t|h_\tau)$ defined by

$$\tilde{j}_k(E_l)\equiv LT[j_k(t|h_\tau)]=\int_0^\infty dt\,\exp(-st)j_k(t|h_\tau) , \tag{2.36}$$

with $h_T\equiv h_{eE}+\eta v+H_B$ and

$$\langle\lambda_1|Y_z|\lambda_2\rangle=\frac{f(\varepsilon_{\lambda_1})-f(\varepsilon_{\lambda_2})}{\varepsilon_{\lambda_1}-\varepsilon_{\lambda_2}}\langle\lambda_1|j_l|\lambda_2\rangle . \tag{2.37}$$

Here ε_λ is the energy eigenvalue of the Hamiltonian h_e satisfying

$$h_e|\lambda\rangle=\epsilon_\lambda|\lambda\rangle . \tag{2.38}$$

Then, the field-dependent conductivity formula can be rewritten from Eqs. (2.35) and (2.37) as

$$\sigma_{kl}(E_l)=\Omega^{-1}\sum_{\lambda_1,\lambda_2}\frac{f(\varepsilon_{\lambda_1})-f(\varepsilon_{\lambda_2})}{\varepsilon_{\lambda_1}-\varepsilon_{\lambda_2}}j_{l\lambda_1\lambda_2}\langle\tilde{j}_{k\lambda_2\lambda_1}(E_l)\rangle_B , \tag{2.39}$$

where $X_{\lambda_2\lambda_1}=\langle\lambda_2|X|\lambda_1\rangle$ for any operator X. The central problem of evaluation of Eq. (2.39) is the evaluation of the configuration over the background fields. Especially, the main task is then to give a suitable expansion method for the operators $\tilde{j}_{k\lambda_2\lambda_1}(E_l)$ in Eq. (2.39), which will be outlined in the following section.

III. FIELD-DEPENDENT SELF-ENERGY OPERATOR

In order to obtain the field-dependent self energy we will present two representations: a closed-form representation and a continued-fraction-form representation.

A. The closed form

For the calculation of $\tilde{j}_{k\lambda_2\lambda_1}(E_l)$, we define the projection operators P_0 and P_0' for the states $|\lambda_1\rangle$ and $|\lambda_2\rangle$ as

$$P_0X=(X_{\lambda_2\lambda_1}/j_{k\lambda_2\lambda_1})j_k , \tag{3.1}$$

$$P_0'=1-P_0 , \tag{3.2}$$

where X is any operator.

Following Mori,[45] we separate $j_k(t|h_T)$ into the projective and vertical components with respect to the j_k axis as

$$j_k(t|h_T)=P_0j_k(t|h_T)+P_0'j_k(t|h_T) =Z_{0\lambda_2\lambda_1}(t|h_T)j_k +\int_0^t dt_1 Z_{0\lambda_2\lambda_1}(t_1|h_T)f_1'(t-t_1|h_T) , \tag{3.3}$$

where

$$Z_{0\lambda_2\lambda_1}(t|h_T)\equiv j_{k\lambda_2\lambda_1}(t|h_T)/j_{k\lambda_2\lambda_1} , \tag{3.4}$$

$$f_1'(t|h_T)\equiv\exp(iL_1t/\hbar)f_1' , \tag{3.5}$$

$$f_1'\equiv iL_1j_k/\hbar , \tag{3.6}$$

$$L_1\equiv P_0'L_T , \tag{3.7}$$

$$L_T\equiv L_{eE}+\eta L_v+L_B . \tag{3.8}$$

Here $L_{eE}(\equiv L_e+L_E)$, L_v, and L_B are Liouville operators corresponding to the single-electron Hamiltonian h_e+h_E, the scattering potential v, and the background (phonon and impurity) Hamiltonian, respectively.

In order to obtain $\tilde{j}_{k\lambda_2\lambda_1}(E_l)$ or $\tilde{Z}_{0\lambda_2\lambda_1}(E_l)$, we differentiate Eq. (3.4) as

$$\frac{d}{dt}Z_{0\lambda_2\lambda_1}(t|h_T)=i\omega_{0\lambda_2\lambda_1}Z_{0\lambda_2\lambda_1}(t|h_T)+\int_0^t dt_1\Delta_{0\lambda_2\lambda_1}(t-t_1|h_T)Z_{0\lambda_2\lambda_1}(t_1|h_T) \quad (3.9a)$$

$$=i\omega_{0\lambda_2\lambda_1}Z_{0\lambda_2\lambda_1}(t|h_T)+\int_0^t dt_1 Z_{1\lambda_2\lambda_1}(t-t_1|h_T)\Delta_{0\lambda_2\lambda_1}Z_{0\lambda_2\lambda_1}(t_1|h_T)\ . \quad (3.9b)$$

Here

$$\omega_{0\lambda_2\lambda_1}\equiv(L_Tj_k/\hbar)_{\lambda_2\lambda_1}j_{k\lambda_2\lambda_1}=(E_{\lambda_2}-E_{\lambda_1})/\hbar\ , \quad (3.10)$$

$$\Delta_{0\lambda_2\lambda_1}(t|h_T)\equiv f_{1\lambda_2\lambda_1}(t|h_T)/j_{k\lambda_2\lambda_1}$$

$$\equiv Z_{1\lambda_2\lambda_1}(t|h_T)\Delta_{0\lambda_2\lambda_1}\ , \quad (3.11)$$

$$f_1(t|h_T)=iL_Tf_1'(t|h_T)/\hbar\ , \quad (3.12)$$

$$Z_{1\lambda_2\lambda_1}(t|h_T)\equiv f_{1\lambda_2\lambda_1}(t|h_T)/f_{1\lambda_2\lambda_1}\ , \quad (3.13)$$

$$\Delta_{0\lambda_2\lambda_1}\equiv f_{1\lambda_2\lambda_1}/j_{k\lambda_2\lambda_1}\ , \quad (3.14)$$

where we have used[11] $\langle v_{\lambda_2\lambda_2}-v_{\lambda_1\lambda_1}\rangle_B=0$ in Eq. (3.10).

Then, the LT of Eqs. (3.9a) and (3.9b) leads to

$$\tilde{Z}_{0\lambda_2\lambda_1}(E_l)\equiv\tilde{j}_{k\lambda_2\lambda_1}(E_l)/j_{k\lambda_2\lambda_1}$$

$$=[s-i\omega_{0\lambda_2\lambda_1}+\tilde{\Sigma}_{0\lambda_2\lambda_1}(E_l)]^{-1}\ . \quad (3.15)$$

Here $\tilde{\Sigma}_{0\lambda_2\lambda_1}(E_l)$, often called the field-dependent self-energy operator, is defined as

$$\tilde{\Sigma}_{0\lambda_2\lambda_1}(E_l)=-\tilde{\Delta}_{0\lambda_2\lambda_1}(E_l) \quad (3.16a)$$

$$=-\tilde{Z}_{1\lambda_2\lambda_1}(E_l)\Delta_{0\lambda_2\lambda_1}\ , \quad (3.16b)$$

where $\tilde{\Delta}_{0\lambda_2\lambda_1}(E_l)$ and $\tilde{Z}_{1\lambda_2\lambda_1}(E_l)$ are the Laplace transform of Eqs. (3.11) and (3.13), respectively. Considering Eqs. (3.5)–(3.8), (3.11), (3.12), and (3.16a), and taking into account the relation

$$P_0(L_{eE}+L_B)G_{0B}P_0'X=[(L_{eE}+L_B)G_{0B}P_0'X]_{\lambda_2\lambda_1}=0\ ,$$

we obtain

$$\tilde{\Sigma}_{0\lambda_2\lambda_1}(E_l)=(i\hbar j_{k\lambda_2\lambda_1})^{-1}$$

$$\times\left\langle\sum_{N=1}^{\infty}[(\eta L_vG_{0B}P_0')^NL_Tj_k]_{\lambda_2\lambda_1}\right\rangle_B\ , \quad (3.17)$$

where $G_{0B}=(-i\hbar s-L_{eE}-L_B)^{-1}$ and we have used the relation $(A-B)^{-1}=A^{-1}\sum_{m=0}^{\infty}(BA^{-1})^m$ for any operators A and B. Now the self-energy operator $\tilde{\Sigma}_{0\lambda_2\lambda_1}(E_l)$ has been expanded with respect to L_v corresponding to the scattering potential. Equation (3.17) is the general formula for the field-dependent self-energy operator given in a closed expansion form for electron-impurity and electron-phonon systems, which is applicable to the weak-coupling case since we have taken the relation $(A-B)^{-1}=A^{-1}\sum_{m=0}^{\infty}(BA^{-1})^m$. Equation (3.17) is identical to the result[35] obtained by the Argyres-Sigel projection-operator method.[48]

B. The continued-fraction form

In order to obtain $\tilde{Z}_{1\lambda_2\lambda_1}(E_l)$ of Eq. (3.16b) we define the projection operators P_1 and P_1' as

$$P_1X=(X_{\lambda_2\lambda_1}/f_{1\lambda_2\lambda_1})f_1\ , \quad (3.18)$$

$$P_1'=1-P_1\ . \quad (3.19)$$

By utilizing these operators we separate $f_1(t|h_T)$ of Eq. (3.12) into the projective and vertical components with respect to the f_1 axis as

$$f_1(t|h_T)=P_1f_1(t|h_T)+P_1'f_1(t|h_T)$$

$$=Z_{1\lambda_2\lambda_1}(t|h_T)f_1$$

$$+\int_0^t Z_{1\lambda_2\lambda_1}(t_1|h_T)f_2'(t-t_1|h_T)dt_1\ , \quad (3.20)$$

where

$$f_2'(t|h_T)\equiv\exp(iL_2t/\hbar)f_2'\ , \quad (3.21)$$

$$f_2'\equiv iL_2f_1/\hbar\ , \quad (3.22)$$

$$L_2\equiv P_1'L_TP_0'\ . \quad (3.23)$$

In order to obtain $\tilde{Z}_{1\lambda_2\lambda_1}(E_l)$, we differentiate Eq. (3.13) as

$$\frac{d}{dt}Z_{1\lambda_2\lambda_1}(t|h_T)=i\omega_{1\lambda_2\lambda_1}Z_{1\lambda_2\lambda_1}(t|h_T)+\int_0^t dt_1\Delta_{1\lambda_2\lambda_1}(t-t_1|h_T)Z_{1\lambda_2\lambda_1}(t_1|h_T) \quad (3.24a)$$

$$=i\omega_{1\lambda_2\lambda_1}Z_{1\lambda_2\lambda_1}(t|h_T)+\int_0^t dt_1Z_{2\lambda_2\lambda_1}(t-t_1|h_T)\Delta_{1\lambda_2\lambda_1}Z_{1\lambda_2\lambda_1}(t_1|h_T)\ , \quad (3.24b)$$

where

$$\omega_{1\lambda_2\lambda_1}\equiv(L_TP_0'f_1/\hbar)_{\lambda_2\lambda_1}/f_{1\lambda_2\lambda_1}\ , \quad (3.25)$$

$$\Delta_{1\lambda_2\lambda_1}(t|h_T)\equiv f_{2\lambda_2\lambda_1}(t|h_T)/f_{1\lambda_2\lambda_1}$$

$$\equiv Z_{2\lambda_2\lambda_1}(t|h_T)\Delta_{1\lambda_2\lambda_1}\ , \quad (3.26)$$

$$f_2(t|h_T)=iL_TP_0'f_2'(t|h_T)/\hbar\ , \quad (3.27)$$

$$Z_{2\lambda_2\lambda_1}(t|h_T)\equiv f_{2\lambda_2\lambda_1}(t)/f_{2\lambda_2\lambda_1}\ , \quad (3.28)$$

$$\Delta_{1\lambda_2\lambda_1}\equiv f_{2\lambda_2\lambda_1}/f_{1\lambda_2\lambda_1}\ . \quad (3.29)$$

Then the LT of Eq. (3.24) leads to

$$\tilde{Z}_{1\lambda_2\lambda_1}(E_l)\equiv\tilde{f}_{1\lambda_2\lambda_1}(E_l)/f_{l\lambda_2\lambda_1}$$

$$=[s-i\omega_{1\lambda_2\lambda_1}+\tilde{\Sigma}_{1\lambda_2\lambda_1}(E_l)]^{-1}\ . \quad (3.30)$$

Here $\tilde{\Sigma}_{1\lambda_2\lambda_1}(E_l)$ is the first-order field-dependent self-energy operator in the continued-fraction forms given by

$$\tilde{\Sigma}_{1\lambda_2\lambda_1}(E_l)=-\tilde{\Delta}_{1\lambda_2\lambda_1}(E_l) \tag{3.31a}$$

$$=-\tilde{Z}_{2\lambda_2\lambda_1}(E_l)\Delta_{1\lambda_2\lambda_1} . \tag{3.31b}$$

We now see that $\tilde{\Delta}_{1\lambda_2\lambda_1}(E_l)$, the LT of Eq. (3.26), is given in a closed-form expansion while $\tilde{Z}_{2\lambda_2\lambda_1}(E_l)$, the LT of Eq. (3.28), can be given in a continued-fraction manner via the successive projection operators onto the $f_2, f_3, f_4, \ldots$ axes as follows.

In order to obtain the general form for $\tilde{Z}_{j\lambda_2\lambda_1}(E_l)$ we define the projection operators P_j and P_j' onto the f_j axis as

$$P_jX=(X_{\lambda_2\lambda_1}/f_{j\lambda_2\lambda_1})f_j , \tag{3.32}$$

$$P_j'=1-P_j . \tag{3.33}$$

Thus we have

$$\begin{aligned} f_j(t|h_T)&=iL_T\prod_{m=0}^{j-2}P_m'f_j'(t|h_T)/\hbar \\ &=P_jf_j(t|h_T)+P_j'f_j(t|h_T) \\ &=Z_{j\lambda_2\lambda_1}(t|h_T)f_j \\ &\quad+\int_0^t Z_{j\lambda_2\lambda_1}(t_1|h_T)f_{j+1}'(t-t_1|h_T)dt_1 , \end{aligned} \tag{3.34}$$

where the notation $\prod_{m=0}^{j-2}P_m'$ means $P_0'P_1'P_2'\cdots P_{j-2}'$ and

$$Z_{j\lambda_2\lambda_1}(t|h_T)=f_{j\lambda_2\lambda_1}(t|h_T)/f_{j\lambda_2\lambda_1} , \tag{3.35}$$

$$f_{j+1}'(t|h_T)\equiv\exp(iL_{j+1}t/\hbar)f_{j+1}' , \tag{3.36}$$

$$f_{j+1}'\equiv iL_{j+1}f_j/\hbar , \tag{3.37}$$

$$L_{j+1}\equiv P_j'L_T\prod_{m=0}^{j-1}P_m' . \tag{3.38}$$

Then the time derivative of $Z_{j\lambda_2\lambda_1}(t|h_T)$ leads to

$$\frac{d}{dt}Z_{j\lambda_2\lambda_1}(t|h_T)=i\omega_{j\lambda_2\lambda_1}Z_{j\lambda_2\lambda_1}(t|h_T)+\int_0^t dt_1\Delta_{j\lambda_2\lambda_1}(t-t_1|h_T)Z_{j\lambda_2\lambda_1}(t_1|h_T) \tag{3.39a}$$

$$=i\omega_{j\lambda_2\lambda_1}Z_{j\lambda_2\lambda_1}(t|h_T)+\int_0^t dt_1 Z_{j+1\lambda_2\lambda_1}(t-t_1|h_T)\Delta_{j\lambda_2\lambda_1}Z_{j\lambda_2\lambda_1}(t_1|h_T) , \tag{3.39b}$$

where

$$\omega_{j\lambda_2\lambda_1}\equiv\left[L_T\prod_{m=0}^{j-1}P_m'f_j/\hbar\right]_{\lambda_2\lambda_1}\bigg/f_{j\lambda_2\lambda_1} , \tag{3.40}$$

$$\begin{aligned}\Delta_{j\lambda_2\lambda_1}(t|h_T)&\equiv f_{J+1\lambda_2\lambda_1}(t|h_T)/f_{j\lambda_2\lambda_1} \\ &\equiv Z_{j+1\lambda_2\lambda_1}(t|h_T)\Delta_{j\lambda_2\lambda_1} ,\end{aligned} \tag{3.41}$$

$$f_{j+1}(t|h_T)=iL_T\prod_{m=0}^{j-1}P_m'f_{j+1}'(t|h_T)/\hbar , \tag{3.42}$$

$$\Delta_{j\lambda_2\lambda_1}\equiv f_{j+1\lambda_2\lambda_1}/f_{j\lambda_2\lambda_1} . \tag{3.43}$$

The LT of Eq. (3.39) leads to

$$\tilde{Z}_{j\lambda_2\lambda_1}(E_l)\equiv\tilde{f}_{j\lambda_2\lambda_1}(E_l)/f_{j\lambda_2\lambda_1}$$

$$=[s-i\omega_{j\lambda_2\lambda_1}-\tilde{\Delta}_{j\lambda_2\lambda_1}(E_l)]^{-1} \tag{3.44a}$$

$$=[s-i\omega_{j\lambda_2\lambda_1}-\tilde{Z}_{j+1\lambda_2\lambda_1}(E_l)\Delta_{j\lambda_2\lambda_1}]^{-1} \quad (0\le j\le\infty) , \tag{3.44b}$$

where $\tilde{\Delta}_{j\lambda_2\lambda_1}(E_l)$ and $\tilde{Z}_{j+1\lambda_2\lambda_1}(E_l)$ are the LT of Eqs. (3.41) and (3.43), respectively. Now Eq. (3.44a) is given in a closed-form expansion in the jth continued-fraction representation. By considering Eqs. (3.15), (3.16b), (3.30), (3.31b), and (3.44b), we obtain the general field-dependent self-energy operator given in a continued fraction:

$$\begin{aligned}\tilde{\Sigma}_{0\lambda_2\lambda_1}(E_l)&=\frac{-\Delta_{0\lambda_2\lambda_1}}{s-i\omega_{1\lambda_2\lambda_1}+\tilde{\Sigma}_{1\lambda_2\lambda_1}(E_l)} \\ &=\cfrac{-\Delta_{0\lambda_2\lambda_1}}{s-i\omega_{1\lambda_2\lambda_1}-\cfrac{\Delta_{1\lambda_2\lambda_1}}{s-i\omega_{2\lambda_2\lambda_1}-\cfrac{\Delta_{2\lambda_2\lambda_1}}{s-i\omega_{3\lambda_2\lambda_1}-\cfrac{\Delta_{3\lambda_2\lambda_1}}{s-i\omega_{4\lambda_2\lambda_1}-\ddots}}}} ,\end{aligned} \tag{3.45}$$

where $\Delta_{0\lambda_2\lambda_1}$, $\Delta_{1\lambda_2\lambda_1}, \ldots$ and $\omega_{1\lambda_2\lambda_1}, \omega_{2\lambda_2\lambda_1}, \ldots$ can be easily obtained from Eqs. (3.40) and (3.43). We see that Eq. (3.45) is applicable to the strong-coupling case. Considering Eqs. (2.39), (3.10), and (3.15) we can express the field-dependent static conductivity tensor as

$$\sigma_{kl}(E_l)=\frac{\hbar}{i\Omega}\sum_{1,2}\frac{f(\varepsilon_1)-f(\varepsilon_2)}{\varepsilon_1-\varepsilon_2} \times\frac{j_{l12}j_{k21}}{-i\hbar s-E_2+E_1-\langle i\hbar\tilde{\Sigma}_{021}(E_l)\rangle_B} . \tag{3.46}$$

It should be noted that the $|\lambda_i\rangle(\equiv|i\rangle)$ and the $E_i(i=1,2,3,\ldots)$ are, respectively, the eigenstates and eigenvalues of h_{eE} satisfying Eq. (2.34). The real and imaginary parts of $i\hbar\tilde{\Sigma}_{021}(E_l)$ defined by

$$i\hbar\tilde{\Sigma}_{021}(E_l)\equiv\tilde{\nabla}_{021}(E_l)+i\tilde{\Gamma}_{021}(E_l) \tag{3.47}$$

are the line shift and linewidth, respectively, for the transition between states $|1\rangle$ and $|2\rangle$. Both of these quantities are functions of temperature and the external electric field. Physically the inclusion of the field in the self-energy operator accounts for the intracollisional field effect, i.e., the accelerating effect of the electric field. The self-energy results in the lifetime broadening, which is responsible for the spectral broadening of line shapes. Therefore, the ICFE effect of the interaction in each collision event and the CB effect by scattering are studied theoretically by examining the real part of the conductivity tensor.

IV. EXPLICIT EXPRESSION FOR THE SELF-ENERGY

In this section we shall derive an explicit expression for the field-dependent self-energy for both the weak-coupling and the strong-coupling cases given in Eqs. (3.17) and (3.45), respectively. The central interest in the evaluation of Eqs. (3.17) and (3.45) is averaging over the background (impurity and phonon) configurations.

A. Weak-coupling case

For the second order of the scattering potential in Eq. (3.17) we obtain for the impurity scatterings

$$i\hbar\tilde{\Sigma}_{021}(E_l)=\left\langle\sum_3\left[\frac{|v_{23}|^2}{E_1-E_3-i\hbar s}+\frac{|v_{31}|^2}{E_3-E_2-i\hbar s}\right]\right\rangle_{\rm imp} , \tag{4.1}$$

where we have used the relations

$$\sum_3 (P_0'X)_{31}=\sum_{3\neq 2}X_{31} , \tag{4.2}$$

$$(G_0X)_{ij}=(-i\hbar s-E_i+E_j)^{-1}X_{ij} \tag{4.3}$$

and dropped[11,48] the vertex correction terms involving v_{ii}. Equation (4.1) is good for sufficiently weak scattering, which is identical with that of Suzuki[11] obtained for low-impurity density.

For the phonon scatterings, we have

$$i\hbar\tilde{\Sigma}_{021}(E_l)=(l/j_{k21})\langle\{(L_vG_{op}P_0'L_vj_k)_{21}+[L_vG_{op}P_0'L_vG_{op}P_0'(L_{eE}+L_p)j_k]_{21}\}\rangle_{\rm ph} \tag{4.4}$$

for the second order of the scattering potential, where

$$G_{op}=(-i\hbar s-L_{eE}-L_p)^{-1} . \tag{4.5}$$

We define the phonon state as[49]

$$|q\rangle\equiv|n_{\rm q1},n_{\rm q2},n_{\rm q3},\ldots,n_{\rm qi},\ldots\rangle \tag{4.6}$$

and take into account the following relations:

$$b_{\rm q}|q\rangle=\sqrt{n_{\rm q}}|q-1\rangle , \tag{4.7}$$

$$b_{\rm q}^\dagger|q\rangle=\sqrt{n_{\rm q}+1}|q+1\rangle , \tag{4.8}$$

$$\langle q|q\pm 1\rangle=0 , \tag{4.9}$$

$$\langle q|(G_{op}X)_{ij}|q'\rangle =\frac{\langle q|X_{ij}|q'\rangle}{-i\hbar s-E_i+E_j-\langle q|H_p|q\rangle+\langle q'|H_p|q'\rangle} \tag{4.10}$$

for any operator X. Here the matrix element is given with respect to both the electron states ($|i\rangle,|j\rangle$) and the phonon states ($|q\rangle,|q'\rangle$). Then with the help of Eqs. (4.2) and (4.6)–(4.10) the field-dependent self-energy in Eq. (4.4) leads to

$$i\hbar\tilde{\Sigma}_{021}(E_l)=\sum_{\rm q}\sum_3\left[(1+n_{\rm q})\left[\frac{|\gamma_{\rm q23}|^2}{E_1-E_3-\hbar\omega_{\rm q}-i\hbar s}+\frac{|\gamma_{\rm q31}^{+}|^2}{E_3-E_2+\hbar\omega_{\rm q}-i\hbar s}\right] +n_{\rm q}\left[\frac{|\gamma_{\rm q23}^{+}|^2}{E_1-E_3+\hbar\omega_{\rm q}-i\hbar s}+\frac{|\gamma_{\rm q31}|^2}{E_3-E_2-\hbar\omega_{\rm q}-i\hbar s}\right]\right] , \tag{4.11}$$

where $n_{\mathbf{q}}=[\exp(\beta\hbar\omega_{\mathbf{q}})-1]^{-1}$ is the phonon distribution function and the vertex correction terms involving $\gamma_{\mathbf{q}ii}$ have been dropped.[11,48] It should be noted that the matrix elements of the interaction term in Eqs. (4.1) and (4.11) depend generally on the electric field, and the field dependence on the electronic transition rates is introduced through the electron states and the energy denominators. Equation (4.11) is identical to that of Suzuki[11] and Ryu *et al.*[35]

B. Strong-coupling case

By considering Eq. (3.45) given in a continued-fraction manner we can obtain the general formula for the strong-coupling case. In order to obtain the field-dependent self-energy $i\hbar\tilde{\Sigma}_{021}(E_l)$, we must evaluate the quantities Δ_{021} and ω_{121} given in Eqs. (3.14) and (3.25), respectively:

$$\Delta_{021}\equiv f_{121}/j_{k21}=-(L_TP_0'L_Tj_k/\hbar^2)_{21}/jk_{21} , \quad (4.12)$$

$$\omega_{121}\equiv(L_TP_0'f_1/\hbar)_{21}/f_{121} = -(L_TP_0'L_TP_0'L_Tj_k/\hbar^3)_{21}/f_{121} . \quad (4.13)$$

These quantities are contained in Eq. (3.45) and should be averaged over the background (impurity and phonon) configurations. The simplest way to do so is to average the numerator and the denominator separately in approximation.

For the impurity scatterings we have

$$\Delta_{021}=-(A'+B')/\hbar^2 , \quad (4.14)$$

$$\omega_{121}=\frac{(C'+D')}{\hbar(A'+B')} , \quad (4.15)$$

where we have used Eq. (4.2) and utilized the fact that any terms including an odd number of v disappear in the impurity average, and

$$A'\equiv\sum_3 v_{23}v_{32} , \quad (4.16)$$

$$B'\equiv\sum_3 v_{13}v_{31} , \quad (4.17)$$

$$C'\equiv\sum_3 (E_3-E_1)v_{23}v_{32} , \quad (4.18)$$

$$D'\equiv\sum_3 (E_2-E_3)v_{13}v_{31} . \quad (4.19)$$

Here we have dropped[11,48] the vertex correction terms involving v_{ii}. Then we obtain from Eqs. (3.45), (4.14), and (4.15)

$$i\hbar\tilde{\Sigma}_{021}(E_l)=\left\langle\sum_3\left[\frac{|v_{23}|^2}{E_1-E_3-i\hbar s+\theta_{il}-i\hbar\tilde{\Sigma}_{121}(E_l)}+\frac{|v_{31}|^2}{E_3-E_2-i\hbar s+\theta_{i2}-i\hbar\tilde{\Sigma}_{121}(E_l)}\right]\right\rangle_{\text{imp}} , \quad (4.20)$$

where $i\hbar\tilde{\Sigma}_{121}(E_l)$ in the denominator is the field-dependent high-order term given in Eq. (3.45) and

$$\theta_{il}=\frac{\{[(E_3-E_1)A'-C']+[(E_3-E_1)B'-D']\}}{(A'+B')} , \quad (4.21)$$

$$\theta_{i2}=\frac{\{[(E_2-E_3)A'-C']+[(E_2-E_3)B'-D']\}}{(A'+B')} . \quad (4.22)$$

Equation (4.20) is the general formula for the strongly interacting electron-impurity scattering case which is similar to Sawaki's expression[36] based on the Stark-ladder-representation approach. If the quantities θ_{i1}, θ_{i2}, and the high-order self-energy $i\hbar\tilde{\Sigma}_{121}(E_l)$ are neglected, Eq. (4.20) is reduced to Eq. (4.1) obtained up to the second-order terms for the weak-coupling case.

For the phonon scatterings we have from Eqs. (4.12) and (4.13)

$$\Delta_{021}=-(A+B+C+D)/\hbar^2 , \quad (4.23)$$

$$\omega_{121}=\frac{(F+G+H+I)}{\hbar(A+B+C+D)} , \quad (4.24)$$

where

$$A\equiv\sum_{\mathbf{q}}\sum_3(1+n_{\mathbf{q}})\gamma_{\mathbf{q}23}\gamma^+_{\mathbf{q}32} , \quad (4.25)$$

$$B\equiv\sum_{\mathbf{q}}\sum_3 n_{\mathbf{q}}\gamma^+_{\mathbf{q}23}\gamma_{\mathbf{q}32} , \quad (4.26)$$

$$C\equiv\sum_{\mathbf{q}}\sum_3 n_{\mathbf{q}}\gamma^+_{\mathbf{q}13}\gamma_{\mathbf{q}31} , \quad (4.27)$$

$$D\equiv\sum_{\mathbf{q}}\sum_3(1+n_{\mathbf{q}})\gamma_{\mathbf{q}13}\gamma^+_{\mathbf{q}31} , \quad (4.28)$$

$$F\equiv\sum_{\mathbf{q}}\sum_3(1+n_{\mathbf{q}})\gamma_{\mathbf{q}23}\gamma^+_{\mathbf{q}32}(E_3-E_1+\hbar\omega_{\mathbf{q}}) , \quad (4.29)$$

$$G\equiv\sum_{\mathbf{q}}\sum_3 n_{\mathbf{q}}\gamma^+_{\mathbf{q}23}\gamma_{\mathbf{q}32}(E_3-E_1-\hbar\omega_{\mathbf{q}}) , \quad (4.30)$$

$$H\equiv\sum_{\mathbf{q}}\sum_3 n_{\mathbf{q}}\gamma^+_{\mathbf{q}13}\gamma_{\mathbf{q}31}(E_2-E_3+\hbar\omega_{\mathbf{q}}) , \quad (4.31)$$

$$I\equiv\sum_{\mathbf{q}}\sum_3(1+n_{\mathbf{q}})\gamma_{\mathbf{q}13}\gamma^+_{\mathbf{q}31}(E_2-E_3-\hbar\omega_{\mathbf{q}}) . \quad (4.32)$$

Here we have used Eqs. (4.2) and (4.6)–(4.9) and dropped[11,48] the vertex correction terms involving $\gamma_{\mathbf{q}ii}$ Then considering Eqs. (3.45), (4.23), and (4.24) we obtain

$$i\hbar\tilde{\Sigma}_{021}(E_l)=\sum_{\mathbf{q}}\sum_{3}\left[(1+n_{\mathbf{q}})\left[\frac{|\gamma_{\mathbf{q}23}|^2}{E_1-E_3-\hbar\omega_{\mathbf{q}}-i\hbar s+\theta_{p1}(\hbar\omega_{\mathbf{q}})-i\hbar\tilde{\Sigma}_{121}(E_l)}+\frac{|\gamma^{+}_{\mathbf{q}31}|^2}{E_3-E_2+\hbar\omega_{\mathbf{q}}-i\hbar s+\theta_{p2}(\hbar\omega_{\mathbf{q}})-i\hbar\tilde{\Sigma}_{121}(E_l)}\right]\right.$$
$$\left.+n_{\mathbf{q}}\left[\frac{|\gamma^{+}_{\mathbf{q}23}|^2}{E_1-E_3+\hbar\omega_{\mathbf{q}}-i\hbar s+\theta_{p3}(\hbar\omega_{\mathbf{q}})-i\hbar\tilde{\Sigma}_{121}(E_l)}+\frac{|\gamma_{\mathbf{q}31}|^2}{E_3-E_2-\hbar\omega_{\mathbf{q}}-i\hbar s+\theta_{p4}(\hbar\omega_{\mathbf{q}})-i\hbar\tilde{\Sigma}_{121}(E_l)}\right]\right], \tag{4.33}$$

where $i\hbar\tilde{\Sigma}_{121}(E_l)$ in the denominator is the field-dependent high-order term given in Eq. (3.45) and

$$\theta_{p1}(\hbar\omega_{\mathbf{q}})=\{[(E_3-E_1+\hbar\omega_{\mathbf{q}})A-F]+[(E_3-E_1+\hbar\omega_{\mathbf{q}})B-G]$$
$$+[(E_3-E_1+\hbar\omega_{\mathbf{q}})C-H]+[(E_3-E_1+\hbar\omega_{\mathbf{q}})D-I]\}/(A+B+C+D)\,, \tag{4.34}$$

$$\theta_{p2}(\hbar\omega_{\mathbf{q}})=\{[(E_2-E_3-\hbar\omega_{\mathbf{q}})A-F]+[(E_2-E_3-\hbar\omega_{\mathbf{q}})B-G]$$
$$+[(E_2-E_3-\hbar\omega_{\mathbf{q}})C-H]+[(E_2-E_3-\hbar\omega_{\mathbf{q}})D-I]\}/(A+B+C+D)\,, \tag{4.35}$$

$$\theta_{p3}(\hbar\omega_{\mathbf{q}})=\theta_{p1}(-\hbar\omega_{\mathbf{q}})\,, \tag{4.36}$$

$$\theta_{p4}(\hbar\omega_{\mathbf{q}})=\theta_{p2}(-\hbar\omega_{\mathbf{q}})\,. \tag{4.37}$$

If $\theta_{p1}(\hbar\omega_{\mathbf{q}})$, $\theta_{p2}(\hbar\omega_{\mathbf{q}})$, $\theta_{p3}(\hbar\omega_{\mathbf{q}})$, $\theta_{p4}(\hbar\omega_{\mathbf{q}})$, and $i\hbar\tilde{\Sigma}_{121}(E_l)$ are neglected, Eq. (4.33) is reduced to Eq. (4.11) for the weak-coupling case. Equation (4.33) is the general formula for the phonon scattering which is similar to that of Sawaki.[36] Real and imaginary parts of Eqs. (4.1), (4.11), (4.20), and (4.33) give the line shift and linewidth, respectively. By using Eqs. (3.46) and (3.47), the steady-state nonlinear conductivity is given by

$$\mathrm{Re}\{\sigma_{kl}(E_l)\}=(\hbar/\Omega)\sum_{1,2}j_{l12}j_{k21}\frac{f(\varepsilon_1)-f(\varepsilon_2)}{\varepsilon_1-\varepsilon_2}\frac{\tilde{\Gamma}_{021}(E_l)}{[E_1-E_2-\tilde{\nabla}_{021}(E_l)]^2+\tilde{\Gamma}^2_{021}(E_l)}\,, \tag{4.38}$$

where Re means "the real part of." $\tilde{\Gamma}_{021}(E_l)$ and $\tilde{\nabla}_{021}(E_l)$, respectively, can be calculated from Eqs. (4.1) and (4.11) for a weak-coupling case or Eqs. (4.20) and (4.33) for a strong-coupling case as follows:

$$\tilde{\Gamma}_{021}(E_l)\equiv\mathrm{Im}[i\hbar\tilde{\Sigma}_{021}(E_l)]=[\tilde{\Gamma}_{021}(E_l)]_{\mathrm{imp}}+[\tilde{\Gamma}_{021}(E_l)]_{\mathrm{ph}}$$
$$=\pi\sum_{3}\langle[|v_{23}|^2\delta(E_1-E_3)+|v_{31}|^2\delta(E_3-E_2)]\rangle_{\mathrm{imp}}$$
$$+\pi\sum_{\mathbf{q}}\sum_{3}\{(n_{\mathbf{q}}+1)[|\gamma_{\mathbf{q}23}|^2\delta(E_1-E_3-\hbar\omega_{\mathbf{q}})+|\gamma^{+}_{\mathbf{q}31}|^2\delta(E_3-E_2+\hbar\omega_{\mathbf{q}})]$$
$$+n_{\mathbf{q}}[|\gamma^{\dagger}_{\mathbf{q}23}|^2\delta(E_1-E_3+\hbar\omega_{\mathbf{q}})+|\gamma_{\mathbf{q}31}|^2\delta(E_3-E_2-\hbar\omega_{\mathbf{q}})]\}\,, \tag{4.39}$$

$$\tilde{\nabla}_{021}(E_l)\equiv\mathrm{Re}\{i\hbar\tilde{\Sigma}_{021}(E_l)\}=[\tilde{\nabla}_{021}(E_l)]_{\mathrm{imp}}+[\tilde{\nabla}_{021}(E_l)]_{\mathrm{ph}}$$
$$=\sum_{3}\left\langle\left[|v_{23}|^2P\left[\frac{1}{E_1-E_3}\right]+|v_{31}|^2P\left[\frac{1}{E_3-E_2}\right]\right]\right\rangle_{\mathrm{imp}}$$
$$+\sum_{\mathbf{q}}\sum_{3}\left\{(n_{\mathbf{q}}+1)\left[|\gamma_{\mathbf{q}23}|^2P\left[\frac{1}{E_1-E_3-\hbar\omega_{\mathbf{q}}}\right]+|\gamma^{+}_{\mathbf{q}31}|^2P\left[\frac{1}{E_3-E_2+\hbar\omega_{\mathbf{q}}}\right]\right]\right.$$
$$\left.+n_{\mathbf{q}}\left[|\gamma^{+}_{\mathbf{q}23}|^2P\left[\frac{1}{E_1-E_2+\hbar\omega_{\mathbf{q}}}\right]+|\gamma_{\mathbf{q}31}|^2P\left[\frac{1}{E_3-E_2-\hbar\omega_{\mathbf{q}}}\right]\right]\right\} \tag{4.40}$$

for weak coupling and

$$\begin{aligned}\tilde{\Gamma}_{021}(E_l)&=[\tilde{\Gamma}_{021}(E_l)]_{\rm imp}+[\tilde{\Gamma}_{021}(E_l)]_{\rm ph}\\&=\sum_3\left[\left\langle\frac{|v_{23}|^2\tilde{\Gamma}_{121}(E_l)}{[E_1-E_3+\theta_{i1}-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right.\right.\\&\qquad\left.\left.+\frac{|v_{31}|^2\tilde{\Gamma}_{121}(E_l)}{[E_3-E_2+\theta_{i2}-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right\rangle_{\rm imp}\right]\\&\quad+\sum_{\rm q}\sum_3\left[(n_{\rm q}+1)\left[\frac{|\gamma_{\rm q23}|^2\tilde{\Gamma}_{121}(E_l)}{[E_1-E_3-\hbar\omega_{\rm q}+\theta_{p1}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right.\right.\\&\qquad\left.+\frac{|\gamma_{\rm q31}^{+}|^2\tilde{\Gamma}_{121}(E_l)}{[E_3-E_2+\hbar\omega_{\rm q}+\theta_{p2}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right]\\&\qquad+n_{\rm q}\left[\frac{|\gamma_{\rm q23}^{+}|^2\tilde{\Gamma}_{121}(E_l)}{[E_1-E_3+\hbar\omega_{\rm q}+\theta_{p3}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right.\\&\qquad\left.\left.+\frac{|\gamma_{\rm q31}|^2\tilde{\Gamma}_{121}(E_l)}{[E_3-E_2-\hbar\omega_{\rm q}+\theta_{p4}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right]\right],\end{aligned}\tag{4.41}$$

$$\begin{aligned}\tilde{\nabla}_{021}(E_l)&=[\tilde{\nabla}_{021}(E_l)]_{\rm imp}+[\tilde{\nabla}_{021}(E_l)]_{\rm ph}\\&=\sum_3\left[\left\langle\frac{|v_{23}|^2[E_1-E_3+\theta_{i1}-\tilde{\nabla}_{121}(E_l)]}{[E_1-E_3+\theta_{i1}-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right.\right.\\&\qquad\left.\left.+\frac{|v_{31}|^2[E_3-E_2+\theta_{i2}-\tilde{\nabla}_{121}(E_l)]}{[E_3-E_2+\theta_{i2}-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right\rangle_{\rm imp}\right]\\&\quad+\sum_{\rm q}\sum_3\left[(n_{\rm q}+1)\left[\frac{|\gamma_{\rm q23}|^2[E_1-E_3-\hbar\omega_{\rm q}+\theta_{p1}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]}{[E_1-E_3-\hbar\omega_{\rm q}+\theta_{p1}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right.\right.\\&\qquad\left.+\frac{|\gamma_{\rm q31}^{+}|^2[E_3-E_2+\hbar\omega_{\rm q}+\theta_{p2}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]}{[E_3-E_2+\hbar\omega_{\rm q}+\theta_{p2}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right]\\&\qquad+n_{\rm q}\left[\frac{|\gamma_{\rm q23}|^2[E_1-E_3+\hbar\omega_{\rm q}+\theta_{p3}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]}{[E_1-E_3+\hbar\omega_{\rm q}+\theta_{p3}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right.\\&\qquad\left.\left.+\frac{|\gamma_{\rm q31}|^2[E_3-E_2-\hbar\omega_{\rm q}+\theta_{p4}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]}{[E_3-E_2-\hbar\omega_{\rm q}+\theta_{p4}(\hbar\omega_{\rm q})-\tilde{\nabla}_{121}(E_l)]^2+\tilde{\Gamma}_{121}^2(E_l)}\right]\right]\end{aligned}\tag{4.42}$$

for strong coupling, where $\tilde{\Gamma}_{121}(E_l)$ and $\tilde{\nabla}_{121}(E_l)$ in Eq. (4.42), respectively, are the real part and imaginary part of the high-order self-energy $[-i\hbar\tilde{\Sigma}_{121}(E_l)]$ in Eq. (3.45). To obtain Eqs. (4.39) and (4.40), we have used the Dirac identity

$$\lim_{s\to 0^+}(x\pm is)^{-1}=P(1/x)\mp i\pi\delta(x)\ ,\tag{4.43}$$

where P denotes Cauchy's principle-value integral. The symbols Re and Im in Eqs. (4.39) and (4.40) denote, respectively, the real and the imaginary parts of the quantity. We see that the field-dependent conductivity in Eq. (4.38) exhibits a Lorentz-like line shape. The quantities $\tilde{\nabla}_{021}(E_l)$ and $\tilde{\Gamma}_{021}(E_l)$ play the role of the line shift and the half-width, respectively. $\tilde{\Gamma}_{021}(E_l)$ gives the reciprocal of the relaxation time. It should be noted that both of these quantities are functions of temperature and the external electric field.

V. CONCLUSION

We have derived expressions of the nonlinear dc conductivity and the field-dependent self-energies for the systems of electrons in interaction with impurities and phonons. For the derivation of nonlinear dc conductivity tensor, Tani's theory[42] of nonlinear response has been utilized.

The perturbation has been dealt with by the two techniques based on the Mori-type method[45] of calculation. One is a closed-form representation which is applicable to the weak scattering case, the other is a continued-fraction form representation which is applicable to the strong scattering case. The results obtained for the weak electron-background coupling case are identical to those of Suzuki[11] based on the resolvent superoperator approach and of Ryu *et al.*[35] based on Argyres-Sigel's projection method,[50] and the results for the strong-coupling case are similar to the expression of Sawaki[36] based on the Stark-ladder-representation method.

There are several important issues under continuous study, including electron-electron interactions, spin-orbit coupling effects, the multiband model, and hot-electron effects. All these works are left for future studies.

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