Effect of magnetic fields on exciton binding energies in type-II GaAs-AlAs quantum-well structures

Spiros V. Branis, J. Cen, and K. K. Bajaj
Department of Physics, Emory University, Atlanta, Georgia 30322
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We calculate the binding energies of both the light-hole and heavy-hole excitons in type-II GaAs-AlAs quantum wells in the presence of a magnetic field applied parallel to the axis of growth. Two methods are applied and compared: a variational approach and a perturbation method. The exciton binding energies are calculated assuming infinite potential barriers. The behavior of the exciton binding energies as functions of well widths and the magnetic field is discussed. For a given value of the magnetic field, the exciton binding energies are found to be larger than the zero-magnetic-field case. Results obtained from both methods are compared.

I. INTRODUCTION

Type-II GaAs-AlAs quantum-well (QW) structures have drawn considerable interest, both experimentally and theoretically in recent years.[1–15] For narrow well sizes [typically GaAs-layer thickness < 30 Å (Refs. 1–4)], the band-edge configuration at the GaAs-AlAs heterojunction may become "staggered" or type II. In this case, an electron and a hole are confined in spatially separate wells (see Fig. 1). There have been a few theoretical studies in type-II QW systems. Duggan and Ralph[5] have calculated the exciton binding energies variationally, in the type-II configuration, using infinite potential barriers for both the electrons and the holes. The calculated values are comparable to those in type-I quantum wells. Matsuura and Shinozuka[8] studied the same problem by using a variational approach by incorporating slightly different trial wave functions, and found similar results to those obtained by Duggan and Ralph. Salmassi and Bauer[9] studied the electron-hole exchange interaction in type-II QW's and have also calculated the exciton binding energies by a variational method, assuming finite potential barriers. The nonzero overlap between electron and hole wave functions boosts the exciton binding energies by about 10–15 % more than those obtained by using infinite barriers.[5,8] Degani and Farias[13] have calculated the exciton binding energy in type-II quantum wells in the presence of a static electric field using a variational approach. Recently, Branis and Bajaj[15] have calculated the exciton binding energies by using a perturbation-variational expansion method;[16] the calculated energies are somewhat lower than those obtained by using a variational calculation,[5] similar to the perturbation-variational results in type-I QW's.[16]

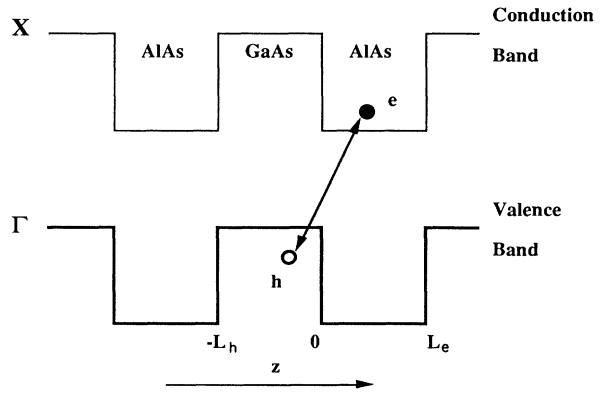

FIG. 1. Schematic energy-band diagram for the electron-hole pair in type-II QW structure.

The application of a magnetic field parallel to the growth direction is expected to modify exciton binding energies in type-II GaAs-AlAs QW's. The magnetic-field factor comes in addition to the effects of the electron-hole confinements in the direction of growth and Coulomb coupling. Recently, Hodge *et al.*[11] have measured the energy of $1s \rightarrow 2p_+$ transition of a heavy-hole exciton in type-II GaAs-AlAs QW's in the presence of a magnetic field using photoinduced far-infrared absorption spectroscopy. Zhang and Bajaj[17] have calculated the exciton binding energies of both the heavy-hole and light-hole excitons for the type-II system in the presence of magnetic field applied parallel to the direction of growth, by using a variational approach, based on a trial exciton wave function, which is a combination of hydrogenic and Gaussian eigenstates in two dimensions.

In this paper, we report a formalism to calculate the binding energies of both the heavy-hole and light-hole excitons in type-II GaAs-AlAs QW's as a function of the size of AlAs layer (or GaAs layer), in the presence of a magnetic field directed along the growth axis. We use two different approaches: a variational method with appropriate trial wave functions[18,19] and a perturbation one. The results from both these methods are compared and are found to agree rather well, for various magnetic-field strengths and GaAs or AlAs well widths. We have assumed infinite potential barriers for both electrons and holes for the sake of illustration.

II. THEORY

The energy band-edge configuration for the type-II system and the corresponding coordinate system are shown

in Fig. 1. The holes are confined in the GaAs layer while the electrons reside in the indirect-band-gap AlAs layer. Assuming perfect confinement for both electrons and holes (infinite-potential-barrier model), we replace the different dielectric constants of GaAs and AlAs materials with their average static dielectric constant $\epsilon_0=12.3$. This eliminates any image-charge corrections. A constant, uniform magnetic field B is applied perpendicular to the layers (in the growth direction). The Hamiltonian for the electron-hole system is[20]

$$H=H_e(-i\hbar\nabla+(e/c)\mathbf{A})-H_h(i\hbar\nabla+(e/c)\mathbf{A}) +V_e(z_e)+V_h(z_h)-\frac{e^2}{\epsilon_0|\mathbf{r}_e-\mathbf{r}_h|}\ , \tag{1}$$

where the potential wells for the electron $V_e(z_e)$ and for the hole $V_h(z_h)$ are assumed to be infinite, and the range of z coordinates is $0\le z_e<L_e$ and $-L_h\le z_h\le 0$; $\mathbf{A}=(\mathbf{r}\times\mathbf{B})/2$ is the vector potential of the magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ and $\mathbf{r}_e$ and $\mathbf{r}_h$ are the electron and hole coordinates. The electron Hamiltonian H_e is described in the effective-mass approximation, while the hole Hamiltonian H_h is the 4×4 Kohn-Luttinger Hamiltonian.[21] We assume parabolic hole bands in the x-y plane and in the z direction and by neglecting the off-diagonal terms, we ignore the coupling between heavy-hole and light-hole bands. Following a standard procedure, the total Hamiltonian of the exciton associated with either the heavy-hole or the light-hole band can be expressed as (in cylindrical polar coordinates)

$$H=-\frac{\hbar^2}{2\mu_\pm}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right]-\frac{\hbar^2}{2(m_e)_l}\frac{\partial^2}{\partial z_e^2} +V_e(z_e)-\frac{\hbar^2}{2(m_\pm)_l}\frac{\partial^2}{\partial z_h^2}+V_h(z_h) -\frac{e^2}{\epsilon_0[\rho^2+(z_e-z_h)]^{1/2}}+\frac{\hbar\omega_c}{2}\hat{L}_z+\frac{\mu_\pm\omega_c^2}{8}\rho^2\ , \tag{2}$$

where $\hat{L}_z$ is the z component of the angular momentum (in units of $\hbar$), $\omega_c=eB/\mu_\pm c$ is the cyclotron frequency, and $(m_e)_l$ and $(m_\pm)_l$ are the effective masses of electron and (heavy or light) holes, respectively, in the direction of the growth [the transverse masses are $(m_e)_t$ and $(m_\pm)_t$]. All the mass values can be expressed in terms of Luttinger parameters[21] γ_1 and γ_2 as in[22]

$$\frac{1}{(m_\pm)_l}=\frac{1}{m_0}(\gamma_1\mp2\gamma_2)\ ,\quad \frac{1}{(m_\pm)_t}=\frac{1}{m_0}(\gamma_1\pm\gamma_2)\ ,\quad \frac{1}{\mu_\pm}=\frac{1}{(m_e)_t}+\frac{1}{(m_\pm)_t}\ , \tag{3}$$

where m_0 is the free-electron mass, and $\mu_\pm$ is the transverse reduced mass of the electron-hole pair in the x-y plane.

We can scale all lengths in terms of the exciton Bohr radius $\alpha_\pm=\epsilon_0\hbar^2/\mu_\pm e^2$ and the energies in terms of Rydberg constant $R_\pm=e^4\mu_\pm/2\epsilon_0^2\hbar^2=e^2/2\epsilon_0\alpha_\pm$ to obtain the dimensionless form of the Hamiltonian in Eq. (2),

$$H=-\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right]-\frac{\mu_\pm}{(m_e)_l}\frac{\partial^2}{\partial z_e^2}+V_e(z_e) -\frac{\mu_\pm}{(m_\pm)_l}\frac{\partial^2}{\partial z_h^2}+V_h(z_h)-\frac{2}{[\rho^2+(z_e-z_h)^2]^{1/2}} +\gamma\hat{L}_z+\frac{\gamma^2}{4}\rho^2\ . \tag{4}$$

The parameter γ is the dimensionless ratio of the half of the cyclotron energy $\hbar\omega_c$ over the Rydberg constant $\mathcal{R}_\pm$:

$$\gamma=\frac{\hbar\omega_c}{2\mathcal{R}_\pm}=\frac{e\hbar B}{2\mu_\pm c\mathcal{R}_\pm}=\frac{\alpha_\pm^2}{\alpha_c^2}\ , \tag{5}$$

where $\alpha_c=(\hbar/\mu_\pm\omega_c)^{1/2}$ is the cyclotron radius.

A. Variational approach

We now calculate the energy of the ground state $1s$ of the Hamiltonian described by Eq. (4) following a variational approach. The Hamiltonian H [see Eq. (2)] can be grouped into three terms, namely, the electron part H_e, the hole part H_h, and the exciton part $H_{\rm exc}(B)$,

$$H=H_e+H_h+H_{\rm exc}(B)\ , \tag{6}$$

where

$$H_e(z_e)=-\frac{\hbar^2}{2(m_e)_l}\frac{\partial^2}{\partial z_e^2}+V_e(z_e)\ , \tag{7a}$$

$$H_h(z_h)=-\frac{\hbar^2}{2(m_h)_l}\frac{\partial^2}{\partial z_h^2}+V_h(z_h)\ , \tag{7b}$$

$$H_{\rm exc}(B)=-\frac{\hbar^2}{2\mu_\pm}\left[\frac{\partial}{\rho\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right] -\frac{e^2}{\epsilon_0[\rho^2+(z_e-z_h)^2]^{1/2}}+\gamma\mathcal{R}_\pm L_z+\frac{\mathcal{R}_\pm\gamma^2}{4\alpha_\pm^2}\rho^2\ . \tag{7c}$$

In the variational approach, we write the total wave function $\psi(\mathbf{r}_e,\mathbf{r}_h)$ in the following form to express the explicit dependence on z_e,z_h and on the relative distance $\mathbf{r}=\mathbf{r}_e-\mathbf{r}_h$:[23,24]

$$\psi(\mathbf{r}_e,\mathbf{r}_h)=F_e(z_e)F_h(z_h)\Phi(\mathbf{r})\ , \tag{8}$$

where $\Phi(\mathbf{r})$ is the wave function describing the internal state of an exciton, $F_e(z_e)$ is the first electron subband wave function, and $F_h(z_h)$ the first hole subband wave function. The two wave functions in the z direction are determined by the following two Schrödinger equations:

$$H_eF_e(z_e)=E_eF_e(z_e)\ , \tag{9a}$$

where

$$E_e=\frac{\hbar^2}{2(m_e)_l}\left[\frac{\pi}{L_e}\right]^2\ ,\quad F_e(z_e)=\left[\frac{2}{L_e}\right]^{1/2}\sin\left[\frac{\pi z_e}{L_e}\right] \tag{9b}$$

and

$$H_h F_h(z_h) = E_h F_h(z_h) , \tag{9c}$$

where

$$E_h = \frac{\hbar^2}{2(m_h)_l}\left[\frac{\pi}{L_h}\right]^2 ,$$
$$F_h(z_h) = \left[\frac{2}{L_h}\right]^{1/2} \sin\left[\frac{\pi z_h}{L_h}\right] . \tag{9d}$$

For narrow potential wells considered here, effects of the coupling between neighboring subbands on exciton binding energies are expected to be small, therefore it is sufficient to assume an exciton to be associated with a single electron subband and a single hole subband.

Next we express the exciton wave function $\Phi(\mathbf{r})$ in terms of Gaussian orbitals and use a variational calculation to determine the expansion parameters and the exciton binding energy

$$\Phi(\rho,\phi;z=z_e-z_h) = \sum_{i=1}^{n} c_i R_i(\rho,\phi)\xi_i(z) , \tag{10}$$

where c_i are the expansion coefficients, $R_i(\rho,\phi)$ and $\xi_i(z)$ are, respectively, the basis functions in the x-y plane and in the z direction,

$$R_i(\rho,\phi) = \frac{e^{im\phi}}{\sqrt{2\pi}}\rho^{|m|}\exp[-(\alpha_i+\beta)\rho^2] \quad (m=0,\pm1,\pm2,\ldots) , \tag{11a}$$

$$\xi_i(z) = \exp[-(\alpha_i'+\delta)z^2] , \tag{11b}$$

where β and δ are the variational parameters and α_i and α_i' $(i=1,\ldots,n)$ are sets of constants that can be determined by expanding hydrogenic wave functions in series of anisotropic Gaussian orbitals.[18,19] The parameters β and δ are varied to adjust the Gaussian basis functions to minimize the total energy E. The Hamiltonian for the exciton corresponds to an anisotropic hydrogen problem, which is reflected in the choice of the trial exciton wave function.

One observes that the system is invariant under rotations about the z axis (magnetic-field axis), i.e., the z component (m) of the total angular momentum is a good quantum number. We then solve for the eigenvalues and eigenfunctions of the Hamiltonian in Eq. (6) using a standard variational approach by minimizing the total energy. In the absence of the Coulomb interaction, the total energy E is just the sum of E_e, E_h, and the Landau level energy γ. The Coulomb interaction between the electron and hole lowers E and leads to the formation of the exciton. The exciton binding energy E_B is defined as

$$E_B = E_e + E_h + \gamma - E . \tag{12}$$

As mentioned in the Introduction, Zhang and Bajaj[17] have calculated the exciton binding energies of both the heavy-hole and the light-hole excitons for the type-II system in the presence of magnetic field applied parallel to the direction of growth, by using a variational approach, based on a trial wave function of the following form:

$$\Psi(\rho,\phi,z_e,z_h) = F_e(z_e)F_h(z_h)\Phi(\rho,\phi) , \tag{13a}$$

where

$$F_e(z_e) = A_1\exp(-\alpha_1 z_e)\sin(\pi z_e/L_e) , \tag{13b}$$

$$F_h(z_h) = B_1\exp(\beta_1 z_h)\sin(\pi z_h/L_h) , \tag{13c}$$

while

$$\Phi(\rho,\phi) = C_1\exp(-\delta_1\rho-\lambda_1\rho^2) \tag{13d}$$

describes the internal motion of the exciton. Here, α_1, β_1, δ_1, and λ_1 are variational parameters and A_1, B_1, and C_1 are normalization constants. We discuss and compare our results with those of Zhang and Bajaj[17] in the next section.

B. Perturbation approach

For the perturbation method, we rewrite the total Hamiltonian [Eq. (6)] as follows:

$$H = H_0 + H' = H_e + H_h + H_B^0 + H' , \tag{14}$$

where H_e and H_h are given in Eqs. (7a) and (7b), H_B^0 corresponds to a two-dimensional motion of a particle of mass $\mu_\pm$ in the presence of a magnetic field in the z direction,

$$H_B^0 = -\frac{\hbar^2}{2\mu_\pm}\left[\frac{\partial}{\rho\partial\rho}\rho\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right]+\gamma\mathcal{R}_\pm L_z+\frac{\gamma^2}{4\alpha_\pm^2}\rho^2 , \tag{15}$$

and H' is a perturbation for the system

$$H' = -\frac{e^2}{\epsilon_0[\rho^2+(z_e-z_h)^2]^{1/2}} . \tag{16}$$

For a ground state, the energy perturbation series can be written as follows:

$$E_{\text{pertb}} = E_0 + \Delta E = E_{0,0} + E_1 + E_2 + \cdots \tag{17}$$

where

$$H_0\psi_{0,0} = E_{0,0}\psi_{0,0} , \tag{18a}$$

$$E_1 = \langle\psi_{0,0}|H'|\psi_{0,0}\rangle , \tag{18b}$$

$$E_2 = \sum_{n,m}{}' \frac{|\langle\psi_{0,0}|H'|\psi_{n,m}\rangle|^2}{E_{0,0}-E_{n,m}} , \tag{18c}$$

etc.

The solutions for the unperturbed part H_0 are exactly known. The electron and hole motion in the z direction is a one-dimensional problem with infinite potential barriers [see Eqs. (9a) and (9b)], while H_B^0 corresponds to a two-dimensional (2D) particle motion in a uniform magnetic field and has been discussed by Landau and Lifshitz.[25] The eigenvectors for the unperturbed Hamiltonian H_0 are

$$\psi_{n,m}=F_e(z_e)F_h(z_h)\psi^B_{n,m}(\rho,\phi)$$
$$=\left[\frac{4}{L_eL_h}\right]^{1/2}\sin\left[\frac{\pi z_e}{L_e}\right]\sin\left[\frac{\pi z_h}{L_h}\right]\psi^B_{n,m}(\rho,\phi)\ , \tag{19}$$

where

$$\psi^B_{n,m}(\rho,\phi)=\frac{1}{\sqrt{2\pi}}\exp(im\phi)\left[\frac{1}{\alpha_c^{(1+|m|)}}\right]\times\left[\frac{n!}{2^{|m|}(n+|m|)!^3}\right]^{1/2}\times\exp(-\rho^2/4\alpha_c^2)\rho^{|m|}L^{|m|}_{n+|m|}(\rho^2/2\alpha_c^2) \tag{20}$$

and the eigenenergies are given by

$$E_0=E_e+E_h+E^B_{n,m}=\frac{\hbar^2\pi^2}{2(m_e)_lL_e^2}+\frac{\hbar^2\pi^2}{2(m_\pm)_lL_h^2}+\hbar\omega_c\left[n+\frac{|m|}{2}+\frac{m}{2}+\frac{1}{2}\right] \tag{21}$$

where $n=0,1,2,\ldots$ and $L^{|m|}_{n+|m|}(\rho^2/2\alpha_c^2)$ are the associated Laguerre polynomials, while $|m|\le n$. For the ground state [lowest subband electron, subband hole, and the lowest Landau level ($n=m=0$)], the first-order perturbation correction to the ground state energy is

$$E_1=2\mathcal{R}_\pm\left[\frac{\gamma\pi}{2}\right]^{1/2}\frac{4}{L_eL_h}\times\int_0^{L_e}dz_e\int_{-L_h}^{0}dz_h\sin^2\left[\frac{\pi z_e}{L}\right]\sin^2\left[\frac{\pi z_h}{L}\right]\times\exp(\beta^2)\mathrm{erfc}(\beta)\ , \tag{22}$$

where $\beta=(z_e-z_h)/2^{1/2}\alpha_c\ge 0$ and $\mathrm{erfc}(\beta)$ is the complementary error function.[26] In the second-order perturbation theory we have calculated analytically the corrections up to the first three leading terms

$$E_2=\frac{|\langle\psi_{0,0}|H'|\psi_{1,0}\rangle|^2}{E_{0,0}-E_{1,0}}+\frac{|\langle\psi_{0,0}|H'|\psi_{2,0}\rangle|^2}{E_{0,0}-E_{2,0}}+\frac{|\langle\psi_{0,0}|H'|\psi_{3,0}\rangle|^2}{E_{0,0}-E_{3,0}}\ . \tag{23}$$

The expressions for the matrix elements are given as

$$\langle\psi_{0,0}|H'|\psi_{1,0}\rangle=-2\mathcal{R}_\pm\left[\frac{\gamma\pi}{2}\right]^{1/2}\frac{4}{L_eL_h}\int_0^{L_e}dz_e\int_{-L_h}^{0}dz_h\sin^2\left[\frac{\pi z_e}{L_e}\right]\sin^2\left[\frac{\pi z_h}{L_h}\right]\left[\exp(\beta^2)\mathrm{erfc}(\beta)(\beta^2+\tfrac{1}{2})-\beta/\sqrt{\pi}\right]\ , \tag{24a}$$

$$\langle\psi_{0,0}|H'|\psi_{2,0}\rangle=-2\mathcal{R}_\pm\left[\frac{\gamma\pi}{2}\right]^{1/2}\frac{4}{L_eL_h}\times\int_0^{L_e}dz_e\int_{-L_h}^{0}dz_h\sin^2\left[\frac{\pi z_e}{L_e}\right]\sin^2\left[\frac{\pi z_h}{L_h}\right]\left[\exp(\beta^2)\mathrm{erfc}(\beta)(\beta^4/2+3\beta^2/2+\tfrac{3}{8})-(\beta/2\sqrt{\pi})(\tfrac{5}{2}+\beta^2)\right]\ , \tag{24b}$$

and

$$\langle\psi_{0,0}|H'|\psi_{3,0}\rangle=-2\mathcal{R}_\pm\left[\frac{\gamma\pi}{2}\right]^{1/2}\frac{4}{L_eL_h}\times\int_0^{L_e}dz_e\int_{-L_h}^{0}dz_h\sin^2\left[\frac{\pi z_e}{L_e}\right]\sin^2\left[\frac{\pi z_h}{L_h}\right]\times\left[\exp(\beta^2)\mathrm{erfc}(\beta)(\beta^6/6+5\beta^4/4+15\beta^2/8+\tfrac{5}{16})-(\beta/6\sqrt{\pi})(\tfrac{33}{4}+7\beta^2+\beta^4)\right]\ . \tag{24c}$$

The binding energy for the exciton is given by

$$E_B=-E_1-E_2\ . \tag{25}$$

III. RESULTS—DISCUSSION

We have calculated the values of the binding energies E_B of the heavy-hole and the light-hole excitons as a function of AlAs-layer thickness (or GaAs-layer thickness) using variational and perturbation approaches, for various magnetic-field strengths. The values of the various physical parameters involved in the GaAs-AlAs QW's that have been used in our calculations are $(m_e)_l=1.1m_0$, $(m_e)_t=0.19m_0$, $\gamma_1=6.93$, $\gamma_2=2.15$;[27] for heavy-hole mass [$(m_+)_l=0.38m_0$, $(m_+)_t=0.11m_0$]; for light-hole mass [$(m_-)_l=0.089m_0$, $(m_-)_t=0.21m_0$]. The reduced mass in the x-y plane for heavy-hole exciton is $\mu_+=0.07m_0$ and for the light-hole exciton $\mu=0.1m_0$.

In Figs. 2 and 3 we display the dependence of heavy-hole exciton binding energy E_B as a function of the AlAs

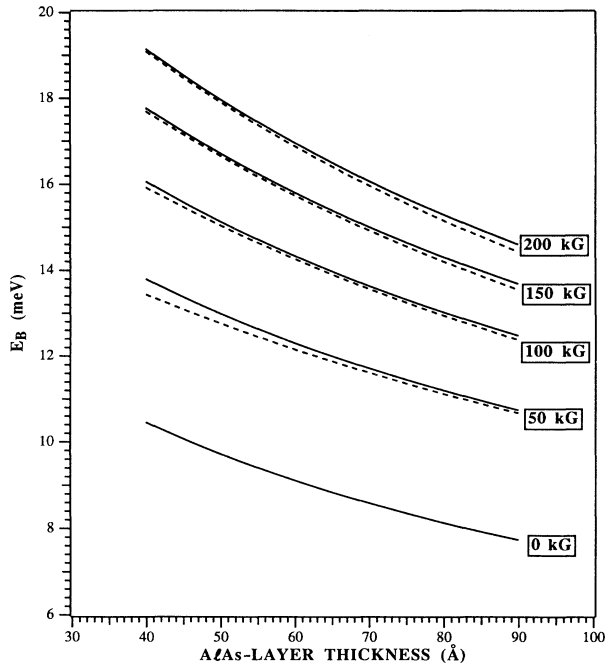

FIG. 2. Binding energy (E_B) of a heavy-hole exciton as a function of AlAs-layer thickness for GaAs-layer width $L_h = 20$ Å for several values of the magnetic field. Perturbation approach, — — —. Variational calculation, ——.

(electron) layer width for two different GaAs-layer thicknesses (20 and 28 Å, respectively), for various values of the magnetic field. For $B = 0$, smaller AlAs widths increase the exciton binding energy, due to the approach of electrons and holes closer to the interface. The variational approach, gives good results for very small magnetic fields, by using Gaussian eigenstate expansion for the trial wave function. Finite magnetic fields compress the excitonic wave function by reducing the cyclotron radius compared to heavy-hole exciton Bohr radius ($\alpha_+ = 65$ Å); especially for $B \sim 76$ kG, which corresponds to $\gamma = 1$ or $\alpha_+ = \alpha_c$, the crossing point between strong and weak fields. The variational and perturbation results are quite close to each other at large fields ($B > 100$ kG). For small fields ($20 \leq B \leq 76$ kG), the perturbation method still holds quite successfully (maximum difference between the two methods ~ 0.3 meV), while for very small fields ($B < 10$ kG), the perturbation method fails to reproduce results close to variational ones (for example, for $B = 5$ kG and GaAs width of 20 Å, the difference between the results of the variational and perturbation methods varies from 1.7 meV for AlAs width $L_e = 40$ Å to 0.4 meV for 90 Å). This is not surprising since at low magnetic fields the wave function of the exciton is more hydrogenic than Gaussian in character.

In Figs. 4 and 5, we display the light-hole exciton energy E_B versus the AlAs- (electron) layer width for two different GaAs-layer thicknesses (20 and 28 Å, respectively), for various values of the magnetic field. Again the be-

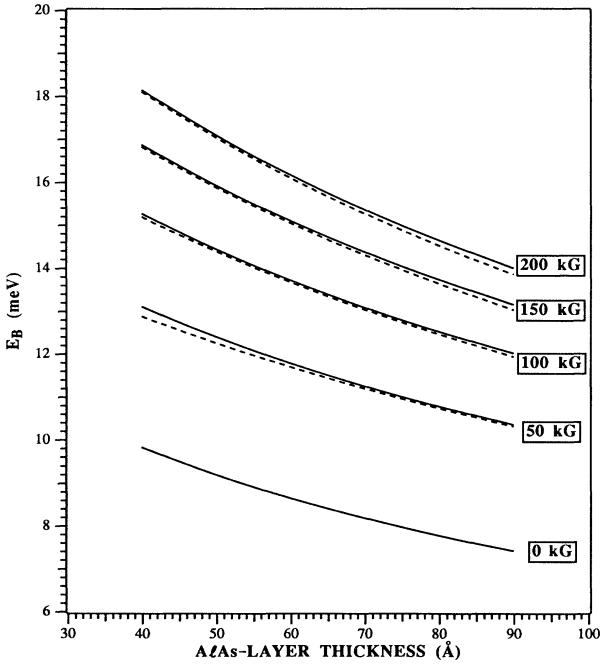

FIG. 3. Binding energy (E_B) of a heavy-hole exciton as a function of AlAs-layer thickness for GaAs-layer width $L_h = 28$ Å for several values of the magnetic field. Perturbation approach, — — —. Variational calculation, ——.

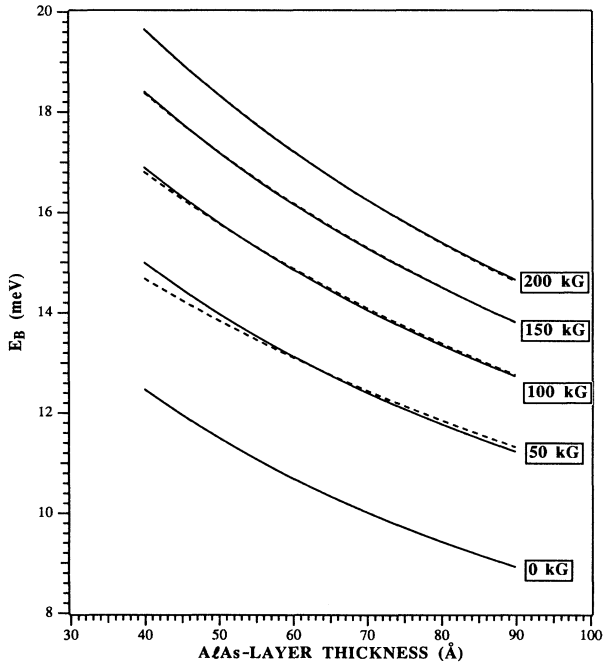

FIG. 4. Binding energy (E_B) of a light-hole exciton as a function of AlAs-layer thickness for GaAs-layer width $L_h = 20$ Å for several values of the magnetic field. Perturbation approach, — — —. Variational calculation, ——.

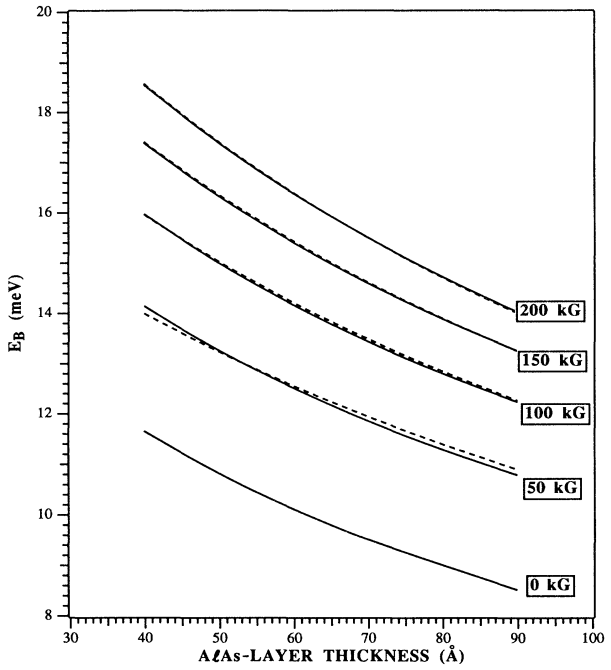

FIG. 5. Binding energy (E_B) of a light-hole exciton as a function of AlAs-layer thickness for GaAs-layer width $L_h = 28$ Å for several values of the magnetic field. Perturbation approach, — — —. Variational calculation, ——.

havior of the exciton binding energy is essentially similar to that for the heavy-hole exciton. The $\gamma = 1$ case corresponds to $B \sim 155$ kG. The crossing of the perturbation results over the variational ones in low magnetic fields, with increased magnitude relative to variational ones (less than 0.1 meV), is attributed to the way the E_B is calculated by the variational method, and does not have any physical meaning.

We compare our results in the case of $B = 0$ with those of Duggan and Ralph,[5] using the same physical parameters. Our values are larger than theirs, because in their case, the exciton trial wave function $\Phi(\rho,\phi,z)$ is proportional to $\exp(-\delta\rho)$ and thus has only one variational parameter. For finite magnetic fields, we compare our results with those of Zhang and Bajaj[17] and find that our values are slightly smaller than theirs.

Hodge *et al.*[11] have measured the energy of $1s \rightarrow 2p_+$ transition of a heavy-hole exciton in type-II GaAs-AlAs QW's in the presence of a magnetic field using photoinduced far-infrared absorption spectroscopy. It is not possible for us to compare our results directly with those of Hodge *et al.*,[11] since we do not calculate the energy of the $2p_+$ level of a heavy-hole exciton. However, we find that the values of the binding energies of the $1s$ state of the heavy-hole exciton that we calculated, for the values of GaAs- and AlAs-layer thicknesses and magnetic fields used by them, are somewhat lower than their measured values of $1s \rightarrow 2p_+$ transition. This is partly due to the fact that we assume infinite potential barriers in our calculations and ignore the effect of the electron and hole wave-function overlap, which enhances the binding energies.

IV. CONCLUSIONS

We have calculated the binding energies for both the heavy-hole and light-hole excitons in type-II GaAs-AlAs QW's in the presence of a magnetic field applied parallel to the direction of growth, for various AlAs and GaAs quantum-well sizes. We have used two methods—a variational approach and a perturbation method—and have assumed infinite potential barriers. For a given set of AlAs- and GaAs-layer widths, the exciton binding energy increases with increasing magnetic field. We compare the results from both methods and find that these are quite close to each other even for small magnetic fields.

ACKNOWLEDGMENTS

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