Interface plasmon modes of coupled semi-infinite superlattices

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The energy dispersion relation of the local interface plasmon modes of two coupled semi-infinite periodic arrays of quantum wells separated by a distance d has been calculated. An exact solution is obtained when the two semi-infinite superlattices have equivalent periodicities and dielectric constants, but different densities. For the separation equal to zero limit, one interface plasmon mode is found. This mode exists only for wave vectors greater than a critical value depending upon the ratio of the densities of the two superlattices. For finite separation, two interface plasmon modes are found corresponding to the symmetric and antisymmetric combinations of the individual superlattice plasmon modes. These modes are found to exist only for separations less than some critical value depending upon the ratio of the separation distance to the superlattice periodicity.

I. INTRODUCTION

In recent years, there has been a large research effort in the physics of quantum-well structures resulting from recent advancements in crystal-growth techniques presented by molecular-beam epitaxy and metalorganochemical vapor deposition. Additional structures and phenomena, involving quantum confinement, reduce dimensionality, and superlattices, have been predicted and experimentally observed. Extensive investigations have also been carried out on the collective excitations of an infinite semiconductor superlattice consisting of a periodic array of doped quantum wells.[1-4] For an infinite superlattice, the two-dimensional plasmons excitations of the individual quantum wells are coupled together by the long-range Coulomb force. Because of the periodicity of the superlattice, Bloch-type propagating wave solutions for the plasmon dispersion were shown to exist. In the case of a semi-infinite superlattice, Giuliani and Quinn[5,6] investigated the possibility of surface plasmon modes. They predicted the existence of surface plasmons between a semi-infinite superlattice and an adjoining bulk material if the dielectric constants of the two were different. Surface plasmons were found to exist only for wave vectors larger than some critical wave vector depending upon the ratio of the dielectric constants.[5-7]

In this work, we investigate the local interface plasmon modes of two semi-infinite semiconductor superlattices separated by a distance d. In general, the two superlattices can have different periodicities, densities, and dielectric constants. The formalism that we present in this paper pertains to the general case, although we specialize the results to the case where the dielectric constants and the periodicities of the two superlattices are the same, but the superlattices have different densities. In this limit, we have obtained exact solutions for the interface plasmon modes. The different densities of the superlattices give rise to two distinctly different continuum plasmon bands for the individual superlattices. The coupling of plasmon excitations of the superlattices results in a unique set of interface plasmon modes. In the case considered here with equivalent dielectric constants, the isolated semi-infinite superlattices do not have Giuliani-Quinn-type[5,6] surface plasmon modes which require dielectric discontinuities.

II. THEORY

In this paper, we calculate the energy dispersion relation for the local interface plasmon modes of two semi-infinite semiconductor superlattices separated by a distance d. The system we are studying is depicted in Fig. 1. In the figure, we show two semi-infinite superlattices; the one on the left-hand side (superlattice labeled 1 in region 1) has periodicity b and the one on the right-hand side (superlattice labeled 3 in region 3) has periodicity a. The two semi-infinite superlattices are separated by a distance d. The quantum wells of each superlattice are uniformly doped and have an equivalent two-dimensional electron density (three-dimensional dopant density times the well width), n_1 for superlattice 1, and n_3 for superlattice 3. Region 1 has dielectric constant ϵ_1, region 2 has ϵ_2, and region 3 has ϵ_3. The quantum wells are labeled with the index j. The wells are assumed to be far enough apart so that wave-function overlap is negligible. In addition, the model is specialized to the case where the quantum-well widths approach zero giving rise to arrays of two-dimensional electron sheets. This model has been successfully used in the past for the plasmon eigenmodes of the infinite and semi-infinite superlattices.[1-7] The effects of finite well thickness on the plasmon dispersion relation of the superlattice were previously investigated[8] and shown to be only a few percent correction to the zero-width results.

To find the interface plasmon excitations, one solves Maxwell's equations coupled to the density response of the two-dimensional electron sheets. The density response to an applied potential is treated within the framework of linear response theory. We apply the boundary conditions of continuity of the tangential component of the electric field and discontinuity of the normal component of the displacement vector by $4\pi p$ across

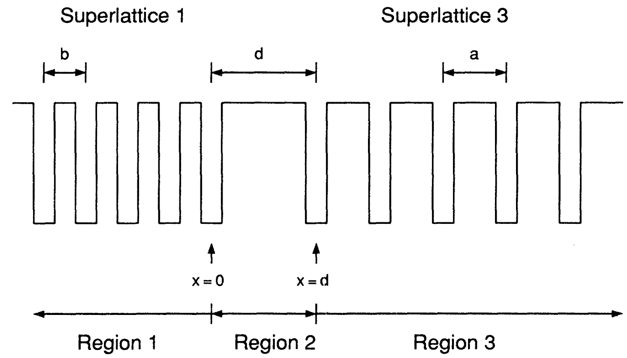

FIG. 1. Two superlattices of quantum wells with periodicities a and b separated by a distance d.

each interface. For a p-polarized electromagnetic wave, the solution for the electric field in region 3 ($z>d$) must be of the following form in order to satisfy Maxwell's equations:

$$\begin{aligned} &E_x=0\ ,\\ &E_y=e^{i(qy-\omega t)}(E_{3,j}^{+}e^{i\beta_3 z}+E_{3,j}^{-}e^{-i\beta_3 z})\ , \\ &E_z=-q\beta_3^{-1}e^{i(qy-\omega t)}(E_{3,j}^{+}e^{i\beta_3 z}-E_{3,j}^{-}e^{-i\beta_3 z})\ , \end{aligned} \tag{1}$$

in region 2 ($0<z<d$),

$$\begin{aligned} &E_x=0\ ,\\ &E_y=e^{i(qy-\omega t)}(E_2^{+}e^{i\beta_2 z}+E_2^{-}e^{i\beta_2 z})\ , \\ &E_z=-q\beta_2^{-1}e^{i(qy-\omega t)}(E_2^{+}e^{i\beta_2 z}-E_2^{-}e^{-i\beta_2 z})\ , \end{aligned} \tag{2}$$

and in region 1 ($z<0$),

$$\begin{aligned} &E_x=0\ ,\\ &E_y=e^{i(qy-\omega t)}(E_{1,j}^{+}e^{-i\beta_1 z}+E_{1,j}^{-}e^{i\beta_1 z})\ , \\ &E_z=q\beta_1^{-1}e^{i(qy-\omega t)}(E_{1,j}^{+}e^{-i\beta_1 z}-E_{1,j}^{-}e^{i\beta_1 z})\ , \end{aligned} \tag{3}$$

where β_n is related to q by $\beta_n^2=\epsilon_n\omega^2/c^2-q^2$, n refers to the specific region, and j labels the quantum well. For the semi-infinite superlattices, Eq. (1) is defined for $ja<z<(j+1)a$ in region 3, where z is measured from ja and the $j=0$ well is at $z=d$. Likewise, in region 1, Eq. (3) is defined for $-jb<z<-(j+1)b$ where j is taken to be positive, z is measured from $-jb$, and the $j=0$ well is at $z=0$. Here, the z direction is along the superlattice, and the x and y directions are perpendicular to the superlattice (along the quantum wells).

To solve for the interface plasmon modes of the coupled semi-infinite superlattices, we make the ansatz of decaying solutions in regions 1 and 3 of the form

$$E_{1,j}^{+,-}=E_{1,0}^{+,-}e^{-\alpha_1 jb}\ , \tag{4}$$

$$E_{3,j}^{+,-}=E_{3,0}^{+,-}e^{-\alpha_3 ja}\ , \tag{5}$$

where α_1 and α_3 are the inverse decay lengths in the respective superlattices. Applying the continuity of the tangential component of the electric field and discontinuity of the normal component of the displacement vector by $4\pi\rho$ in region 3 gives

$$E_{3,j}^{+}e^{i\beta_3 a}+E_{3,j}^{-}e^{-i\beta_3 a}=E_{3,j+1}^{+}+E_{3,j+1}^{-}\ , \tag{6}$$

$$(E_{3,j+1}^{+}-E_{3,j+1}^{-})-(E_{3,j}^{+}e^{i\beta_3 a}-E_{3,j}^{-}e^{i\beta_3 a}) = -\frac{4\pi\rho[(j+1)a]}{\epsilon_3 q\beta_3^{-1}}\ , \tag{7}$$

where $\rho(j)$ is determined from linear response theory to be

$$\rho_3(j)=\frac{ie^2\chi_3^0(q,\omega)(E_{3,j}^{+}+E_{3,j}^{-})}{q}\ . \tag{8}$$

Here $\chi_3^0(q,\omega)$ is the susceptibility of the two-dimensional electron gas. In the random-phase approximation[9] (RPA) $\chi_3^0(q,\omega)$ is given by

$$\chi_3^0(q,\omega)=2\sum_p \frac{f_{p+q}-f_p}{\epsilon_{p+q}-\epsilon_p-\omega}\ , \tag{9}$$

where the subscript on χ refers to the specific semi-infinite superlattice. For $\omega\gg qv_f$, where v_f is the Fermi velocity, $\chi_3^0(q,\omega)$ is to a good approximation

$$\chi_3^0(q,\omega)=\frac{n_3 q^2}{m\omega^2}\ . \tag{10}$$

Using the decaying solution ansatz of Eq. (5) in Eqs. (6) and (7), we find

$$E_{3,0}^{+}e^{i\beta_3 a}+E_{3,0}^{-}e^{-i\beta_3 a}=e^{-\alpha_3 a}(E_{3,0}^{+}+E_{3,0}^{-})\ , \tag{11}$$

$$[e^{-\alpha_3 a}(1+\gamma_3)-e^{i\beta_3 a}]E_{3,0}^{+} -[e^{-\alpha_3 a}(1-\gamma_3)-e^{i\beta_3 a}]E_{3,0}^{-}=0\ , \tag{12}$$

where

$$\gamma_3=\frac{4\pi ie^2}{\epsilon_3\beta_3^{-1}q^2}\chi_3^0(q,\omega)\ . \tag{13}$$

Solving Eqs. (11) and (12) simultaneously, the relation for ω, the plasmon energy, is determined to be

$$\begin{aligned} \omega^2&=(\omega_{p,3}^{2\mathrm{D}})^2\frac{\sinh(qa)}{\cosh(qa)-\cosh(\alpha_3 a)} \\ &=(\omega_{p,3}^{2\mathrm{D}})^2\frac{\sinh(qa)}{\cosh(qa)-\frac{1}{2}(x_3+1/x_3)}\ , \end{aligned} \tag{14}$$

where $x_3=e^{\alpha_3 a}$ is the inverse decay factor into superlattice 3. $\omega_{p,3}^{2\mathrm{D}}$ is the single-well two-dimensional (2D) plasmon dispersion energy,[9]

$$(\omega_{p,3}^{2\mathrm{D}})^2=\frac{2\pi n_3 e^2 q}{\epsilon_3 m}\ , \tag{15}$$

where n_3 is the two-dimensional density of the quantum well. Identical equations are obtained for the superlattice plasmon 1 where every symbol with subscript 3 is replaced with subscript 1. Equation (14) is the same equation previously derived for the bulk superlattice plasmon energy except that the wave vector $\mathbf{k}$ along the superlattice is replaced by i times α.[1-3] If one replaces $\cosh(\alpha a)$ by $\cos(ka)$, then one obtains a band of plasmons along the k direction (the direction of k is along the superlattice axis, whereas q is parallel to the two-dimensional sheet of electrons). This corresponds to traveling-wave solutions where Eq. (5) becomes $E_{3,j}^{+,-}=E_{3,0}^{+,-}e^{i\alpha_3 ja}$. For a well-defined excitation, the plasmon dispersion for the local mode plasmon must lie outside the bulk plasmon bands. The edges of the bulk plasmon bands are determined by $\cos(ka)$ equal to plus or minus one.

Applying the boundary conditions at the interfaces of region 2, at $z=0$ and d, gives the additional equations

$$E_{1,0}^{+}+E_{1,0}^{-}=E_2^{+}+E_2^{-} , \tag{16}$$

$$E_2^{+}e^{i\beta_2 d}+E_2^{-}e^{-i\beta_2 d}=E_{3,0}^{+}E_{3,0}^{-} , \tag{17}$$

$$\left[\frac{\epsilon_2}{\epsilon_1}\right](E_2^{+}-E_2^{-})+(E_{1,0}^{+}-E_{1,0}^{-})=-\gamma_1(E_2^{+}+E_2^{-}) , \tag{18}$$

$$(E_{3,0}^{+}-E_{3,0}^{-})-\left[\frac{\epsilon_2}{\epsilon_3}\right](E_2^{+}e^{i\beta_2 d}-E_2^{-}e^{-i\beta_2 d}) = -\gamma_3(E_{3,0}^{+}+E_{3,0}^{-}) . \tag{19}$$

Equations (11) and (12), the corresponding equations for superlattice 1, and Eqs. (16)–(19) must be solved simultaneously for the interface plasmon modes. In order to investigate the bare plasmon modes uncoupled to the radiation field, it is appropriate to take the nonretarded limit, $\omega \ll qc$. Solving these eight equations in this limit, after some amount of algebra, leads to the following equations to be solved:

$$x_3+\frac{1}{x_3}2\cosh(qa)=\gamma_3\sinh(qa) , \tag{20}$$

$$x_1+\frac{1}{x_1}2\cosh(qb)=\gamma_1\sinh(qb) , \tag{21}$$

$$\left[[x_1-\cosh(qb)\frac{\sinh(qd)}{\sinh(qb)}+\frac{\epsilon_2}{\epsilon_1}(\cosh(qd)\right] \times\left[[x_3-\cosh(qa)]\frac{\sinh(qd)}{\sinh(qa)}+\frac{\epsilon_2}{\epsilon_3}\cosh(qd)\right]=\frac{\epsilon_2\epsilon_2}{\epsilon_1\epsilon_3} , \tag{22}$$

where γ_3 is defined by Eq. (13), and $x_1=e^{\alpha_1 a}$ and $x_3=e^{\alpha_3 a}$. The solution of these equations results in a fourth-order equation in x_3 that in general has an analytic solution, although rather complicated. After obtaining x_3, substituting into Eq. (20) gives the energy of the localized plasmon excitation. It is also necessary to solve for x_1 [determined by substituting x_3 into Eq. (22)] since to have a physically acceptable solution, one must show that both x_1 and x_3 are larger than positive one or less than minus one. In the general case of arbitrary periodicities and dielectric constants for the two superlattices, Eqs. (20)–(22) are solved numerically for the four roots for x_3. However, there are a number of interesting physical limits for which we have determined exact solutions that we now discuss.

III. ANALYSIS AND RESULTS

A. The separation between the superlattices is zero

First, we consider the limit where d is exactly zero, and the superlattices have equal periodicities and equal dielectric constants. That is, $d=0$, $a=b$, and $\epsilon_1=\epsilon_3$. Then Eq. (22) reduces to

$$\epsilon_1[x_1-\cosh(qb)]=-\epsilon_3[x_3-\cosh(qa)] . \tag{23}$$

In this case, Eqs. (20)–(22) can be readily solved. Although the superlattices have equal periodicities they are still nonequivalent since their densities are different (β is arbitrary), and consequently, the bulk plasmon bands have different bandwidths. In this limit, Eqs. (20)–(22) result in a cubic equation to be solved for x_3 with the only physically acceptable root

$$x_3=\frac{2}{1+\beta}\cosh(qa) , \tag{24}$$

where

$$\beta=(\epsilon_1/\epsilon_3)(n_3/n_1)=(n_3/n_1) .$$

The other roots are $x_3=e^{qa}$ and $x_3=e^{-qa}$, and are not allowed since both x_1 and x_3 must be greater than unity in order to have a decaying solution (for $x_3=e^{qa}$, $x_1=e^{-qa}$ which is less than unity and consequently not allowed). The solution for x_1 corresponding to x_3 of Eq. (24) is

$$x_1=\frac{2\beta}{1+\beta}\cosh(qa) . \tag{25}$$

The constraints on x_1 and x_3 lead to the condition that qa must be greater than some critical value of $(qa)_{\text{crit}}$ in order for a localized solution to exist. For $\beta\geq 1$, $(qa)_{\text{crit}}$ must satisfy

$$2\cosh(qa)_{\text{crit}}-1=\beta . \tag{26}$$

For $\beta=1$, interface plasmons exist for all qa [that is, $(qa)_{\text{crit}}$ is zero]. For $\beta=2$, $(qa)_{\text{crit}}$ is 0.962, for $\beta=4$, $(qa)_{\text{crit}}$ is 1.567. As β increases, so does the required $(qa)_{\text{crit}}$ for the existence of the mode. Using x_3 in Eq. (14) leads to an explicit solution for the plasmon dispersion,

$$\omega^2=(\omega_{p3}^{2\text{D}})^2\frac{(1+2\beta)\sinh(qa)\cosh(qa)}{2\beta\cosh^2(qa)-\frac{1}{2}(\beta+1)^2} . \tag{27}$$

For $\beta=1$ (equivalent densities, $n_1=n_3=n$), this corresponds to a superlattice with the quantum well at $x=0$ having a density of $2n$. In this case, Ref. 10 showed that a local mode exists with an energy dispersion given by Eq. (27) for $\beta=1$. This mode is directly analogous to a local phonon mode[11] in a vibrational superlattice where one mass is different from the others. For the local plasmon mode ($\beta=1$), we find, $x_1=x_3=\cosh(qa)$ and $\omega^2=(\omega_{p1}^{2\text{D}})^2 2\coth(qa)$.

For arbitrary β and in the limit of large qa (the wells become isolated),

$$x_1 = \frac{\beta}{1+\beta} e^{qa} \tag{28}$$

and

$$x_3 = \frac{1}{1+\beta} e^{qa} . \tag{29}$$

In this limit, the plasmon dispersion simplifies to

$$\omega^2 = (\omega_{p1}^{2D})^2 (1+\beta) . \tag{30}$$

Since $\beta = n_3/n_1$ this is just the result for a single isolated quantum well with density $n_1 + n_3$ as expected.

In Fig. 2 we plot the interface plasmon energy versus qa for $\beta = 1$, 2, and 4. The regions between the dashed lines in the figure correspond to the energies of the continuum of the bulk plasmon states for the isolated superlattices. For $\beta = 2$ and 4, the upper pair corresponds to the bulk plasmon modes associated with superlattice 3, the lower pair with superlattice 1. For $\beta = 1$, the continuum of bulk plasmon states is the same for both superlattices. The energies are measured in reduced units of

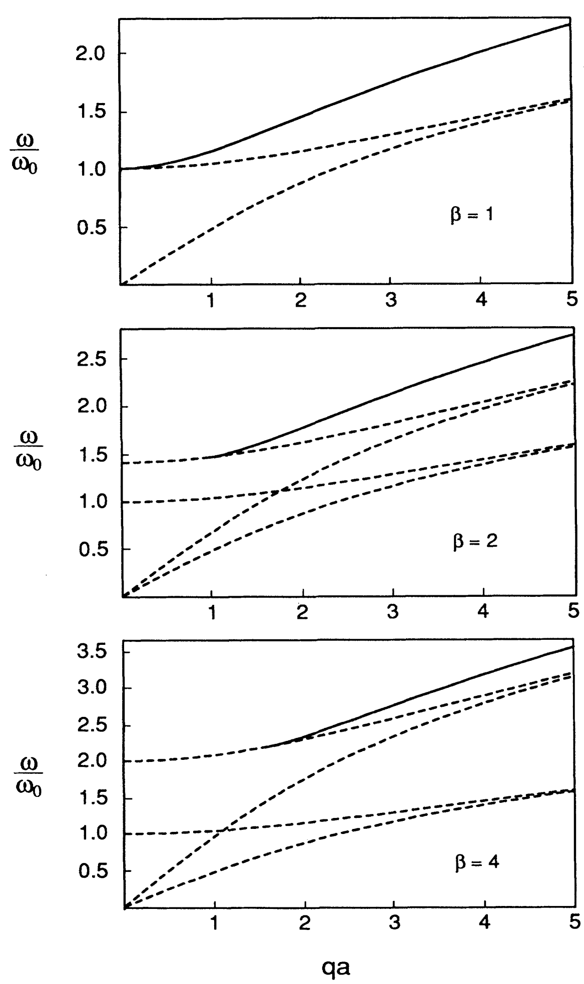

FIG. 2. Plots of the interface plasmon energies vs qa for $\beta = 1,2,4$, and zero separation.

$$\omega_0 = \left[\frac{4\pi (n_1/a) e^2}{m \epsilon_1} \right]^{1/2} ,$$

relative to superlattice 1. In the d equal zero limit, only one interface plasmon mode exists. This corresponds to the symmetric mode arising from the coupling between the two superlattices. For $\beta = 1$, we find a local mode at the quantum well located at $z = 0$ as discussed above. For $\beta = 2$ and 4, the interface plasmon mode exists only for wave vectors larger than $(qa)_{\rm crit}$. The $(qa)_{\rm crit}$ values for the existence of a decaying solution occur at 0, 0.97, and 1.57 for $\beta = 1$, 2, and 4, respectively, in agreement with Eq. (26). Below, we show that for finite d, two interface plasmon modes exist, corresponding to symmetric and antisymmetric modes.

B. Identical superlattices separated by finite distance

Next we consider another physically important limit that also has an exact solution. We consider two equivalent semi-infinite superlattices with identical periodicities a embedded in a media with equal dielectric constants, doped with the same carrier densities but separated by a distance d. That is, $a = b$, $\epsilon_1 = \epsilon_2 = \epsilon_3$, and $\beta = 1$, but d is arbitrary. In this case, we can also solve Eqs. (20)–(22) exactly. The resultant quartic equation has two unphysical solutions for x_3, $x_3 = e^{qa}$ and $x_3 = e^{-qa}$. The quadratic equation can then be solved for x_3, and subsequently x_1, using Eq. (22), with the results

$$x_1 = e^{\alpha_1 a} = x_3 = e^{\alpha_3 a} = \frac{e^{qd} \pm e^{2qa}}{e^{qa}(e^{qd} \pm 1)} , \tag{31}$$

where the plus sign is for the higher-energy mode (symmetric mode) and the minus sign is for the lower-energy mode (antisymmetric mode). For d equal to zero, Eq. (31) reduces to our previous results of Sec. III A. From the condition that x_3 must be greater than unity or less than minus one in order to have a decaying solution, we find that localized plasmon modes exist only for $d < a$. The distance between the superlattices must be less than the superlattice periodicity. For the lower-energy modes, x_1 and x_3 are negative indicating that the solution is of the form $(-1)^j$ times Eqs. (4) and (5). This indicates that the phase changes by π every lattice spacing. The equation for the energy of the interface plasmon excitations is found to be

$$\omega^2 = (\omega_{p1}^{2D})^2 (1 \pm e^{-qd}) \left[\frac{e^{2qa} \pm e^{qd}}{e^{2qa} - 1} \right] , \tag{32}$$

where the plus sign is the higher-energy mode and the minus sign the lower-energy mode. If we specialize to the case where $a \gg d$, the superlattice periodicity is much larger than the separation and we find

$$x_1 = x_3 = \frac{\pm e^{qa}}{e^{qd} \pm 1} \tag{33}$$

and

$$\omega^2 = (\omega_{p1}^{2D})^2 (1 \pm e^{-qd}) . \tag{34}$$

For qd approaching zero, the plasmon dispersion for the higher-energy mode becomes

$$\omega^2=2(\omega_{p1}^{2D})^2 , \tag{35}$$

and for the lower-energy mode,

$$\omega^2=(\omega_{p1}^{2D})^2 qd . \tag{36}$$

These results agree with those of Das Sarma and Madhukar[12] who considered the case of two quantum wells coupled together by the long-range Coulomb interaction. They found a low-energy acousticlike mode (ω proportional to q) identical to Eq. (36). For the quantum wells of the superlattices far apart compared to their separation ($a \gg d$), our results correspond exactly to the two coupled-quantum-well results.

In Fig. 3 we plot the interface plasmon energies versus qa for separations, $d=0.01a$, $0.1a$, $0.5a$, and $0.7a$. The upper and lower energy interface modes arising from the symmetric and antisymmetric linear combination of isolated superlattice plasmon eigenstates are evident at all separations depicted in the figure. For $d=0.01a$, the upper mode is very close to the $d=0$, $\beta=1$ mode of Fig. 2. However, a low-lying acousticlike excitation is now observed in agreement with Eq. (36). This mode vanishes for d equal to zero as observed in Fig. 2. As d increases the upper mode decreases in energy and the lower mode increases towards the continuum states. There is no $(qa)_{\text{crit}}$ for the equal density $\beta=1$ case; the upper mode exists for all qa. However, as shown following Eq. (31), there is a critical separation distance d at which both the upper and lower modes cease to exist. At $d=a$, there are no allowed interface plasmon modes. This result agrees with the d approaching infinite limit where one has isolated semi-infinite superlattices. In this case, Giuliani-Quinn-type[5,6] surface plasmons for the semi-infinite superlattice do not exist because there is no dielectric discontinuity.

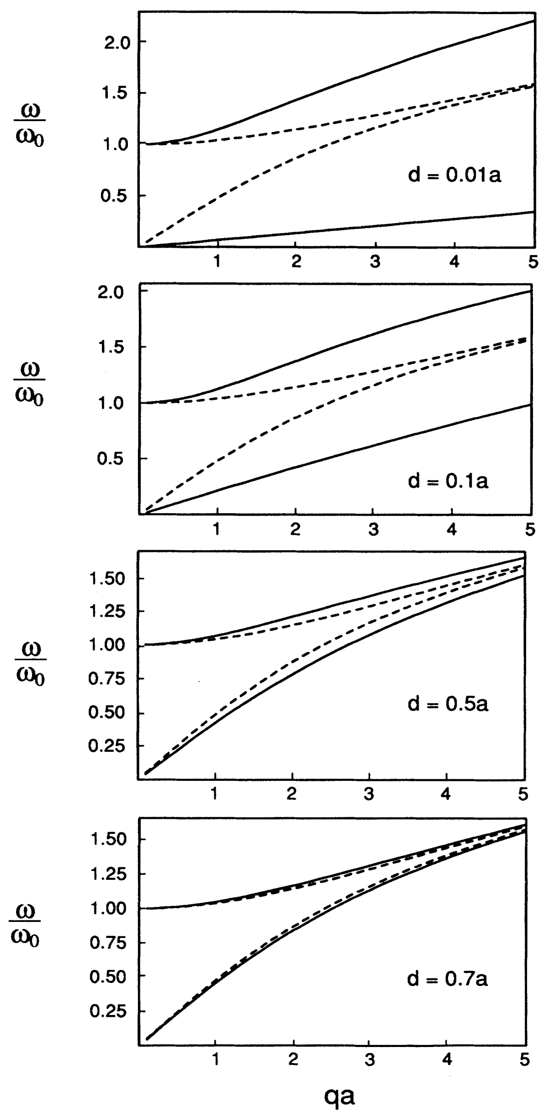

FIG. 3. Plots of the interface plasmon energies vs qa for $\beta=1$ and separations $d=0.01a$, $0.1a$, $0.5a$, and $0.7a$.

C. Superlattices with different doping densities and finite separation

Next, we consider the case in which the semi-infinite superlattices have identical periodicities and dielectric constants, but different doping densities (β not equal to unity) and are separated by a finite distance d. Again, we have solved this case exactly. The solution to the quartic equation resulting from Eqs. (20)–(22) for x_3 can be reduced to the following quadratic equation, with the other two roots being unphysical:

$$ax_3^2+bx_3+c=0 , \tag{37}$$

where

$$a=e^{4qa}-e^{2qa+2qd}-e^{4qa+2qd}+e^{2qa+4qd} , \tag{38}$$

$$\begin{aligned}b=&-2e^{5qa}+e^{qa+2qd}+e^{3qa+2qd}+e^{5qa+2dq}-2e^{qa+4qd}\\&+\beta(-e^{qa+2qd}+2e^{3qa+2qd}-e^{5qa+2qd}) ,\end{aligned} \tag{39}$$

$$c=e^{6qa}+e^{4qd}-e^{2qa+2qd}-e^{4qa+2qd} . \tag{40}$$

Although the solution to the quadratic equation, expressed above, is trivial, the equation for the roots is rather involved and is not explicitly given. The reader can easily solve the quadratic for x_3 and substitute this into Eq. (14) to determine the plasmon energy ω. In the limit that qa approaches infinity and qd approaches zero, the quantum wells are far apart and the superlattices are in close proximity. This limit corresponds to the interaction of two quantum wells, with densities n_1 and n_3 separated by a distance d. In this case, we find x_3 for the higher-energy mode

$$x_3=\frac{e^{qa}}{1+\beta} , \tag{41}$$

and for the lower-energy mode,

$$x_3 = -\frac{(1+\beta)e^{qa}}{2qd} . \tag{42}$$

With respective plasmon energies

$$\omega^2 = (\omega_{p3}^{2D})^2 \left[\frac{1+\beta}{\beta} \right] = (\omega_{p1}^{2D})^2 + (\omega_{p3}^{2D})^2 , \tag{43}$$

$$\omega^2 = (\omega_{p3}^{2D})^2 \left[\frac{2qd}{1+\beta} \right] . \tag{44}$$

These results for the plasmon energies agree with the results of Ref. 12 for the case of two different quantum wells interacting by the Coulomb interaction.

In the limit that qd becomes large (but $qa \gg qd$) and $\beta \geq 2$, we find for the higher-energy mode

$$x_3 = \frac{1}{\beta - 1} e^{qa - 2qd} , \tag{45}$$

and for the lower energy mode,

$$x_1 = -\frac{\beta}{\beta - 1} e^{qa - 2qd} . \tag{46}$$

The results for x_3 for the upper mode and x_1 for the lower mode are presented above because these are the solutions that become unphysical as the separation d approaches the superlattice periodicity a. From Eqs. (45) and (46), we see that for $\beta \geq 2$, x_3 approaches unity, and x_1 approaches minus unity when $d \approx a/2$. Although the expansion above becomes suspect when d approaches $a/2$, it does give approximately the critical value for d for the existence of the interface plasmon modes. A precise condition determining d is given by Eqs. (48) and (50) discussed below. Although other limiting forms follow directly from Eq. (37), they are rather involved and will not be presented here. In the following, we present our numerical results for the interface plasmon excitations in Figs. 4 and 5.

In Fig. 4 we plot the plasmon dispersion for $\beta = 2$ (the density of superlattice 3 is two times that of superlattice 1) versus qa for the separations $d = 0.01a$, $0.1a$ $0.3a$, and $0.4a$. We find the existence of two interface modes corresponding to the symmetric (higher energy) and antisymmetric (lower energy) combination of plasmon eigenstates associated with each individual superlattice. The higher-energy mode does not exist for qa less than a critical value. The critical value for qa, below which the higher-energy mode ceases to exist, is $(qa)_{\mathrm{crit}} = 0.97$ for $d = 0.01a$, $(qa)_{\mathrm{crit}} = 1.09$ for $d = 0.1a$, $(qa)_{\mathrm{crit}} = 1.56$ for $d = 0.3a$, and $(qa)_{\mathrm{crit}} = 2.07$ for $d = 0.4a$. The critical qa for $d = 0.01a$ is very close to the d equals zero results of Sec. III A. Note that as the separation d increases so does the critical value of qa. For $d \approx 0.5a$ (not shown), both the upper and lower energy modes are very close to the continuum. For $d > 0.5a$, we find that the upper mode ceases to exist. The lower mode ceases to exist for $d > 0.65a$. These conditions will be discussed further below.

In Fig. 5 we plot the plasmon dispersion for $\beta = 4$ (the density of superlattice 3 is four times that of superlattice 1) versus qa for the separations $d = 0.01a$, $0.1a$, $0.2a$, and $0.3a$. Again, we find the existence of two modes corresponding to the symmetric and antisymmetric eigenstates. The higher-energy mode exists only for wave vectors greater than a critical value; $(qa)_{\mathrm{crit}} = 1.60$ for $d = 0.01a$, $(qa)_{\mathrm{crit}} = 1.84$ for $d = 0.1a$, $(qa)_{\mathrm{crit}} = 2.23$ for $d = 0.2a$, $(qa)_{\mathrm{crit}} = 3.06$ for $d = 0.3a$, and $(qa)_{\mathrm{crit}} = 5.50$ for $d = 0.4a$ (not shown). For $d = 0.5a$, there is no solution for the higher-energy interface mode. The critical value for qa is larger in this case, where the density of superlattice 3 is four times the density of superlattice 1, as compared to the ratio of two in Fig. 4. The lower mode ceases to exist for d greater than approximately $0.6a$.

The conditions determining the critical values of qa for the existence of the interface plasmon modes can be determined from Eq. (37). In order to have a well-defined plasmon mode, both x_1 and x_3 must be greater than positive one or both x_1 and x_3 must be less than negative one.

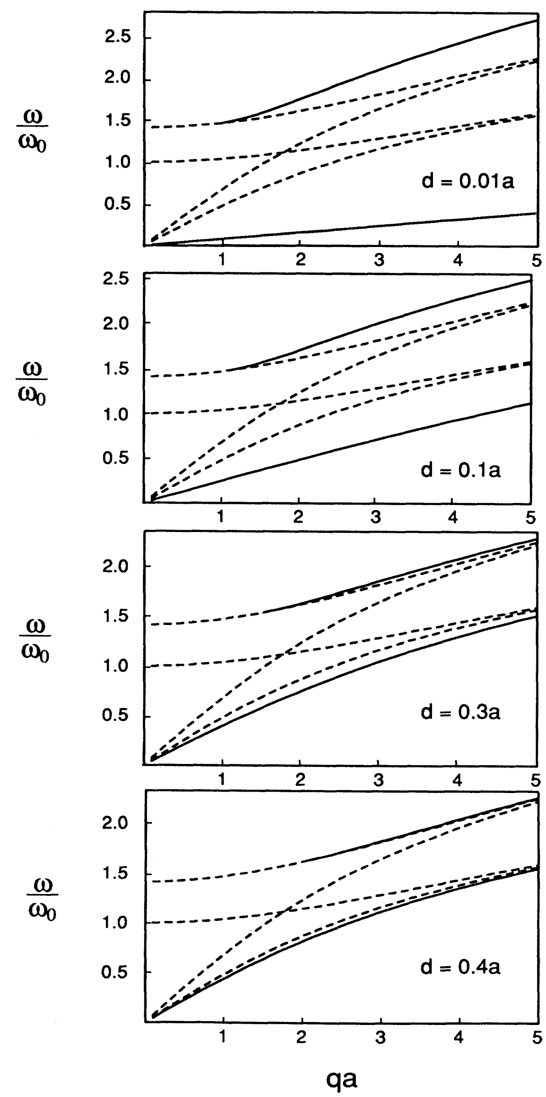

FIG. 4. Plots of the interface plasmon energies vs qa for $\beta = 2$ and separations $d = 0.01a$, $0.1a$, $0.3a$, and $0.4a$.

For the upper mode, $(qa)_{\rm crit}$ is set by having x_3 equal to unity (for $\beta > 1$). This gives

$$a + b + c = 0 , \tag{47}$$

where a, b, and c are define by Eqs. (38)–(40). Solving Eq. (47) for β,

$$\beta = \frac{(e^{qa_{\rm crit}} + e^{2qd})(e^{3qa_{\rm crit}} + e^{2qd})}{e^{qa_{\rm crit}+3qd}(1+e^{qa_{\rm crit}})^2} . \tag{48}$$

In Fig. 6(a), we plot β vs $(qa)_{\rm crit}$ for $d = 0.4a$, $0.5a$, $0.6a$, $0.7a$, $0.8a$ and $0.9a$. For $d < 0.5a$, the curves increase exponentially following Eq. (45) as depicted in the figure for $d = 0.4a$. A solution for $(qa)_{\rm crit}$, above which the upper interface plasmon is allowed, always exist for $d < 0.5a$. The critical values for the wave vectors as determined from Eq. (48) agree with those found in Figs. 4 and 5. As d approaches $0.5a$, $(qa)_{\rm crit}$ approaches infinity for $\beta > 2$. For $d = 0.5a$ and for $\beta > 2$, no solution for $(qa)_{\rm crit}$ exists indicating that there is no allowed upper interface plasmon mode. For $1 < \beta < 2$, two solutions for $(qa)_{\rm crit}$ exist for a given β and the upper interface plasmon mode is allowed between the smaller and larger $(qa)_{\rm crit}$ values.

For the lower interface plasmon mode, $(qa)_{\rm crit}$ is set by having x_1 equal to minus unity,

$$a - b + c = 0 . \tag{49}$$

Again, solving for β,

$$\beta = -\frac{e^{qa_{\rm crit}+2qd}(-1+e^{qa_{\rm crit}})^2}{(e^{qa_{\rm crit}} - e^{2qd})(e^{3qa_{\rm crit}} - e^{2qd})} . \tag{50}$$

In Fig. 6(b), we plot β vs $(qa)_{\rm crit}$ for $d = 0.55a$, $0.60a$, $0.65a$, $0.70a$, and $0.75a$. For a given β, the solution gives a $(qa)_{\rm crit}$ below which the lower interface plasmon mode exists and above which it does not. If $\beta = 2$, then for $d = 0.65a$ we see that there is no solution for $(qa)_{\rm crit}$. Consequently, the lower interface mode is not allowed for $d > 0.65a$ and $\beta = 2$ as discussed in Fig. 4. For other values of β, the value for d above which there is no solu-

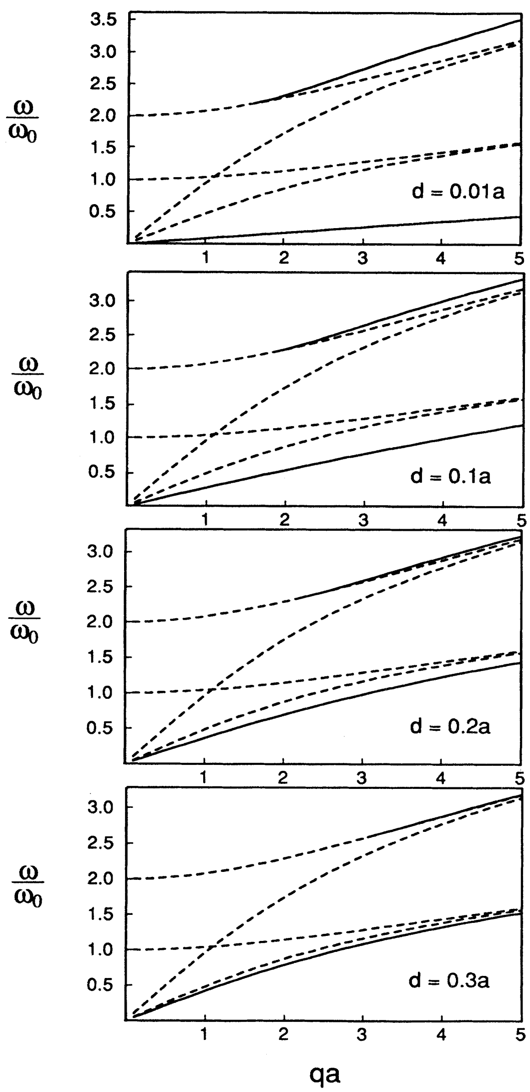

FIG. 5. Plots of the interface plasmon energies vs qa for $\beta = 4$ and separations $d = 0.01a$, $0.1a$, $0.2a$, and $0.3a$.

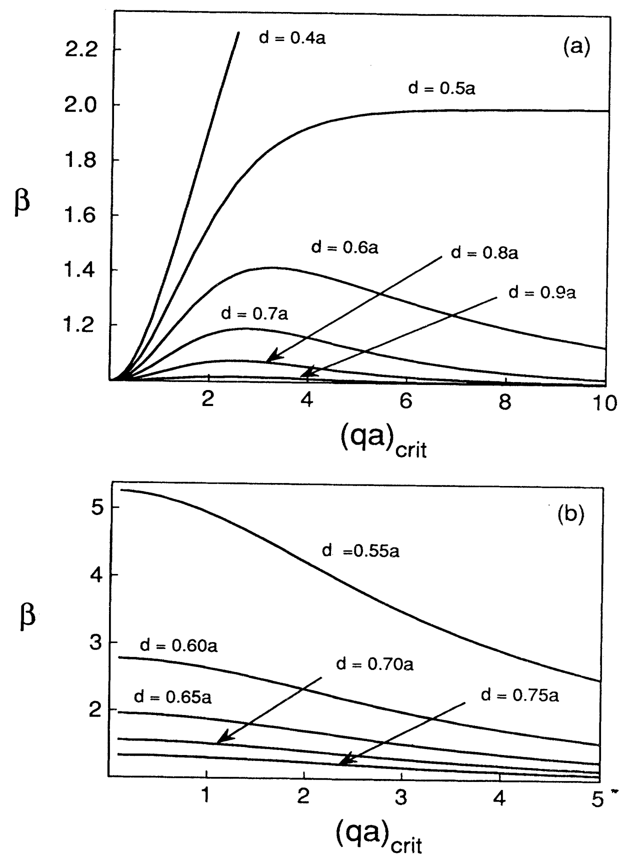

FIG. 6. Plots of β vs $(qa)_{\rm crit}$ determining the critical wave vector for the existence of the interface plasmon modes for various separations d: (a) upper mode, (b) lower mode.

tion can be determined from Fig. 6(b). As d approaches $0.50a$ $(qa)_{\mathrm{crit}}$ goes to infinity. For d less than $0.50a$, the lower acoustic mode is always an allowed solution.

Before concluding, we address the question of the region of validity of the long-wavelength approximation for the susceptibility of the two-dimensional electron gas, Eq. (10). In order for this approximation to be valid, ω must be greater than qv_f. In terms of our reduced units, this condition can be expressed as $\omega/\omega_0 \gg (qa)\sqrt{a_{\mathrm{eff}}/(2a)}$ where $a_{\mathrm{eff}} = \hbar^2\epsilon/me^2$ is the effective Bohr radius. In general, for periodicities greater than the effective Bohr radius (for the GaAs-$Ga_{1-x}Al_xAs$ superlattice, the effective Bohr radius is approximately 100 Å), the long-wavelength approximation is valid. For the results presented here, this condition is easily satisfied, except possibly for the very low-lying acoustic plasmon modes. Assuming that we have an acoustic plasmon solution as expressed by Eq. (36), the above condition reduces to $d \gg a_{\mathrm{eff}}$. For situations in which this is not true, then one must use the exact random-phase approximation (not the $\omega \gg qv_f$ limit) for the susceptibility as derived by Stern.[9] To go beyond the random-phase approximation, higher-order exchange-correlation effects can be included by the standard diagrammatic approach. It is known that exchange-correlation effects are important corrections to the subband energies and optical response function for two-dimensional systems.[13,14] These effects have been approximately included for the two-dimensional plasmon energy by Rajagopal,[15] Beck and Kumar,[16] and Johnson[17] and can be important corrections to the results reported in papers which use the random-phase approximation.

IV. CONCLUSION

We have shown the existence of localized interface plasmon modes for two coupled semi-infinite superlattices separated by a distance d. For the case of equal periodicities and dielectric constants of the two superlattices, exact results for the interface plasmon modes are derived. For $d=0$, only the symmetric eigenstate exists and it exists only for qa above a certain value depending on the ratio of the density of the two superlattices. For d finite, two modes exist corresponding to a symmetric and antisymmetric combination of plasmon states of the individual superlattices. The lower-energy mode (antisymmetric mode) is acousticlike ($\omega \approx q$). As the separation d increases, these modes merge into the continuum, and are not allowed at a critical value of d. Conditions for the existence of the interface plasmon modes as a function of the separation of the superlattices were derived.

For the most general case, with arbitrary parameters for the doping density, periodicities, and dielectric constants of the two semi-infinite superlattices, one must resort to the solution of the fourth-order equation arising from Eqs. (20)–(22).

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