PHYSICAL REVIEW B VOLUME 44, NUMBER 19 15 NOVEMBER 1991-I

Dielectric formalism for a quasi-one-dimensional electron gas. III. Arrays of quantum wires

Yu. M. Sirenko
Institute of Semiconductors, Academy of Sciences of the Ukranian Soviet Socialist Republic, Kiev, U.S.S.R

P. Vasilopoulos
Concordia University, Department of Physics, 1455 de Maisonneuve Boulevard W., Montréal, Québec, Canada H3G 1M8

I. I. Boiko
Institute of Semiconductors, Academy of Sciences of the Ukranian Soviet Socialist Republic, Kiev, U.S.S.R
(Received 5 June 1991)

Quantum transport equations for the one-particle distribution function, pertinent to one-dimensional or two-dimensional periodic arrays of quasi-one-dimensional electron gases (quantum wires), are derived from first principles. The electrons are assumed to interact weakly with an external system and/or with each other. The lateral electron confinement is modeled with a square or parabolic well and the vertical one with a triangular or square well. Screening is treated dynamically and the collision integrals are expressed in terms of the dielectric functions and potential correlators. The results are valid for periods large enough that tunneling between the wires can be neglected. The derived energy and momentum relaxation frequencies, with the help of a drifted Fermi-Dirac distribution function, are given in a form suitable for applications. The momentum relaxation frequency and the mobility are evaluated for an array of quantum wires in interaction with volume and sheet impurities at different distances from the array and it is shown that the Coulomb coupling between the quantum wires can have pronounced effects on both quantities. Both types of the considered lateral confinement lead to similar results with small quantitative differences when only the lowest lateral subband is occupied.

I. INTRODUCTION

In previous papers,[1,2] hereafter referred to as I and II, we developed the quantum kinetic equation for a quasi-one-dimensional electron gas (Q1DEG) in a single quantum wire (QW). The relevant collision integrals as well as the relaxation frequencies for some important cases were expressed in terms of the dielectric functions of the scattering system and of the Q1DEG. The dielectric functions and the correlators of the scattering potentials were calculated for several realistic systems.

In this paper we generalize our theory to the case of a superlattice (SL) of QW's which recently has been the subject of several experimental[3] and theoretical[4,5] investigations. The latter works dealt mainly with collective excitations[4] and the dynamical conductivity[5] for electron-phonon interaction. Here we present a systematic dc transport study, taking into account electron-impurity and electron-electron interaction, and give explicit expressions for the relaxation frequencies and mobilities beyond the Drude model.[6] We treat screening dynamically and take into account remote sheet and volume impurities. To our knowledge these aspects have not been studied before. We consider both one-dimensional (1D) and two-dimensional (2D) arrays of QW's with a simple or complex SL period. Two effects are important: the Coulomb interaction, which affects the screening, and the formation of minibands due to tunneling. In this paper we consider only the first effect assuming that the overlap between the wave functions of two successive QW's is small, i.e., we assume that the separation between neighboring QW's is large. The paper is organized as follows. In the next section we present briefly the derivation of the quantum kinetic equation and of the relaxation frequencies using the formalism of I and II. Both 1D and 2D arrays of QW's are considered. In Sec. III we evaluate the mobility of a Q1DEG due to scattering by impurities. Concluding remarks are presented in the last section and the Appendixes detail some of the results used in the text.

II. TRANSPORT EQUATIONS

A. A single array of QW's

We consider an array of identical QW's arranged periodically along the y direction, with period ℓ, as shown in Fig. 1. A more sophisticated system that can be considered is one with several QW's in each SL period or cell with different or alike particles in each QW. In what follows the kind of particles or wires will be denoted by p. Using the notation of I, we denote the Hamiltonian of the free p particle by H_p^0 and its wave function in the nth cell by

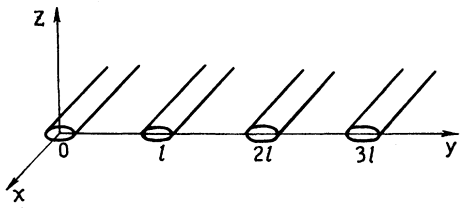

FIG. 1. One-dimensional array of quantum wires.

$$\Psi^p_{nA}(\mathbf{r}) = \psi^p_\alpha(x)\chi^p_a(\rho - \rho_n); \tag{1}$$

here $\rho = (y, z)$, $\rho_n = (n\ell, 0)$, α labels the "longitudinal" eigenfunction along the QW, a the corresponding transverse quantity, and $A = (\alpha, a)$. The corresponding eigenvalue $\hbar\omega^p_A = \hbar\omega^p_a + \hbar\omega^p_\alpha$ does not depend on n. Since we do not consider tunneling the quantum index a is discrete.

With this wave function the total Hamiltonian can be written as [cf. Eq. (1) of I]

$$\begin{aligned} H = H_s &+ \sum_{npA} \hbar\omega^p_A a^\dagger_{npA} a_{npA} \\ &+ \sum_{npAA'} e_p(\varphi^{\mathrm{ex}}_{npAA'} + \varphi^s_{npAA'}) a^\dagger_{npA} a_{npA'} \\ &+ \sum_{nn'pp'AA'BB'} (1 - \tfrac{1}{2}\delta_{p,p'}) V^{nn'pp'}_{AA'B'B} a^\dagger_{npA} a^\dagger_{n'p'A'} \\ &\times a_{n'p'B'} a_{npB}. \end{aligned} \tag{2}$$

Here H_s is the Hamiltonian of the external system, e_p is the charge of the p particle, $\varphi^{\mathrm{ex}} = \varphi^{\mathrm{ex}}(\mathbf{r}, t)$ is the potential of the external electric field (when it is not included in H^0), and $\varphi^s(\mathbf{r}, t)$ is a potential created by the external scattering system. The last term denotes the interparticle interaction; the $a^\dagger$'s and a's are the creation and annihilation operators, respectively.

The procedure for deriving transport equations, detailed in I, is as follows. We start with the equation of motion for the microscopic one-particle density matrix $\rho^{np}_{AB} = a^\dagger_{npB} a_{npA}$. Averaging over the statistical ensemble we obtain the equation for the one-particle distribution function $f^p_{AB} \equiv < \rho^{np}_{AB} >$ in the form

$$\begin{aligned} i\hbar \frac{\partial f^p_{AB}}{\partial t} = \sum_\Gamma [&(H^0_p + e_p\varphi^{\mathrm{ex}} + e_p < \varphi >)^p_{A\Gamma} f^p_{\Gamma B} \\ &- f^p_{A\Gamma}(H^0_p + e_p\varphi^{\mathrm{ex}} + e_p < \varphi >)^p_{\Gamma B}] \\ &+ i\hbar\, \mathrm{St} f^p_{AB}, \end{aligned} \tag{3}$$

where $\mathrm{St} f^p_{AB}$ is the collision integral given by

$$\mathrm{St} f^p_{AB} = \frac{e_p}{i\hbar} < \{\delta\varphi^{np}_{A\Gamma}, \delta\rho^{np}_{\Gamma B}\} - \{\delta\rho^{np}_{A\Gamma}, \delta\varphi^{np}_{\Gamma B}\} > . \tag{4}$$

We have omitted the index n in Eq. (3) and on the left-hand side of Eq. (4) because the QW's are assumed identical; the notation $(\ )^p_{AB}$ in Eq. (3) indicates the matrix element with respect to the basis functions given by Eq. (1) with $n = 0$. The function f^p_{AB} is normalized by the condition $2\sum_A f^p_{AA} = N_p$ where N_p is the total particle number.

To obtain an explicit expression for the collision integral (4) we use the equation for the fluctuating part of the density matrix, $\delta\rho^{np}_{AB}$, in the collisionless approximation, cf. Eq. (13) of I, and the following expresssion for the potential fluctuations:

$$\begin{aligned} \delta\varphi(\omega, q_x, \rho) = \delta\varphi_s(\omega, q_x, \rho) \\ + \int \frac{d^2\rho'}{\epsilon_s(\omega, q_x, \rho, \rho')} \sum_{np} e_p \delta N_{np}(\omega, q_x, \rho'). \end{aligned} \tag{5}$$

Here N_{np} is the number of p particles in the nth cell and $\epsilon_s(\omega, q_x, \rho, \rho')$ is the dielectric function of the external system giving the linear response of the potential, at point ρ, to the test charge eN placed at point ρ'. By virtue of the periodicity we have $\epsilon_s(\omega, q_x, \rho + \rho_n, \rho' + \rho_n) = \epsilon_s(\omega, q_x, \rho, \rho')$.

Solving together the equations for $\delta\rho^{np}_{AB}$ and $\delta\varphi^{np}_{AB}$ and taking discrete Fourier transforms over the number n of cells, as defined in Appendix A, we arrive at the analog of Eq. (26) of I:

$$\begin{aligned} \delta\varphi_K(\omega, q_x, q_y) = \sum_{K'} \Bigg(&\frac{\delta\varphi^0_{K'}(\omega, q_x, q_y)}{\epsilon_{KK'}(\omega, q_x, q_y)} \\ &+ R_{KK'}(\omega, q_x, q_y)\delta\varphi^s_{K'}(\omega, q_x, q_y) \Bigg), \end{aligned} \tag{6}$$

where $K = (p, a, b)$ and

$$\frac{1}{\epsilon_{KK'}(\omega, q_x, q_y)} = \sum_{K''} \frac{R_{KK''}(\omega, q_x, q_y)}{\epsilon^s_{K''K'}(\omega, q_x, q_y)}, \tag{7}$$

$$R_{KK'}(\omega, q_x, q_y) = P^{-1}_{KK'}(\omega, q_x, q_y), \tag{8}$$

$$P_{KK'}(\omega, q_x, q_y) = \delta_{KK'} + \frac{\Delta\epsilon_{K'}(\omega, q_x)}{\epsilon^s_{KK'}(\omega, q_x, q_y)}. \tag{9}$$

The value $\Delta\epsilon_K(\omega, q_x)$ is given by Eq. (23) of I and the quantities $1/\epsilon^s_{KK'}(\omega, q_x, q_y)$ and $\delta\varphi_K(\omega, q_x, q_y)$ are the discrete Fourier transforms of

$$\frac{1}{\epsilon^s_{KK'}(\omega, q_x, n)}$$

$$= \int \frac{\chi^{*p}_a(\rho - \rho_n)\chi^p_b(\rho - \rho_n)\chi^{p'}_{a'}(\rho')\chi^{*p'}_{b'}(\rho')}{\epsilon_s(\omega, q_x, \rho, \rho')} d^2\rho\, d^2\rho' \tag{10}$$

and

$$\delta\varphi_K(\omega, q_x, n) = \int \chi^{*p}_a(\rho - \rho_n)\chi^p_b(\rho - \rho_n)\delta\varphi(\omega, q_x, \rho) d^2\rho, \tag{11}$$

respectively. As for $\delta\varphi^0_K(\omega, q_x, n)$ it is defined with the help of Eq. (26′) of I. For some particular cases the expression $1/\epsilon^s_{KK'}(\omega, q_x, q_y)$ is evaluated in Appendix B.

Taking the inverse Fourier transform of Eq. (6) we obtain the values $\delta\varphi^{np}_{AB}$ and $\delta\rho^{np}_{AB}$ which determine the collision integral, cf. Eq. (4). Assuming the absence of initial correlations between the p particles and between different QW's we can write the diagonal components of the collision integral, $\mathrm{St}f^p_A \equiv \mathrm{St}f^p_{AA}$, as

$$\mathrm{St}f^p_A = \mathrm{St}_{ps} f^p_A + \sum_{p'} \mathrm{St}_{pp'} f^p_A. \tag{12}$$

The interparticle collision integral, $\mathrm{St}_{pp'} f^p_A$, is given by Eq. (34) of I and the transition probability $W^{pp'}_{AB',BA'}$ by Eq. (35) of I with q relaced by q_x and $1/|\epsilon_{KK'}(\omega,q)|^2$ by

$$S_{KK'}(\omega,q_x) = \sum_n |\epsilon_{KK'}(\omega,q_x,n)|^{-2} = \frac{\ell}{2\pi}\int_{-\pi/\ell}^{\pi/\ell} \frac{dq_y}{|\epsilon_{KK'}(\omega,q_x,q_y)|^2}. \tag{13}$$

As for $\mathrm{St}_{ps} f^p_A$, which expresses particle collisions with the external system and the relevant transition probability W_{AB}, they are given by Eqs. (36) and (37) of I but now the relevant correlators and the polarization function are defined differently. Using the notations $\overline{K} = (p,b,a)$,

$$A^K_{\omega,q_x} = [< \delta\Phi^2_s >^K_{\omega,q_x}; < \delta\Phi_s \delta\Phi_s >^K_{\omega,q_x}; \Pi_K(\omega,q_x)], \tag{14}$$

$$B_{KK'}(\omega,q_x,n) = \Big(\tfrac{1}{2}[< \delta\varphi^s_K \delta\varphi^s_{K'} >_{\omega,q_x,n} + < \delta\varphi^s_{K'} \delta\varphi^s_K >_{\omega,q_x,-n}]; < \delta\varphi^s_K \delta\varphi^s_{K'} >_{\omega,q_x,n}; \frac{i}{2}[1/\epsilon^s_{KK'}(\omega,q_x,n) - 1/\epsilon^{*s}_{K'K}(\omega,q_x,-n)] \Big), \tag{15}$$

we can write

$$A^K_{\omega,q_x} = \sum_{K'K''}\sum_{n'n''} R_{KK'}(\omega,q_x,n') R^*_{KK''}(\omega,q_x,n'') B_{K'K''}(\omega,q_x,n''-n') = \frac{\ell}{2\pi}\int_{-\pi/\ell}^{\pi/\ell} \sum_{K'K''} R_{KK'}(\omega,q_x,q_y) R^*_{KK''}(\omega,q_x,q_y) B_{K'K''}(\omega,q_x,q_y) dq_y. \tag{16}$$

$B_{KK'}(\omega,q_x,q_y)$ is obtained from $B_{KK'}(\omega,q_x,n)$ with n replaced by q_y and $\epsilon^s_{KK'}(\omega,q_x,-n)$ by $\epsilon^{*s}_{KK'}(\omega,q_x,q_y)$. The potential correlators in Eq. (15) are given by

$$< \delta\varphi^s_K \delta\varphi^s_{K'} >= \int d^2\rho \int d^2\rho' \chi^{*p}_a(\boldsymbol{\rho}-\boldsymbol{\rho}_n)\chi^p_b(\boldsymbol{\rho}-\boldsymbol{\rho}_n)\chi^{p'}_{a'}(\boldsymbol{\rho}')\chi^{*p'}_{b'}(\boldsymbol{\rho}') < \delta\varphi_s(\boldsymbol{\rho})\delta\varphi_s(\boldsymbol{\rho}') >_{\omega,q_x}. \tag{17}$$

The quantum kinetic equation as well as the energy and momentum balance equations of I, written for a single QW, have the same form and will not be given here. The only changes for the present SL case occur in the dielectric functions and the potential correlators as explained above. Below we give only the final expressions for the momentum (ν^m) and energy (ν^T) relaxation frequencies which correspond to Eqs. (69) and (70) of I since we will use them in the next section:

$$\begin{pmatrix} \nu^m_{aa'} \\ \nu^T_{aa'} \end{pmatrix} = \frac{2}{\pi^2 n_a}\int_0^\infty \frac{d\omega}{\omega^2}\int_0^\infty dq_x \begin{pmatrix} k_B T q_x^2/m^*_a \\ \omega^2 \end{pmatrix} \frac{\omega_{T_a}\omega_{T_s}}{\sinh(\omega_{T_a})\sinh(\omega_{T_s})} \times \mathrm{Im}\Delta\epsilon^{eq}_{aa}(\omega,q_x)\mathrm{Im}\Delta\epsilon^{eq}_{a'a'}(\omega,q_x) S^{eq}_{aa,a'a'}(\omega,q_x), \tag{18}$$

$$\begin{pmatrix} \nu^m_{as} \\ \nu^T_{as} \end{pmatrix} = \frac{2}{\pi^2 n_a \hbar}\int_0^\infty dq_x \int_0^\infty \frac{d\omega}{\omega^2} \begin{pmatrix} k_B T q_x^2/m^*_a \\ \omega^2 \end{pmatrix} \frac{\omega_{T_a}\omega_{T_s}}{\sinh(\omega_{T_a})\cosh(\omega_{T_s})} \mathrm{Im}\Delta\epsilon^{eq}_{aa}(\omega,q_x) < \delta\Phi^2_s >^{aa}_{\omega,q_x}. \tag{19}$$

In these equations the external system is assumed to be in equilibrium at temperature $T_s, \omega_T = \hbar\omega/2k_BT$, and a drifted Fermi function with effective temperature T_a was used. Each subband a is assumed to have its own drift velocity u_a, its temperature T_a, and its effective mass m^*_a and intersubband transitions are neglected. The properties of the external system are specified by $< \delta\Phi^2_s >$ and its connection with the dielectric function is given by Eq. (A11) of I.

B. System of QW's periodic in two directions

Let us consider a system periodic along the y and z directions with periods ℓ_y and ℓ_ζ, respectively. It is convenient now to enumerate the cells with the numbers n_y and n_ζ and use the notation $\mathbf{n} = (n_y, n_\zeta)$. The case of a simple SL, with one QW per cell in either direction, is shown in Fig. 2. If more than one sort of particles is present we use again the label p. QW's of the

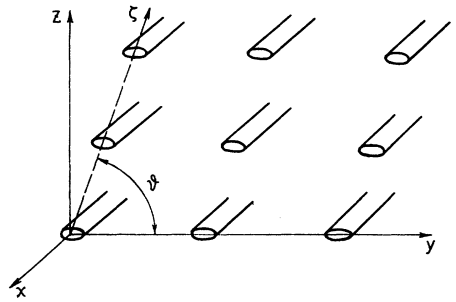

FIG. 2. Two-dimensional array of quantum wires.

same sort p, in different cells, are assumed to be identical. The results of the previous subsection can be taken over with the following changes: n is replaced by $\mathbf{n}$, ρ_n by $\rho_{\mathbf{n}} = n_y \ell_y \mathbf{e}_y + n_\zeta \ell_\zeta \mathbf{e}_\zeta$, and q_y by $\mathbf{Q} = (q_y, q_\zeta)$. Moreover, in Eqs. (13) and (17), $(l/2\pi)\int dq_y$ must be replaced by $A^{-1}\int_{\mathrm{BZ}} d^2Q$, where the integration is over the Brillouin zone and $A = 4\pi^2/\ell_y \ell_\zeta \sin\theta$ is its area.

Finally, we notice that when ℓ_y and ℓ_ζ tend to infinity (or when ℓ does in the case of a SL in one direction) the terms with $\mathbf{n} \neq \mathbf{0}$ (or $n \neq 0$) in the dielectric functions and correlators vanish and we recover the results of a single QW treated in I.

III. SCATTERING OF A DEGENERATE Q1DEG BY IMPURITIES

As an illustration of the general theory presented above we consider the scattering of a Q1DEG, in the SL array with period ℓ, by two impurity configurations as shown in Fig. 3. (i) A sheet of impurities with surface density n_i^s is occupying the plane $z = \lambda_i$. When $\lambda_i = 0$ we have interface impurities. (ii) The half-space $z > \ell_i$ is occupied by impurities of density n_i. This is a model for volume doping with a spacer of width ℓ_i.

We assume that the Q1DEG, in all QW's, occupies only the lowest subbands both in the z and the y direction, i.e., $a = b = 0$. Along the z direction this is the most typical experimental situation for the relevant electron densities. Along the y direction the assumption is made for simplicity and transparency of the results. The QW's parallel to the x axis are assumed to lie in the $z = 0$ plane and to be infinitely thin along z. The confinement along y is modeled with a square well (S), of width W, or with a parabolic well (P) of width λ, cf. Eqs. (14) and (15) of II. The dielectric constants for the regions $z > 0$ and $z < 0$ are denoted by ϵ_1 and ϵ_2, respectively.

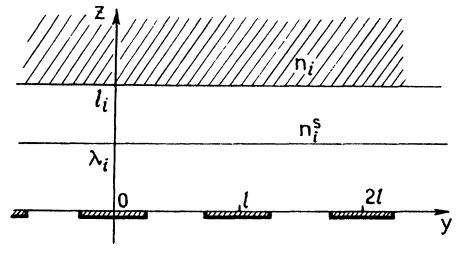

FIG. 3. One-dimensional array of quantum wires parallel to the x axis. Sheet impurities are situated in the plane $z = \lambda_i$ and volume impurities in the region $z \geq \ell_i$.

To evaluate the momentum relaxation frequency ν^m, as given by Eq. (19) we need the quantities $\Delta\epsilon_{00}^{eq}(\omega, q_x)$ and $< \delta\Phi_s^2 >_{\omega,q_x}$ that correspond to one QW. They are given by Eqs. (B9) and (35) of II, respectively, for $a = 0$, $\omega > 0$, $q_x > 0$:

$$\mathrm{Im}\Delta\epsilon_{00}^{eq}(\omega, q_x) \approx \frac{2m^{*2}e^2}{\hbar^3 k_F^2}\omega\delta(q_x - 2k_F) + \frac{q_x e^2}{\hbar}\delta\left(\omega - \frac{\hbar q_x k_F}{m^*}\right), \tag{20}$$

$$< \delta\Phi_s^2 >_{\omega,q_x}^{00} = 2\pi\delta(\omega) < \delta\Phi_s^2 >_{q_x}^{00}, \tag{21}$$

where $k_F = \pi n_e/2$ is the Fermi wave vector and n_e is the electron concentration. Substituting Eqs. (20) and (21) in Eq. (19) we obtain

$$\nu^m = (8e^2 m^*/\pi\hbar^3 n_e) < \delta\Phi_s^2 >_{q_x=2k_F}^{00} . \tag{22}$$

Now with the help of Eqs. (8), (9), (B4), and (B5) we find

$$< \delta\Phi_s^2 >_{q_x}^{00} = \int \frac{dq_y}{2\pi}\left|1 + \frac{\Delta\epsilon(0,q_x)}{\epsilon_{0000}^s(0,q_x,q_y)}\right|^{-2} \times |\Phi_{00}(q_y)|^2 < \delta\varphi_s^2 >_{q_x,q_y}^{z=z'=0}, \tag{23}$$

where

$$\frac{1}{\epsilon_{0000}^s(\omega,q_x,q_y)} = \frac{2\pi}{\ell\bar{\epsilon}}\sum_{c=-\infty}^{\infty}\frac{|\Phi_{00}(q_y - 2\pi c/\ell)|^2}{\sqrt{q_x^2 + (q_y - 2\pi c/\ell)^2}}, \tag{24}$$

and $\bar{\epsilon} = (\epsilon_1 + \epsilon_2)/2$. In obtaining Eq. (23) we used the equalities $\epsilon_{K_z,K_z}^s(\omega,q_x,q_y) = \bar{\epsilon}$ and $< \delta\varphi_s^2 >_{\omega,q_x,q_y}^{K_z,K_z} =< \delta\varphi_s^2 >_{\omega,q_x,q_y}^{z=z'=0}$ which are valid for an infinitely thin Q1DEG occupying the lowest subband. The potential correlator $< \delta\varphi_s^2 >_{q_x,q_y}^{z=z'=0}$, for the considered impurity configuration was calculated previously. Equation (40) of II and Eq. (B3) give

$$< \delta\varphi_s^2 >_{q_x,q_y}^{z=z'=0} = \left(\frac{2\pi e}{\bar{\epsilon}k}\right)^2\left(n_i^s e^{-2k\lambda_i} + \frac{n_i}{2k}e^{-2k\ell_i}\right), \quad k = \sqrt{q_x^2 + q_y^2}. \tag{25}$$

As for the dielectric function of the Q1DEG it is given by Eqs. (B1) and (B2) of II. At low temperatures, i.e., for degenerate statistics, the main contribution to static screening comes from the real part of $\Delta\epsilon$, so in Eq. (23) we can use

$$\mathrm{Re}\Delta\epsilon_{00}(0, 2k_F) = \frac{2m^*e^2}{\pi^2\hbar^2 n_e}\ln\frac{\hbar^2\pi^2 n_e^2}{8m^* k_B T}. \tag{26}$$

Using now Eqs. (22)–(26) and Eqs. (19) and (20) of II for $\Phi^{P,S}(q_y)$, after the substitution $q_y = 2k_F y = \pi n_e y$, we arrive at the following expression for the momentum relaxation frequency ν^m and the mobility μ of the SL array of Q1DEG's interacting with impurities (P stands for parabolic well and S for square well):

$$\nu^m_{P,S} = \frac{e}{m^* \mu_{P,S}} = \frac{32e^4 m^*}{\pi \hbar^3 n_e^2} \int_0^\infty dy \left(\bar{\epsilon} + \frac{\bar{\epsilon}\, \mathrm{Re}\Delta\epsilon_{00}(0, 2k_F)}{\epsilon^s_{0000}(0, 2k_F, 2k_F y)} \right)^{-2} \times A_{P,S}(y)[(n_i^s/S_y^2) e^{-2\pi n_e \lambda_i S_y} + (n_i/2\pi n_e S_y^3) e^{-2\pi n_e \ell_i S_y}], \quad (27)$$

where

$$\frac{\bar{\epsilon}}{\epsilon^s_{0000}(0, 2k_F, 2k_F y)} = \frac{2}{n_e \ell} \sum_{c=-\infty}^{\infty} \frac{A_{P,S}(y_c)}{S_{y_c}},$$

$$y_c = y - 2c/n_e \ell, \quad (28)$$

$$A_P(y) = e^{-(\pi \lambda n_e y)^2/2}, \quad (29)$$

$$A_S(y) = \left(\frac{2}{\pi n_e W y} \frac{\sin(\pi n_e W y/2)}{1 - (n_e W y/2)^2} \right)^2, \quad S_y = \sqrt{1 + y^2}.$$

Two limiting cases can be considered analytically. For large ℓ, such that $n_e \ell \gg 1$, the Coulomb interaction between wires is very small and the QW's can be treated as isolated. In this case the sum in Eqs. (24) and (28) can be transformed into an integral and the dependence on ℓ disappears. The opposite limit $n_e \ell \ll 1$ is satisfied when $n_e \ll n_e^{\max}$, where $n_e^{\max}$ is the maximum n_e that corresponds to one-subband occupation. For a square well $n_e^{\max} = 2/W$ and for a parabolic one $n_e^{\max} = 2\sqrt{2}/\pi\lambda$. In this case the main contribution to the integral in Eq. (27) comes from $y < 1$ and the only significant term in Eq. (28) is the one with $c = 0$. Therefore, the ratio $\bar{\epsilon}/\epsilon^s_{0000}(0, 2k_F, 2k_F y)$ is independent of y and equal to $2/n_e \ell$. It means that only the wires within distances less than or of the order of the de Broglie length, $1/k_F$, of the Q1DEG participate in screening. Carrying out the integration in Eq. (27) for $\ell_i, \lambda_i \gg 1/n_e$ and for $\ell_i, \lambda_i \ll 1/n_e$, we can construct the following interpolation formula valid for arbitrary ℓ_i and λ_i:

$$\nu^m_{P,S} \approx \frac{16e^4 m^*}{\hbar^3 n_e^2} \left(\bar{\epsilon} + \frac{2}{n_e \ell} \mathrm{Re}\Delta\epsilon_{00}(0, 2k_F) \right)^{-2} \times \left(\frac{n_i^s e^{-2\pi n_e \lambda_i}}{1 + \pi\sqrt{n_e \ell_i}} + \frac{(n_i/\pi^2 n_e) e^{-2\pi n_e \ell_i}}{1 + 2\sqrt{n_e \lambda_i}} \right). \quad (30)$$

For a numerical evaluation of ν^m, as given by Eqs. (26)–(29), we use the parameters appropriate to GaAs/Al_xGa_{1-x}As heterojunction wires: $m^* = 0.067 m_0$, $\epsilon = 12.9$. The dependence of ν_s^m on the SL period ℓ is shown in Fig. 4 for volume impurities and in Fig. 5 for sheet impurities. In both figures the parameters are $n_e = 5 \times 10^5$/cm, $W = 50$ Å, and $T = 10$ K. As can be seen the relaxation frequency decreases slightly with decreasing ℓ. In comparison with the case of isolated wires, $\ell \to \infty$, the decrease is about 25% for volume impurities at distance $\ell_i = 0$ from the SL array and about 50% for sheet impurities at $\lambda_i = 0$. For $\ell_i = \lambda_i = 200$ Å the decrease is about 100%. This effect results from the weakening of the scattering potential as the screening becomes stronger

The dependencies of ν^m/n_i^s and of ν^m/n_i on the distances λ_i and ℓ_i of the sheet and volume impurities, respectively, are shown in Fig. 6. The Q1DEG is assumed to occupy the lowest subband in a square well of width $W = 200$ Å, its concentration is $n_e = 10^6$/cm, and

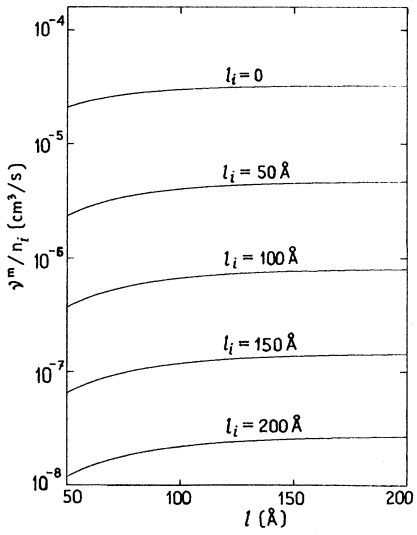

FIG. 4. Relaxation frequency ν^m/n_i as function of the SL period ℓ for scattering by volume impurities situated in the region $z \geq \ell_i$. The electron density is $n_e = 5 \times 10^5/\mathrm{cm}^2$, the wire width $W = 50$ Å, and the temperature $T = 10$ K.

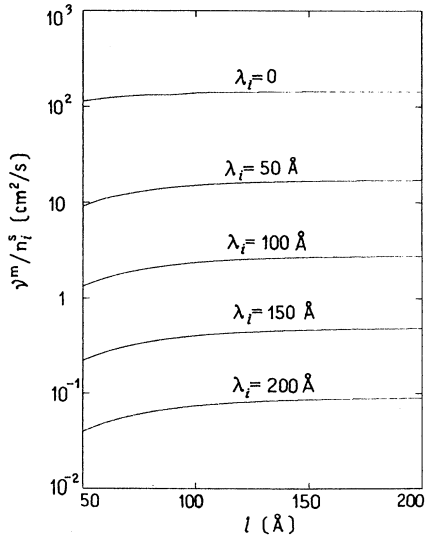

FIG. 5. Relaxation frequency ν^m/n_i^s as function of the SL period ℓ for scattering by sheet impurities situated in the plane $z = \lambda_i$. The parameters are the same as in Fig. 4.

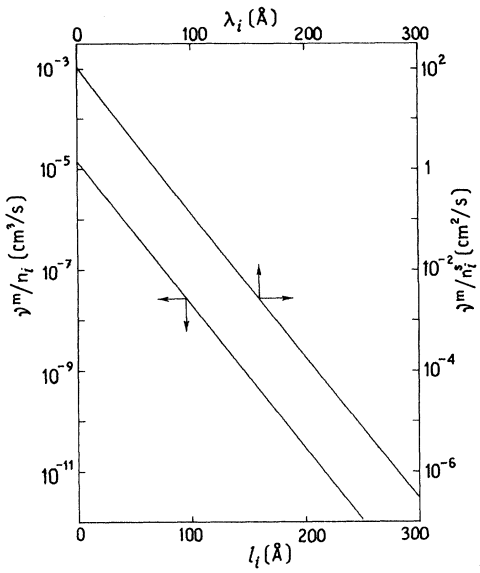

FIG. 6. Lower left panel: Relaxation frequency ν^m/n_i as function of the volume impurities' smallest distance ℓ_i. Upper right panel: Relaxation frequency ν^m/n_i^s as function of the sheet impurities' distance λ_i. The electron density is $n_e = 10^6$/cm, the square well width $W = 200$ Å, and the temperature $T = 30$ K. The results for a parabolic well of width $\lambda = 90$ Å are very close to those shown.

its temperature 30 K. For the same concentration and a parabolic well of an effective width $\lambda = 90$ Å the results are very close to those of Fig. 6 and are not shown for clarity. For planar impurities $\nu_P^m = 0.89\nu_S^m$ at $\lambda_i = 0$; at $\lambda_i = 200$ Å we have $\nu_P^m = 1.09\nu_S^m$. For volume impurities $\nu_P^m = 0.85\nu_S^m$ at $\ell_i = 0$ and $\nu_P^m = 1.08\nu_S^m$ at $\ell_i = 200$ Å. For volume impurities the equality $\nu_S^m = \nu_P^m$ is attained when $\ell_i \approx 80$ Å and for planar impurities when $\lambda_i \approx 70$ Å. All the numerical results can be fitted well with Eq. (30) for both types of confinement.

IV. SUMMARY

In this paper we have extended the previously developed dielectric formalism (cf. I and II) of a single QW to a one-dimensional or two-dimensional array of QW's. The wires were coupled only via the Coulomb interaction and tunneling between them was not considered. The only modifications of the previous formalism occurred in the dielectric functions and the potential correlators as explained in the text.

The general expression for the momentum relaxation frequency and the mobility was evaluated for a SL array of QW's in interaction with planar and volume impurities at different distances from the array. The calculations show that the effect of Coulomb coupling between the QW's can be quite pronounced, of the order of several tens %. The mobility increases exponentially with the distance of the impurities from the array. Finally the results between the square and parabolic lateral confinement differ by about 15%. This small difference is expected because only the lowest subband has been assumed to be occupied. If, depending on the density, several subbands are occupied the differences are expected to be more pronounced since the subbands are not equally in the former case whereas they are in the latter.

ACKNOWLEDGMENTS

The work of one of us (P.V.) was supported by Natural Sciences and and Engineering Research Council (NSERC) of Canada Grant No.URF0116877.

APPENDIX A

The discrete Fourier transformation, used in the text, is defined as

$$A(q_y) = \sum_{n=-\infty}^{\infty} A(n)e^{-iq_y n\ell}, \quad A(\mathbf{Q}) = \sum_{\mathbf{n}} A(\mathbf{n})e^{-i\mathbf{Q}\cdot\boldsymbol{\rho}_n}, \tag{A1}$$

for a 1D or 2D array, respectively. The functions $A(q_y)$ and $A(\mathbf{Q})$ are periodic and have the period of the reciprocal lattice. The inverse transformation is defined by

$$A(n) = \frac{\ell}{2\pi}\int_{-\pi/\ell}^{\pi/\ell} A(q_y)e^{iq_y n\ell}dq_y,$$
$$A(\mathbf{n}) = \frac{1}{B}\int_{\mathrm{BZ}} A(\mathbf{Q})e^{i\mathbf{Q}\cdot\boldsymbol{\rho}_n}d^2Q, \tag{A2}$$

where BZ indicates the Brillouin zone whose area is $B = 4\pi^2/\ell_y\ell_\zeta \sin\theta$.

APPENDIX B

Below we present the relevant dielectric functions and potential correlators (or corrrelation functions) uniform in (i) three and (ii) two directions.

(i) If the external system is uniform in all directions it can be characterized by the usual 3D dielectric function $\epsilon_s(\omega, \mathbf{q})$ and by the potential correlator $< \delta\varphi_s\delta\varphi_s >_{\omega,\mathbf{q}}$, where $\mathbf{q} = (q_x, q_y, q_z)$. Using Eqs. (5), (10), (11), and (A1) we obtain

$$\begin{bmatrix} < \delta\varphi_K^s\delta\varphi_{K'}^s >_{\omega,q_x,q_y} \\ \dfrac{1}{\epsilon_{KK'}^s(\omega,q_x,q_y)} \end{bmatrix} = \frac{1}{2\pi\ell}\sum_n \int \begin{bmatrix} < \delta\varphi_s\delta\varphi_s >_{\omega,\mathbf{q}-\mathbf{g}_n} \\ \dfrac{4\pi}{(\mathbf{q}-\mathbf{g}_n)^2\epsilon^s(\omega,\mathbf{q}-\mathbf{g}_n)} \end{bmatrix} \Phi_K(\mathbf{Q}-\mathbf{g}_n)\Phi_{K'}^*(\mathbf{Q}-\mathbf{g}_n)dq_z, \tag{B1}$$

where $\mathbf{g}_n = (2\pi/\ell)\mathbf{e}_y$, $\mathbf{e}_y$ being the unit vector along the y axis. The functions $\Phi(\mathbf{Q})$ are given, for square and parabolic confinement, by Eqs. (11) and (16)–(20) of II. Using Eq. (B1) and taking into account the periodicity of the functions $R_{K'}(\omega, q_x, q_y)$ in q_y we can rewrite Eq. (17) in the simpler form

$$\begin{bmatrix} < \delta\Phi_s \delta\Phi_s >^K_{\omega,q_x} \\ \Pi_K(\omega, q_x) \end{bmatrix} = \frac{1}{4\pi^2} \int\int \sum_{K'K''} \begin{bmatrix} < \delta\varphi_s \delta\varphi_s >_{\omega,\mathbf{q}} \\ \dfrac{4\pi\, \mathrm{Im}\epsilon_s(\omega, \mathbf{q})}{q^2\, |\epsilon_s(\omega, \mathbf{q})|^2} \end{bmatrix} \times R_{KK'}(\omega, q_x, q_y) R^*_{KK''}(\omega, q_x, q_y) \Phi_{K'}(\mathbf{Q}) \Phi^*_{K''}(\mathbf{Q}) dq_y dq_z . \tag{B2}$$

(ii) If the external system is homogeneous, e.g., in the xy plane and inhomogeneous in the z direction it can be characterized by $\epsilon_s(\omega, q_x, q_y, z, z')$ and by $< \delta\varphi_s \delta\varphi_s >^{z,z'}_{\omega,q_x,q_y}$ defined by Eqs. (26) and (37) of II, respectively. We now assume that the electron eigenfunction has the form $\chi_a(\boldsymbol{\rho}) = Y_{a_y}(y) Z_{a_z}(z)$ and we introduce the following functions [cf. Eq. (28) of II]

$$\begin{bmatrix} < \delta\varphi_{K_z} \delta\varphi_{\overline{K'}_z} >_{\omega,q_x,q_y} \\ \dfrac{1}{\epsilon^s_{K_z K'_z}(\omega, q_x, q_y)} \end{bmatrix} = \int\int \begin{bmatrix} < \delta\varphi_s \delta\varphi_s >^{z,z'}_{\omega,q_x,q_y} \\ \dfrac{1}{\epsilon_s(\omega, q_x, q_y, z, z')} \end{bmatrix} Z^{*p}_{a_z}(z) Z^p_{b_z}(z) Z^{p'}_{a'_z}(z') Z^{*p'}_{b'_z}(z') dz\, dz', \tag{B3}$$

where $K = (K_y, K_z)$, $K_y = (p, a_y, b_y)$, $K_z = (p, a_z, b_z)$. With the help of Eqs. (5), (10), (11), (B3), and Eq. (28) of II we rewrite Eq. (B1) as

$$\begin{bmatrix} < \delta\varphi^s_K \delta\varphi^s_{K'} >_{\omega,q_x,q_y} \\ \dfrac{1}{\epsilon^s_{KK'}(\omega, q_x, q_y)} \end{bmatrix} = \frac{1}{\ell} \sum_n \Phi_{K_y}(q_y - g_n) \Phi^*_{K'_y}(q_y - g_n) \begin{bmatrix} < \delta\varphi^s_{K_z} \delta\varphi^s_{K'_z} >_{\omega,q_x,q_y - g_n} \\ \dfrac{2\pi}{\sqrt{q_x^2 + (q_y - g_n)^2}\, \epsilon^s_{K_z K'_z}(\omega, q_x, q_y - g_n)} \end{bmatrix} . \tag{B4}$$

The functions in square brackets on the right-hand side have been evaluated for some model systems in II and Ref. 7. Using Eq. (B4) and the periodicity of the function $R_{KK'}(\ )$ with respect to q_y we can reduce Eq. (17) to the form

$$\begin{bmatrix} < \delta\Phi_s \delta\Phi_s >^K_{\omega,q_x} \\ \Pi_K(\omega, q_x) \end{bmatrix} = \frac{1}{2\pi} \int \sum_{K'K''} \begin{bmatrix} < \delta\varphi^s_{K'_z} \delta\varphi^s_{K''_z} >_{\omega,q_x,q_y} \\ \dfrac{i\pi}{\sqrt{q_x^2 + q_y^2}} \left(\dfrac{1}{\epsilon^s_{K'_z K''_z}(\omega, q_x, q_y)} - \dfrac{1}{\epsilon^{*s}_{K''_z K'_z}(\omega, q_x, q_y)} \right) \end{bmatrix} \times R_{KK'}(\omega, q_x, q_y) R^*_{KK''}(\omega, q_x, q_y) \Phi_{K'_y}(q_y) \Phi^*_{K''_y}(q_y) dq_y . \tag{B5}$$

[1] I. I. Boiko, Yu. M. Sirenko, and P. Vasilopoulos, Phys. Rev. B **43**, 7216 (1991).

[2] I. I. Boiko, Yu. M. Sirenko, and P. Vasilopoulos, Phys. Rev. B **43**, 7224 (1991).

[3] M. Tanaka, J. Motohisa, and H Sakaki, Surf. Sci. **228**, 408 (1990); M. Tsuchiya, J. M. Gaines, R. M. Yan, R. J. Simes, P. O. Holtz, L. A. Coldren, and P. M. Petroff, Phys. Rev. Lett. **62**, 1320 (1988); T. Fukui and H. Saito, Appl. Phys. Lett. **50**, 824 (1987).

[4] Y. Zhu and S. Zhou, J. Phys. C **21**, 3063 (1988); M. Que and A. Kirczenow, Phys. Rev. B **37**, 7153 (1988).

[5] H. L. Cui and N. J. M. Horing, Phys. Rev. B **40**, 2956 (1989); N. Tzoar and C. Zhang, Phys. Rev. B **32**, 1146 (1985).

[6] S. DasSarma, and X. C. Xie, Phys. Rev. B **35**, 9875 (1987).

[7] I. I. Boiko and Yu. M. Sirenko, Phys. Status Solidi B **159**, 805 (1990).