Current-voltage instabilities in superlattices

B. Laikhtman*
Department of Nuclear Physics, The Weizmann Institute of Science, Rehovot 76100, Israel
(Received 24 April 1991)

A simple model of a series of circuits comprising a nonlinear resistor and a capacitor in parallel is suggested for a description of *I-V*-characteristic instabilities and the formation of high-field domains in superlattices. The model is justified if subbands in a superlattice are destroyed by a high field or scattering and the transport is sequential tunneling. In the case of a wide second subband, the superlattice breaks down in two parts. In the first, the transport is sequential tunneling, but in the second part, electrons propagate in the second subband and the model breaks down there. Simple physical arguments show that the *I-V* characteristics of such a superlattice has to have an S shape, which leads to oscillations and more complicated nonstationary phenomena.

I. INTRODUCTION

Since the pioneering paper of Esaki and Chang,[1] it is well known that if a semiconductor-insulator superlattice is placed in an electric field so high that the average electric potential drop per one period of the superlattice is larger than the width of the first subband, the uniform potential distribution becomes unstable and a high-field domain is formed. The instability and formation of such a domain are attributed to a negative conductance inherent to a semiconductor with narrow bands like that realized in a superlattice.[2–4] A simple phenomenological theory predicts the formation of high-voltage domains propagating with electronic drift velocity in the case of N-shaped *I-V* characteristics and high current domains in the case of S-shaped *I-V* characteristics.[5,6] Usually the origin of negative differential resistance is ascribed to some features of the band structure. This approach presents some problems when applied to the formation of high-voltage domains in superlattices. The main difficulty is that the band structure that accounts for negative differential conductivity appears to be destroyed in the region of the domain. The domain is usually thought of as such a large voltage drop across one of the barriers in the superlattice that it surpasses the width of the first subband and can be equal to or of the order of the gap between the first and the second subbands.[1,7,8] Another feature that prevents a direct analogy between a continuous medium with negative differential resistance and a superlattice is that a high-field domain in a continuous medium drifts while no drift of such a domain has been detected in superlattices.

In the present paper the formation of high-field domains in a superlattice with sequential tunneling is explained with a simple phenomenological model. This model allows one to see clearly the physical reason for the N shape of the *I-V* characteristics and explains the bistability and hysteresis detected in a long-enough superlattice. The model is not adequate if the superlattice is designed in such a way that the transport in the second subband is coherent rather than sequential tunneling. Simple arguments show that in this case one can expect an S-shaped *I-V* characteristics. The theory predicts some nonstationary phenomena.

In Sec. II the stability and the shape of the *I-V* characteristics in the case of sequential tunneling is analyzed. In Sec. III superlattices with a wide second subband, where tunneling is coherent, are considered.

II. N-SHAPED *I-V* CHARACTERISTICS FOR SEQUENTIAL TUNNELING

The simplest way to calculate the *I-V* characteristics of a superlattice is to model it by a series of N identical circuits consisting of a nonlinear resistor and a capacitor in parallel (Fig. 1). It is necessary to make a few remarks about the justification of such a model. The first one concerns the possibility of modeling a superlattice by a series of circuits. Such a model is justified when the transfer of an electron across the superlattice can be considered as a sequence of incoherent tunneling events; instead of subbands one has to consider electron levels in separate wells of the superlattice. This is the case if either the electric potential drop over one superlattice period or the energy uncertainty due to scattering, $\hbar/\tau$, exceeds the width of the first subband[4,8–11] (here τ is the relaxation time). In the first case the electric field destroys the band structure across the superlattice, and the transfer of electrons across the barrier has to be accompanied by scattering. The coherence of the tunneling is not recovered when a high potential drop per period leads to a resonance between the first level on one side of a barrier and the second level on the other side. Levels in the wells between the barriers are not equidistant and there is no resonance between the second and the third levels on different sides of a barrier if there is one between the first

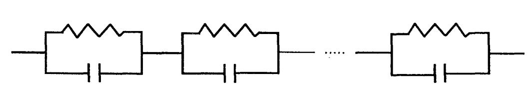

FIG. 1. Equivalent circuit for a superlattice in the case of sequential tunneling.

and the second levels. This means that after the tunneling from the first level to the second level an electron has to relax down to the first level emitting a phonon or photon before the next tunneling. A high potential drop in the region of a high-field domain can provide a resonance between the first and the second levels across only one of the barriers. The coherence of tunneling in the second subband, where electrons come after this barrier, is assumed to be destroyed by scattering (another case is considered in Sec. III).

The case of a short relaxation time implies that scattering is so strong that it destroys the coherence of tunneling events even for a low electric field. It is important that the uncertainty of the energy of the motion across the superlattice arises not only because of inelastic scattering but also due to the elastic scattering since the latter can transfer the energy between different directions of motion.

The second remark is related to the specific choice of the equivalent circuit for one period of superlattice. A physical reason for this choice is very clear. A tunnel junction presents a resistance for a dc current and if tunneling can be neglected the junction is equivalent to a capacitor. The *I-V* characteristics of the resistor (Fig. 2) is identical to that for a single tunnel junction separating two 2D electron gases. Equivalent circuits consisting of resistors and capacitors has been used for tunnel diodes.[12–18] Although theoretical evaluations of the conductance frequency dependence for a tunnel diode (see, e.g., Refs. 19 and 20) show a more complicated behavior than that of two cells consisting of a resistor and a capacitor in parallel, this may result from the fact that these authors were interested mainly in the case of coherent but not sequential tunneling. Here the simple phenomenological model is used for superlattices for a few reasons. The main interest of the present paper is in results concerning possible regimes and their stability for which thin details of dynamics due to the vicinity of the resonance do not make a difference. Also the model is simple enough to allow one to study not only linear but also nonlinear dynamics of the superlattice.

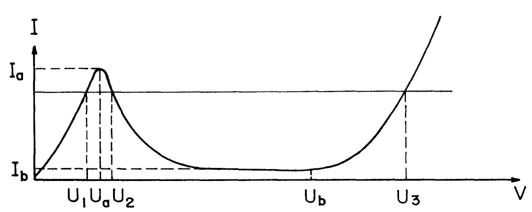

FIG. 2. Typical *I-V* characteristics of a superlattice cell in the case of sequential tunneling. The peak corresponds to the resonance between the first levels in the wells on both sides of a barrier. The sharp increase of the current at high voltage corresponds to the resonance between the first level on one side of the barrier and the second level on the other side. I_a and U_a are the current and voltage in the maximum of the first peak, and I_b and U_b are the current and voltage in the minimum between the resonances. The values U_1, U_2, and U_3 show possible voltage drops across the barrier for a given current across the superlattice.

The last remark concerns the first and the last (in the direction of the electron transport) barriers, which play a specific role in a superlattice. The resistance of the first barrier is nonlinear due to the specific role of conservation laws in the tunneling between 3D and 2D electron gases.[12] The scale of this nonlinearity is different from that of resonant tunneling and in most cases the first barrier does not change the behavior of the superlattice. The resistance of the last barrier is linear if the voltage drop across it is small compared to the height of the barrier. For these reasons it is suitable to consider the first and the last barriers not as parts of the superlattice but as resistors belonging to an external circuit.

For the study of the stability of the superlattice the following features of the *I-V* characteristics in Fig. 2 are important. The *I-V* characteristics has two peaks corresponding to the resonant tunneling from the first subband of the two-dimensional electron gas (2DEG) on one side of the barrier to the first and the second subbands of the 2DEG on the other side. The second peak is higher than the first one and only a part of it is shown in Fig. 2. The values of the voltage and current at the maximum of the first peak are U_a and I_a, and at the minimum between the first and the second peaks are U_b and I_b, respectively. The dynamics of the circuit shown in Fig. 1 is described by the equations

$$C\frac{dV_n}{dt}+J(V_n)=I, \quad n=1,2,\ldots,N \ , \tag{1}$$

$$\sum_{n=1}^{n=N} V_n = V \ , \tag{2}$$

where V_n is the voltage drop across the nth barrier. Stationary states of the superlattice can easily be found if one draws a line $I=\text{const}$ on the plot showing the *I-V* characteristics of one cell (Fig. 2). The crossings of this line with the *I-V* characteristics, U_1, U_2, and U_3 ($U_1<U_a<U_2<U_b<U_3$) show possible voltage drops across one barrier for a given current. Until the total voltage drop $V<NU_a$, all the cells are the regime corresponding to the value U_1. This regime becomes unstable when V surpasses the value NU_a. In this case the voltage drop on one or a few barriers takes the value U_2 or even U_3 provided that

$$(N-1)U_1(I_b)+U_b \le NU_a \ , \tag{3}$$

where $U_1(I_b)$ is the value of U_1 corresponding to the current I_b. The most important criterion for the choice among these regimes is their stability. For the stability study, Eq. (1) has to be linearized by a substitution $V_n=U_n+u_n$, where $U_n=U_1$, U_2, or U_3, u_n is small, and $I=J(U_n)+\delta I$. After linearization, Eqs. (1) and (2) take the form

$$C\frac{du_n}{dt}+J'(U_n)u_n=\delta I \ , \tag{4}$$

$$\sum_{n=1}^{n=N} u_n = 0 \ . \tag{5}$$

Here, $J'(U_n)$ is the derivative of $J(U_n)$ with respect to U_n. The solution of Eqs. (4) and (5) is proportional to $e^{\lambda t}$ where λ satisfies the equation

$$a_1 a_2 \cdots a_N \left[\frac{1}{a_1} + \cdots + \frac{1}{a_N} \right] = 0 , \tag{6}$$

and $a_n = C\lambda + J'(U_n)$.

First of all, if U_n takes the values U_1 or U_3, only then are all a_n positive for a positive λ. This regime is stable, which goes along with an intuitive assumption that a circuit containing elements with positive differential resistances is stable. For the case when k of the voltage drops U_n take the value U_2 and l others take the value U_3, where $k + l \le N$, Eq. (5) is reduced to

$$\begin{aligned}[][C\lambda + J'(U_1)]^{N-k-l-1}&[C\lambda + J'(U_2)]^{k-1}[C\lambda + J'(U_3)]^{l-1} \\ &\times \{NC^2\lambda^2 + [(k+l)J'(U_1) + (N-k)J'(U_2) + (N-l)J'(U_3)]C\lambda \\ &\quad + (N-k-l)J'(U_2)J'(U_3) + kJ'(U_1)J'(U_3) + lJ'(U_1)J'(U_2)\} = 0 . \end{aligned} \tag{7}$$

If $k > 1$, the second factor in Eq. (7) gives $\lambda > 0$, so that the state is unstable. States with $k = 1$ can sometimes be stable. For instance, if $l = 0$, then the last factor in Eq. (7) gives

$$\begin{aligned} \lambda_1 &= -J'(U_3)/C , \\ \lambda_2 &= -[J'(U_1) + (N-1)J'(U_2)]/NC , \end{aligned} \tag{8}$$

and both values can be negative if $J'(U_1) > (N-1)|J'(U_2)|$. This result shows the simple fact that a circuit formed by a nonlinear resistor with a negative differential resistance in series with another resistor with a large positive differential resistance can be stable. But for a superlattice with a large N such a situation is rather exotic and, in general, a state with negative differential resistance of one of the resistors is unstable. The nature of this instability is related to nonuniform fluctuations. If as a result of such a fluctuation $U_n = U_3$ increases, then the tunneling current across the cell decreases. In a long superlattice the total resistance is large and this deviation does not lead to a substantial change of the total current. That is, the decrease of the tunneling current in the cell is accompanied by an increase of the displacement current, which builds up the charge on the cell capacitance and further increases the voltage. The obtained results support a picture that is usually implied in studies on superlattices. According to this picture only one stationary state, $U_n = V/N$, is possible till $V < NU_a$. When V surpasses this critical value an instability develops. The instability leads to an increase of the voltage drop across one of the barriers while the drops across the others decrease. This process eventually draws the superlattice to a stable state where the differential conductance of all the barriers is positive. Further increase of V leads to the jump of one more U_n to U_3 and so on, which results in an oscillatory behavior of the *I-V* characteristics.[1,21,7,8]

It is important to note that in the model considered here there is no space correlation between different cells of the superlattice and the spatial arrangement of the high-field domains cannot be determined. Actually any correlation is destroyed in sequential incoherent tunneling. I am not aware, as well, of any experimental evidence for a specific arrangement of the barriers with a large voltage drop. That is, in the region $NU_a < V < NU_3(I_a)$ the superlattice can contain a few short high-field domains instead of a long one. Without correlation, the arrangement of the domains depends either on the initial stage of instability development and is random, or on features of the specific structure where the barriers may not be identical. Any correlation of the domains in a superlattice with identical barriers (with the accuracy of the order of electron energy uncertainty) can result only from the coherence of the transport in the first or higher subbands.

In the same framework model it is possible to give a simple explanation of the bistability and hysteresis observed in superlattices by Vuong, Tsui, and Tsang.[8] When the voltage increases, the threshold for the instability of the state with k of the voltage drops U_n equals U_3 and other $N-k$ of U_n equals U_1 occurs at $V = (N-k)U_a + kU_3(I_a)$. As a result of the instability, the voltage drop in one more cell jumps from U_a to $U_3(I_a)$. If now the voltage goes down, the new state is stable at least until $(N-k-1)U_1(I_b) + (k+1)U_b$. This threshold is lower than the first one by

$$\begin{aligned} \Delta V = (N-k)&[U_a - U_1(I_b)] \\ &+ k[U_3(I_a) - U_b] + U_1(I_b) - U_b . \end{aligned} \tag{9}$$

The difference $U_a - U_1(I_b)$ is of order of the width of the first resonance while the difference $U_3(I_a) - U_b$ is of order of the gap between the first and the second level, i.e., substantially larger. This means that the range of the bistability has to grow with k. Such a feature was detected in the experiment.[8]

This mechanism for bistability is the same as in a nonlinear resistor with an N-shaped *I-V* characteristics, like that in Fig. 2, in series with a linear resistor. The role of the latter is played by the cells of the superlattice, which are in the regime $U_n = U_1$ or U_3.

Essentially the same mechanism of bistability was observed in double-barrier structures.[22-26] A charge buildup used for the explanation of this bistability is equivalent to the charging of the capacitors in two cells. The mechanism of negative differential conductance in this case was shown by Luryi.[12] However, a substantial complication in the modeling of double-barrier structures by a series of circuits like in Fig. 1 is related to the important role of the depletion and accumulation layers, which can lead to additional nonlinearity of the effective resistances and capacitances.

III. THE MIX OF SEQUENTIAL AND COHERENT TUNNELING AND S-SHAPED *I-V* CHARACTERISTIC

The model considered in the preceding section describes the case when the tunneling between the cells of a superlattice is incoherent. Historically the first studied case was the opposite one.[2] *I-V* characteristics is N-shaped in both of these cases. A quite different shape of the *I-V* characteristics can be obtained in the intermediate case. It can be realized if the first subband is narrower than the energy uncertainty due to scattering but the second one is much wider. Even a simple superlattice of square wells separated by square barriers is a rather flexible system with three adjustable parameters (the period, the ratio of the well and barrier widths, and the height of the barrier), which can be chosen to satisfy these conditions. For a small voltage, when the regime is stable, the *I-V* characteristics of this superlattice is qualitatively the same in both cases. When the total voltage surpasses the first threshold, the voltage drop across one of the barriers jumps from U_a to $U_3(I_a)$. Electrons tunneling across this barrier come directly to a wide second subband (Fig. 3). The resistance of the superlattice for the second subband transport is much smaller than for sequential tunneling through the first levels. Therefore the further increase of the voltage eventually leads to the drop of the total resistance of the superlattice which means an S-shaped *I-V* characteristics. The details of its behavior depend on the mean free path of the electrons in the second subband with respect to the relaxation down to the first level. The most effective relaxation mechanism, emission of LO phonons, can be excluded if parameters of a superlattice are chosen in such a way that the gap between the first and the second subband is less than the energy of LO phonons. Then a number of cases is conceivable.

The simplest case appears when the mean free path is longer than the length of the superlattice. When the voltage applied to the superlattice surpasses its threshold value, NU_a, the jump of the voltage on the kth barrier, in the direction of the electron flow, from U_a to U_3 breaks down the superlattice in two parts (Fig. 3). The first consists of the first k barriers, where the electron transport is sequential tunneling and the threshold voltage for the instability of this part is $(k-1)U_a+U_3(I_a)$. The second part consists of the last $N-k$ barriers, where electrons flow in the second subband and the resistance and the voltage drop are very small compared to the first part. If the first threshold voltage for the whole superlattice, NU_a, is larger than the threshold for its first part, $(k-1)U_a+U_3(I_a)$, then after the value $V=NU_a$ is surpassed the first part of the superlattice is also unstable (it breaks down in two smaller parts and so on). This process terminates only when nearly all the applied voltage drops across the first barrier. After tunneling across this barrier, electrons come to the second subband. The resistance of the superlattice is determined by this barrier and is smaller than the resistance of the superlattice just below the threshold because it is smaller than the resistance of the same barrier below the threshold.

If $(k-1)U_a+U_3(I_a)>\mathrm{N}_a$, then the first part of the superlattice is stable above the first threshold. Further increase of V leads to a breakdown of this first part. The instability threshold for a new part with sequential tun-

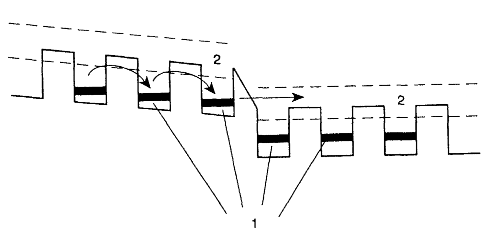

FIG. 3. A high-voltage domain in the case of a wide second subband. Upstream from the high-voltage domain electrons sequentially tunnel through the first levels in the wells (1). After tunneling across the high-voltage domain electrons come to the wide second subband (2).

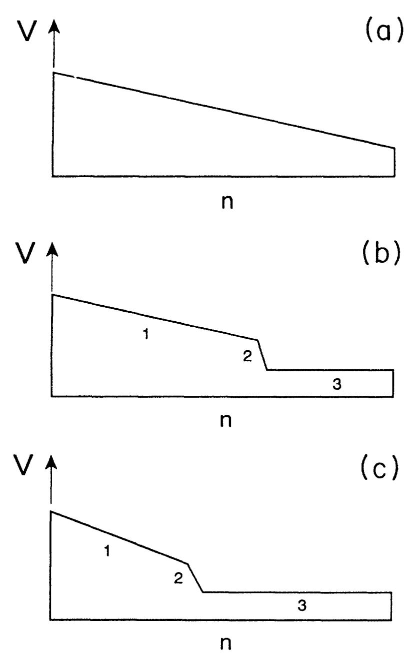

FIG. 4. A voltage distribution in a superlattice in the case of a wide second subband (a) below the first threshold, (b) after the first threshold, and (c) after the second threshold. In the last two cases the superlattice is broken down into the region with the tunneling through the first levels (1), the high-voltage domain (2), and the region with the second subband transport (3).

neling is lower than $(k-1)U_a+U_3(I_a)$ and again the instability develops until the whole applied voltage drops across the first barrier.

The voltage $U_3(I_a)-U_a$ corresponds to the separation between the first level and the bottom of the second subband. Apparently, after the first or the second break $V>U_3(I_a)-U_a$. On the other hand, the regime with the whole voltage drop on the first barrier changes back to the regime below the threshold when this voltage becomes smaller than the separation between the first level and the second subband. This means that there is a hysteresis of the *I-V* characteristics corresponding to its S shape.

It is well known that a circuit consisting of a device with an S-shaped *I-V* characteristics in parallel with a capacitor can be used as a sweep generator. According to the above arguments a superlattice could be used as the main part of such a generator. Generation of high-frequency oscillations was demonstrated on double-barrier structures[13,18,27–33] and can also be obtained in superlattices with an N-shaped *I-V* characteristics. But in those cases a heterostructure device works as an amplifier of one or a few resonator modes. In the case of a superlattice with an S-shaped *I-V* characteristics no resonator is necessary and the frequency of the oscillations depends on the features of the *I-V* characteristics of the superlattice and the external capacitance.

The most complex and, maybe the most interesting case, is that of a superlattice with a wide second subband such that its mean free path is shorter than the length of the superlattice. In such a case when the voltage is high enough to switch electron transport to the second subband a low resistance domain is formed in which the electrons propagate in the second subband. The state with such a domain is unstable because of a high voltage drop across the other part of the superlattice. There is no correlation of tunneling in the direction against the electron motion. But if a new domain starts to be formed not very far from the first one in this direction, that new domain can overlap the first one. As a result the tail of the first domain disappears due to the relaxation of electrons down to the first level, which is equivalent to the shift of the domain (Fig. 4). This shift takes place in the direction opposite to the direction of the electron motion, contrary to the analogous drift of domains in a continuous medium with negative differential conductance.[5] The phenomenon can be much more complicated because of the formation of domains not only upstream from the original one but also downstream. The overlap of two domains can also lead to the transfer of electrons to a third subband. Thus, in this case one can hardly expect regular oscillations but rather a nonstationary regime with a wide frequency spectrum. This nonstationary regime is intrinsic to a superlattice in a sense that it is not necessary to have any external device for its excitation except a voltage source.

IV. SUMMARY

The present work suggests a simple phenomenological model of a superlattice justified in the case of sequential tunneling. The model explains the formation of high-field domains and multistability of *I-V* characteristics. In the case of a wide second subband, the model breaks down for that part of the superlattice where transport takes place in the second subband. Qualitative consideration of this case shows that the *I-V* characteristics has to have an S shape, which leads to oscillations and more complicated nonstationary phenomena.

ACKNOWLEDGMENTS

I appreciate the help of E. Mendez and I. Bar Yosef, who read the manuscript and made many important comments. I also thank M. Heiblum for his hospitality in the Weizmann Institute, where this work was done.

---

*Permanent address: Racah Institute of Physics, The Hebrew University, Jerusalem, Israel.

[1]L. Esaki and L. L. Chang, Phys. Rev. Lett. **33**, 495 (1974).

[2]L. Esaki and R. Tsu, IBM J. Res. Dev. **14**, 61 (1970).

[3]P. A. Lebwohl and R. Tsu, J. Appl. Phys. **41**, 2664 (1970).

[4]R. F. Kasarinov and R. A. Suris, Fiz. Tekh. Poluprovodn. **5**, 797 (1971) [Sov. Phys.—Semicond. **5**, 707 (1971)] **6**, 148 (1972) [**6**, 120 (1972)].

[5]A. F. Volkov and Sh. M. Kogan, Usp. Fiz. Nauk **96**, 633 (1968) [Sov. Phys.–Usp. **11**, 881 (1969)].

[6]H. Kroemer, IEEE Spectrum **5**, 47 (1968).

[7]K. K. Choi, B. F. Levine, R. J. Malik, J. Walker, and C. G. Bethea, Phys. Rev. B **35**, 4172 (1987); B. F. Levine, K. K. Choi, C. G. Bethea, J. Walker, and R. J. Malik, Appl. Phys. Lett. **50**, 1092 (1987); K. K. Choi, B. F. Levine, C. G. Bethea, J. Walker, and R. J. Malik, *ibid.* **50**, 1814 (1987).

[8]T. H. H. Vuong, D. C. Tsui, and W. T. Tsang, Appl. Phys. Lett. **52**, 981 (1988); J. Appl. Phys. **66**, 3688 (1989).

[9]G. H. Döhler, R. Tsu, and L. Esaki, Solid State Commun. **17**, 317 (1975).

[10]R. Tsu and G. Döhler, Phys. Rev. B **12**, 680 (1975).

[11]F. Capasso, K. Mohammed, and A. Y. Cho, IEEE J. Quantum Electron. **QE-22**, 1853 (1986).

[12]S. Luryi, Appl. Phys. Lett. **47**, 490 (1985).

[13]P. D. Coleman, S. Goedeke, T. J. Shewchuk, P. C. Chapin, J. M. Gering, and H. Morkoç, Appl. Phys. Lett. **48**, 422 (1986).

[14]B. Jogai, K. L. Wang, and K. W. Brown, Appl. Phys. Lett. **48**, 1003 (1986).

[15]J. M. Gering, D. A. Crim, D. G. Morgan, P. D. Coleman, W. Kopp, and H. Morkoç, J. Appl. Phys. **61**, 271 (1987).

[16]B. Jogai, K. L. Wang, and K. W. Brown, Appl. Phys. Lett. **48**, 1003 (1986).

[17]W. R. Frensley, Appl. Phys. Lett. **51**, 448 (1987).

[18]T. C. L. G. Sollner, E. R. Brown, W. D. Goodhue, and H. Q. Le, Appl. Phys. Lett. **50**, 332 (1987).

[19]C. Jacobony and P. J. Price, Solid State Commun. **75**, 193 (1990); (unpublished).

[20]K. L. Jensen and F. A. Buot, Phys. Rev. Lett. **66**, 1078 (1991).

[21]Y. Kawamura, K. Wakita, H. Asahi, and K. Kurumada, Jpn. J. Appl. Phys. **25**, L928 (1986).

[22]V. J. Goldman, D. C. Tsui, and J. E. Cunningham, Bull. Am. Phys. Soc. **32**, 833 (1987); Phys. Rev. Lett. **58**, 1256 (1987); **59**, 1623 (1987); T. C. L. G. Sollner, *ibid.* **59**, 1622 (1987).

[23]N. S. Wingreen and J. W. Wilkins, Bull. Am. Phys. Soc. **32**, 833 (1987).

[24]F. W. Sheard and H. A. Toombs, Appl. Phys. Lett. **52**, 1228 (1988).

[25]T. J. Foster, M. L. Leadbeater, L. Eaves, M. Henini, O. H. Hughes, C. A. Payling, F. W. Sheard, P. E. Simmonds, G. A. Toombs, G. Hill, and M. A. Pate, Phys. Rev. B **39**, 6205 (1989).

[26]J. O. Sofo and C. A. Balseiro, Phys. Rev. B **42**, 7292 (1990).

[27]T. C. L. G. Sollner, P. E. Tannenwald, D. D. Peck, and W. D. Goodhue, Appl. Phys. Lett. **45**, 1319 (1984).

[28]T. J. Shewchuk, P. C. Chapin, P. D. Coleman, W. Kopp, R. Fescher, and H. Morkoç, Appl. Phys. Lett. **46**, 508 (1985).

[29]E. R. Brown, T. C. L. G. Sollner, W. D. Goodhue, B. J. Clifton, and P. E. Tannenwald, IEEE Trans. Electron. Dev. **ED-33**, 1864 (1986).

[30]E. R. Brown, T. C. L. G. Sollner, W. D. Goodhue, and C. D. Parker, Appl. Phys. Lett. **50**, 83 (1987).

[31]J. M. Gering, D. A. Crim, D. G. Morgan, P. D. Coleman, W. Kopp, and H. Morkoç, J. Appl. Phys. **61**, 271 (1987).

[32]E. R. Brown, W. D. Goodhue, and T. C. L. G. Sollner, J. Appl. Phys. **64**, 1519 (1988).

[33]C. Kidner, I. Megdi, J. R. East, and G. I. Haddad, IEEE Trans. Microwave Theory Technol. **38**, 864 (1990).