Period doubling and chaos in the Gunn effect

Jiang Zongfu* and Ma Benkun
Department of Physics, Beijing Normal University, Beijing 100875, People's Republic of China
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A model describing the Gunn effect in GaAs is studied. Numerical results show that the model exhibits period doubling and chaos under an externally applied periodic electric field.

Oscillatory instabilities of various types occur in electrical conduction in semiconductors due to nonlinear effects of high applied electric fields, and have been studied for some time.[1,2] Oscillations of the current or the voltage under dc bias can be induced by a variety of mechanisms: (i) circuit-induced oscillations; (ii) transit-time oscillations; (iii) generation and recombination (GR) processes; (iv) breathing of current filaments; (v) helical waves. In recent experiments on high purity GaAs (Refs. 3–6), Ge (Refs. 7–10), InSb (Ref. 11), and Si (Ref. 12). More complex nonlinear phenomena have been found: period doubling, frequency locking, and chaotic oscillation. In the majority of these experiments, the oscillations occurred in the regime of impurity-impact ionization, indicating that generation and recombination processes between localized states and extended band states played a crucial role. Several theoretical models based upon the mechanism of the GR processes for these chaotic phenomena were proposed.[13] However, relatively little work has been reported on chaotic phenomena induced by the other four classes of mechanisms.[14,15] On the other hand, as has been demonstrated in bulk semiconductors, modern electronic device systems associated with the nonlinear carrier transport may be seriously affected by chaotic noise which destroys the reliability of device operation. It is thus important to know how a nonlinear electronic device responds to these external perturbations. In this paper, we will discuss chaotic current oscillations in the Gunn effect, under an external bias, both for a static voltage (dc bias) and for a sinusoidal voltage (ac bias).

It is well known that *n*-type GaAs can exhibit self-sustained current oscillations (Gunn effect) in the microwave frequency range when the applied drift field exceeds a characteristic high-field threshold value. These are transit-time oscillations. Because of inefficient energy relaxation, the electron gas heats up to temperatures well above that of the crystal lattice, and a transfer of carriers from the high-mobility conduction-band minimum to low-mobility satellite valleys takes place. If this transition is fast enough, a bulk negative differential conductivity may arise. The spatially homogeneous electron distribution then becomes unstable, and propagating high-field domains are formed. In the external circuit, the formation and propagation of these domains give rise to current oscillations with a typical frequency of GHz for a μm sample.

That effect can be described by space-charge-dipole-domain dynamics. In one dimension, the Gunn effect is governed by the following equation for the electric field $E(x,t)$:[16]

$$\frac{1}{e'}\frac{\partial}{\partial x}\left[\frac{\partial E}{\partial t}+e'n_0 v_{(E)}+v_{(E)}\frac{\partial E}{\partial x}-D\frac{\partial^2 E}{\partial x^2}\right]=0 \quad (1a)$$

or

$$\frac{\partial E}{\partial t}+e'n_0 v_{(E)}+v_{(E)}\frac{\partial E}{\partial x}-D\frac{\partial^2 E}{\partial x^2}=\frac{4\pi}{\epsilon_0}J(t) . \quad (1b)$$

(1b) is the first integration of (1a) with respect to x. Here $e'=4\pi e/\epsilon_0$, e is the electronic charge, n_0 the equilibrium electron density, ϵ_0 the static dielectric constant, $v(E)$ the electron drift velocity. $J(t)$, which has the meaning of a current density, must be determined in such a way that

$$U=\int_0^L dx\, E(x,t) , \quad (2)$$

where U is the applied potential and L is the sample length. In order to solve Eq. (1) explicitly, it is expanded into a Fourier series[17]

$$E(x,t)=E_0(t)+\sum_{m\neq 0} E_m(t)\exp(imk_0x) , \quad (3)$$

and $v(E)$ is expanded into a power series in E. Equation (1) then becomes the following set of equations.[11]

$$\frac{dE_m(t)}{dt}=(\alpha_m-i\beta_m)E_m(t) - \sum_{m_1+\cdots+m_s=m}\frac{1}{S!}A_{s,m}E_{m_1}\cdots E_{m_s} . \quad (4)$$

The various terms on the right-hand side (rhs) of (4) are defined as follows:

$$\alpha_m=e'n_0v_0^{(l)}-Dk_0^2m^2 , \quad (5)$$

$$\beta_m=mk_0v(E_0) , \quad (6)$$

$$A_{s,m}=e'n_0v_0^{(s)}+imsk_0v_0^{(s-1)} , \quad (7)$$

where $k_0=2\pi/L$ and the derivatives of $v(E)$ are defined by

$$v_0^{(s)}=\left.\frac{d^s v_{(E)}}{dE^s}\right|_{E=E_0} . \quad (8)$$

Inserting (3) into condition (2), one can get $E_0(t)=U/L$, which means that E_0 is an external bias electrical field. Haken[17] described the phase transition of (4) with two modes near threshold by using the adiabatic elimination procedure. For an external bias well above threshold, adiabatic elimination is not applicable because $|e'n_0 v_0^{(l)}|>Dk_0^2 m^2$. In the following we discuss (4) for the post-threshold case, focusing on only the two-mode interaction with diffusion neglected. Decomposing the complex variables E_1 and E_2 into real and imaginary parts as $E_1=x_1+ix_2$, $E_2=x_3+ix_4$, and using the relation $E_{-m}=E_m^*$, Eqs. (4) reduce to

$$
\begin{aligned}
\frac{dx_1}{dt}&=\alpha x_1+\beta_1 x_2+v_1 r_1-v_2 r_2-(w_1 x_1-w_2 x_2)C_1 \ ,\\
\frac{dx_2}{dt}&=-\beta_1 x_1+\alpha x_2+v_2 r_1+v_1 r_2-(w_1 x_2+w_2 x_1)C_1 \ ,\\
\frac{dx_3}{dt}&=\alpha x_3+\beta_2 x_4-2v_2 x_1 x_2+v_4 C_3/2\\
&\quad -(w_1 x_3-w_2 x_4)C_2 \ ,\\
\frac{dx_4}{dt}&=-\beta_2 x_3+\alpha x_4+v_1 x_1 x_2+v_2 C_3/2\\
&\quad -(w_1 x_4+w_2 x_3)C_2 \ .
\end{aligned}
\tag{9}
$$

The terms on the rhs of (9) are defined as follows:

$$
\begin{aligned}
&v_1=-e'n_0 v_0^{(2)} \ , \quad v_2=2k_0 v_0^{(1)} \ ,\\
&w_1=e'n_0 v_0^{(3)} \ , \quad w_2=-6k_0 v_0^{(3)} \ ,\\
&r_1=x_3 x_1+x_4 x_2 \ , \quad r_2=x_4 x_1+x_3 x_2 \ ,\\
&C_1=(x_1^2+x_2^2)/2+x_3^2+x_4^2 \ ,\\
&C_2=(x_3^2+x_4^2)/2+x_1^2+x_2^2 \ , \quad C_3=x_1^2-x_2^2 \ .
\end{aligned}
\tag{10}
$$

To perform the calculation, we take an explicit form for $v(E)$ of the typical sample GaAs,[16]

$$v_{(E)}=[u_1 E+v_v(E/E_a)^4]/[1+(E/E_a)^4] \ , \tag{11}$$

where the parameters $u_1=8000$ cm/V s, $v_v=8.5\times10^6$ cm/s, $E_a=4.0$ kV/cm. When $E_0>E_{th}\approx3.5$ kV/cm, $v(E)$ decreases with E and the differential mobility $v_0^{(1)}$ is negative. The numerical values of the physical parameters of the sample taken from Refs. 14 and 16, are $n_0=10^{15}$ cm^{-3}, the sample length $L=0.01$ cm, the donor level $\Delta E=0.03$ eV, the intervalley energy separation $\Delta=0.35$ eV, the central valley mobility $\mu_1=5000$ cm^2/V s, and the effective mass $m_1=0.08m_0$, the satellite valley effective mass $m_2=1.2m_0$, the lattice constant $a=5.64$ Å. For these values, we have the transit time $T=0.625\times10^{-9}$ s. In the following discussion, we choose $1/T$ as the unit of frequency.

Steady-state solutions of x_{is} in Eq. (9) were numerically

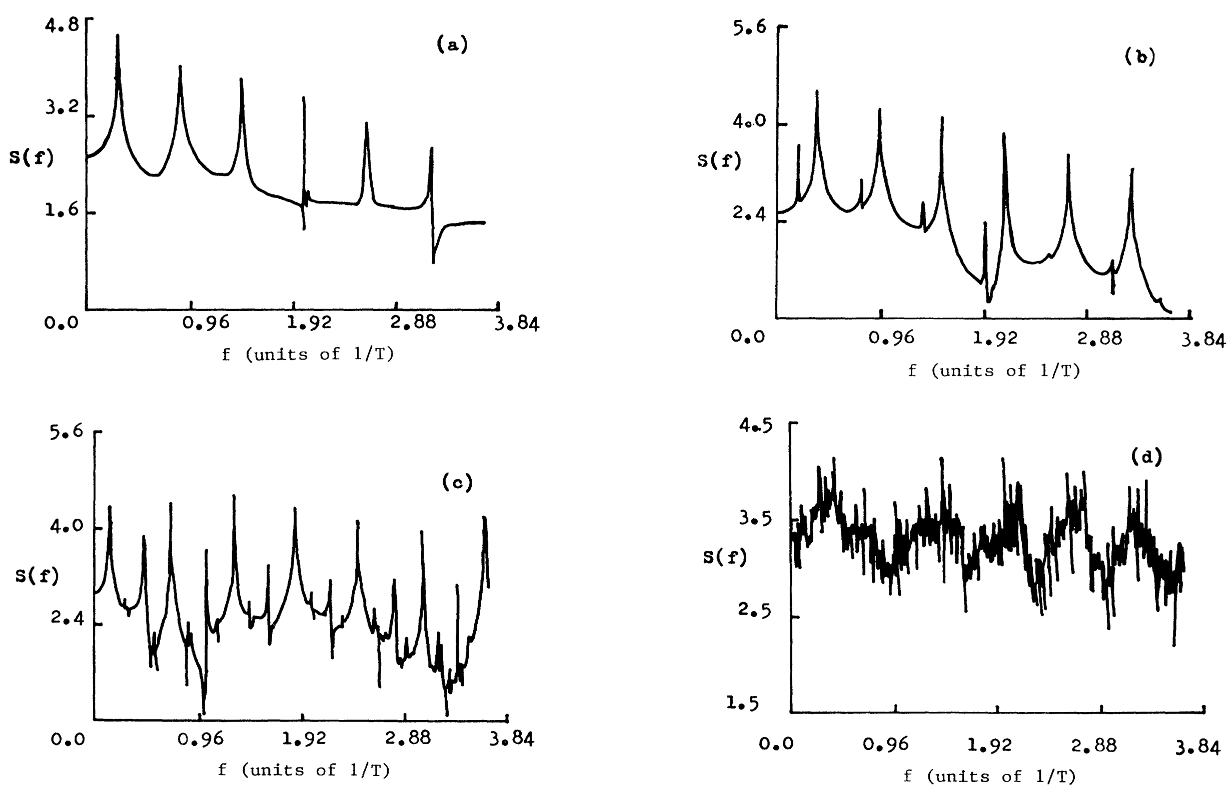

FIG. 1. The power spectrum $S(f)$ of $x_1(t)$ for driving frequency $f_{dr}=1.2/T$ and 4 decreasing dc bias, (a) $E_{dc}=4.290$, (b) $E_{dc}=4.230$, (c) $E_{dc}=4.115$, (d) $E_{dc}=4.110$. The frequency f is given in units of $1/T$.

calculated by setting $dx_{is}/dt=0$. There are only zero solutions ($x_{is}=0$). Performing linear stability analysis, we observe that x_{is} becomes unstable for $\alpha>0$. This can be the case if $v_0^{(l)}$ is negative, i.e., if $E_0>E_{\rm th}$. In the following, we report the numerical result of Eq. (9) under an external electrical field including both dc and ac bias.

$$E_0=E_{\rm dc}+E_{\rm ac}\sin(2\pi f_{\rm dr}t)\ . \tag{12}$$

The onset of a spontaneous oscillation occurs when $E_{\rm dc}>E_{\rm th}$, that is, the Gunn oscillation or Gunn effect. The oscillation frequency decreases as dc bias increases. They are just simply periodic oscillations in the absence of ac bias. If we add ac bias, the chaotic oscillations are observed.

The response of (9) for ac bias amplitude $E_{\rm ac}=0.4$ kV/cm and $f_{\rm dr}=1.2/T$ are shown in Fig. 1 at four successive values of $E_{\rm dc}$. This figure shows the power spectrum $S(f)$ of $x_1(t)$ as $E_{\rm dc}$ decreases from 4.29 to 4.11 kV/cm. Figure 1(a) shows a simple periodic response with $E_{\rm dc}=4.29$. As $E_{\rm dc}$ decreases, oscillations of period two [Fig. 1(b)], four [Fig. 1(c)], and chaotic oscillations [Fig. 1(d)] are successively displayed. For period oscillations, the spectra consist of sharp peaks [Figs. 1(a)–1(c)], while in the chaotic regime a high level of broad-band noise is present [Fig. 1(d)].

Figure 2 shows a phase diagram giving the dynamical state of the system (9) for different dc bias $E_{\rm dc}$ and drive frequence $f_{\rm dr}$. For $E_{\rm dc}>4.3$ kV/cm the current was simply periodic and no excess noise was observed. However, for decreasing $E_{\rm dc}$ at frequencies $f_{\rm dr}>0.66$, the system exhibited a period-doubling cascade to an island of chaotic behavior, as shown in Fig. 2. More complex phase diagrams were obtained for different values of ac bias $E_{\rm ac}$, which included frequency locking and quasiperiodicity.

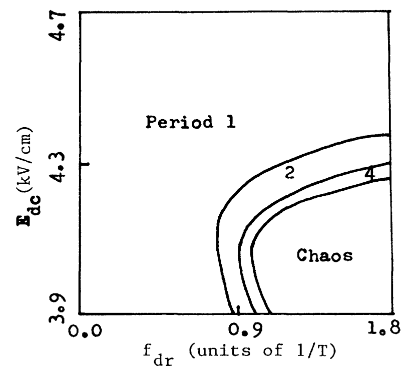

FIG. 2. Phase diagram for the system response at different dc bias $E_{\rm dc}$ and frequencies $f_{\rm dr}$. The region marked 1 denotes simply periodic response, 2 denotes period-2 response, and 4 denotes period-4 and higher response; the chaotic region is also indicated.

In conclusion, we have discussed a model system of coupled-mode equations designed to represent the instability of the electric current flow (Gunn effect). The numerical results of the model (9) exhibit period doubling and chaos under an external applied field E including dc and ac bias. Since Eq. (9) describes the Gunn effect in a GaAs system, the numerical results presented above are expected to be experimentally observable. And an understanding of these phenomena may prove useful in the design and tailoring of low-noise electronic devices.

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*Present address: Institute of Physics, Chinese Academy of Sciences, P.O. Box 603, Beijing 100080, People's Republic of China.

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