Quantum-state representations in a strong quantizing magnetic field: Pairing theory of superconductivity

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(Received 23 May 1991)

Two of the commonly used basis states of an electron in a strong quantizing magnetic field ("orbit center" and "angular momentum"), their interrelation, and physical interpretations are described briefly here. Their relative utility in elucidating various physical aspects is exhibited by examining (1) the exact solution of the linearized superconducting gap equation, and (2) the matrix elements of a plane wave in both representations. It is shown that the two-particle matrix element of the contact interaction, $\bar{V}\delta(\mathbf{r}_1-\mathbf{r}_2)$, in the orbit-center representation is separable in the relative orbit centers. A complete set of solutions of the linearized gap equation is thereby obtained.
PACS number(s): 74.20.$-$z

I. INTRODUCTION

The original Landau solution[1] to the problem of the quantization of electrons moving in a strong constant magnetic field has many interesting, and by now well-known, consequences. Some of the most recent ones will be mentioned here.

There are several equivalent representations of the motion of a particle in the presence of a constant magnetic field which reflect the various ways of expressing the energy (Landau) level degeneracies. Two of these representations, which have been used within different contexts, will be described here. The first basis exploits the translational covariance of the system, with one Cartesian coordinate of the orbit center being the degenerate quantum number, and therefore we refer to this formulation as the "orbit-center" basis. The second basis, which we refer to as the "angular-momentum" basis, exploits the rotational covariance, with the angular momentum along the magnetic-field direction being the degenerate quantum number. For a given Landau level, the angular momentum determines the radial distance of the orbit center from the origin of coordinates.

The latter states were employed in the description of the fractional quantum Hall effect,[2] whereas the former were employed in the calculations required in developing the anyon theory of superconductivity,[3] the pairing theory for projectively translation-invariant states,[4] integer and fractional quantum Hall effects,[5] and in discussing exotic superconducting states in strong magnetic fields.[6-8] Both of these states have been employed in the past in discussing the quantum theory of the galvanomagnetic effect at extremely strong magnetic fields.[9]

The one-particle thermal Green function of this problem is found to be most useful in many of these formulations. This function can be constructed from either of the two representations above. The orbit-center states in such a construction were found to be relatively easy to manipulate, leading to a cylindrically symmetric expression upon summing over the orbit centers. Such an explicit construction using the angular-momentum basis is not as easy a manipulation but can, of course, be done after much algebra. The development of the critical temperature of the superconducting state in the presence of a strong magnetic field was set up using the orbit-center basis and an exact solution of the linearized gap equation was found.[6,7] The nature of the superconducting state below T_c has not yet been fully discussed in the published literature but, in view of the recent work, this issue is of interest, and will be discussed in a separate publication.

The purpose of the present paper is to provide an insight into the utility of these representations in various situations. We discuss the pairing phenomena by examining the known exact solution of the linearized gap equation in both the angular-momentum and orbit-center bases to illustrate these aspects. It will be shown that pairing of states in given Landau levels (N,N') requires their associated angular momenta (m,m') be such that $m+m'=0$ for spin-singlet pairing with a selection rule that m be restricted to a finite set of $(N+N'+1)$ states, given by $m=-N,-N+1,\ldots,-1,0,1,\ldots,N'$, among the infinitely degenerate angular-momentum states. In the orbit center representation, the pairing of the states in a given pair of Landau levels involves all positions of the orbit center. It is shown that the corresponding orbit-center representation of the gap function can be interpreted as a product wave function of the center of mass and the relative orbit-center coordinates: a function of the center of mass of the orbit centers which has the functional form of the lowest Landau level wave function, and a function of the orbit-center difference which has the form of the $N+N'$ Landau level wave function, with no selection rules. The superconducting properties for $T<T_c$ thus requires pairing among several Landau levels so that the approach to T_c from below is consistent with that found previously from the normal-state instability towards the pairing state at T_c.

In order to gain more insight into the nature of correlations among the Landau states, the two-particle matrix element of the contact interaction $\bar{V}\delta(\mathbf{r}_1-\mathbf{r}_2)$ is evaluated in closed form in the orbit-center representation. In this representation, the interaction becomes separable in the relative orbit centers, with a finite rank. Then the linearized superconducting gap equation in the orbit-center representation can be solved completely, thus generalizing the exact solution obtained before.[6] In a separate paper,[10,11] we will present vortex solutions of the Gorkov equations, which generalize the well-known Abrikosov quasiclassical solutions, and meet the consistency condition mentioned above.

In discussions of the effects of impurities as well as interactions, as, for example, in theories of quantum Hall effects and other physical phenomena, matrix elements of plane waves are required. In the orbit-center representation, it is known that this matrix element has a selection rule concerning the positions of the orbit centers and the corresponding component of the plane wave and is a function $J_{N,N'}(Q_x,Q_y)$ which depends on the Landau levels and the components of the wave vector. On the other hand, we find a result for the angular-momentum representation: The matrix element is merely a product

$$J_{N,N'}(Q_x,Q_Y)J^*_{N+m,N'+m'}(-Q_x,-Q_y)$$

(asterisk meaning complex conjugate) but with no selection rule on angular momenta. Here $\mathbf{Q}$ is the wave vector of the plane wave and Q_x,Q_y are its components in a plane perpendicular to the magnetic field.

In Sec. II, we give a brief account of the two representations and the relation between them along with their physical interpretations to establish the notations used in this paper. In Sec. III, we give the analysis of the known exact solution of the gap equation in the two representations. In Sec. IV we fully solve the linearized gap equation for a BCS model in the orbit-center representation. In Sec. V, the matrix elements of a plane wave and their interpretations in the two representations are given. We end the paper with a last section containing a few concluding remarks concerning the significance of the analysis given here.

II. REPRESENTATIONS AND THEIR RELATIONSHIPS

The solutions of the Schrödinger equation for the motion of an electron in a constant magnetic field in the symmetric gauge[1,9]

$$\mathcal{H}\psi=E\psi\ , \tag{1}$$

where

$$\mathcal{H}=\frac{1}{2M}\left[\left[-i\hbar\frac{\partial}{\partial x}-\frac{eH}{2c}y\right]^2+\left[-i\hbar\frac{\partial}{\partial y}+\frac{eH}{2c}x\right]^2+\left[-i\hbar\frac{\partial}{\partial z}\right]^2\right]+V(z) \tag{1a}$$

may be written equivalently in either the orbit-center representation or the angular-momentum representation. Here we include $V(z)$ to allow for the possibility of a confining or periodic potential in the z direction such as those encountered in quantum well, MOSFET, or layer systems. The choice of the symmetric gauge is made here for purposes of illustration. The wave functions in any other gauge may be obtained by an appropriate transformation.

A. Orbit center

The representation which simultaneously diagonalizes the Hamiltonian, the x component of the orbit center given by $\hat{X}=x/2+il^2\partial/\partial y$, and the z component of the linear momentum is

$$\begin{aligned}\langle \mathbf{r}|NX\alpha\rangle&=\psi_{NX\alpha}(\mathbf{r})\\&=\exp(ixy/2l^2)\phi_N(x-X)\\&\quad\times\exp(-iXy/l^2)\xi_\alpha(z)\end{aligned} \tag{2}$$

with

$$\phi_N(x)=\exp(-x^2/2l^2)H_N(x/l)/(2^NN!\pi^{1/2}l)^{1/2}\ , \tag{2a}$$

where $l^2=c\hbar/eH$ is the square of the Larmor radius associated with the constant magnetic field, H, H_N are the Hermite polynomials, N is the Landau quantum member, α is the quantum number of the motion in the z direction with its associated wave function $\xi_\alpha(z)$, $X:(-\infty,\infty)$ is the eigenvalue of $\hat{X}$, the orbit center along the x axis with energy eigenvalues which are degenerate in X,

$$E_{N\alpha}=\hbar\Omega(N+1/2)+\varepsilon_\alpha\ . \tag{3}$$

Here $\Omega=eH/Mc$ is the cyclotron frequency, and ε_α is the energy of the state α.

B. Angular momentum

The representation which simultaneously diagonalizes the Hamiltonian, the angular momentum about the z axis, which in the symmetric gauge is $\hat{L}_z=(x\hat{p}_y-y\hat{p}_x)$, and the z component of the linear momentum is

$$\langle \mathbf{r}|Nm\alpha\rangle=\psi_{Nm\alpha}(\mathbf{r})=\frac{\exp(-im\phi)}{(2\pi)^{1/2}}\frac{1}{l}\left[\frac{\mu!}{(\mu+|m|)!}\right]^{1/2}\exp(-\rho^2/4l^2)(\rho^2/2l^2)^{|m|/2}L_\mu^{|m|}(\rho^2/2l^2)\xi_\alpha(z) \tag{4}$$

with

$$\mu\equiv N+(m-|m|)/2,$$
$$m=-N,-N+1,\ldots,-1,0,1,\ldots,\infty\ . \tag{5}$$

The energy eigenvalues given by (3) are now degenerate with respect to the angular momentum $-\hbar m$. $L_\mu^{|m|}$ is the usual Laguerre polynomial, $\rho=(x,y)$ is a vector in the plane perpendicular to the magnetic field. This displays the cylindrical symmetry of Eq. (1) explicitly whereas the solution (2), does not. Note that the operators $\hat{X}$ and $\hat{L}_z$ are gauge dependent.

The two representations are related to each other by

$$\psi_{NX\alpha}(\mathbf{r})=(-1)^N(2\pi l^2)^{1/2} \times \sum_{m=-N}^{\infty}(-1)^{(m-|m|)/2}\phi_{N+m}(X)\psi_{Nm\alpha}(\mathbf{r}) \,, \tag{6}$$

$$\langle NX\alpha|Nm\alpha\rangle =(2\pi l^2)^{1/2}(-1)^{N+(m-|m|)/2}\phi_{N+m}(X) \,, \tag{6a}$$

where $\phi_{N+m}(X)$ is the harmonic-oscillator function introduced in Eq. (2a). Inversely,

$$\psi_{Nm\alpha}(\mathbf{r})=(-1)^{N+(m-|m|)/2}(2\pi l^2)^{1/2} \times \int_{-\infty}^{\infty}\frac{dX}{2\pi l^2}\phi_{N+m}(X)\psi_{NX\alpha}(\mathbf{r}) \,, \tag{7}$$

exhibiting that the cylindrical symmetry of Eq. (1) is evident only after appropriately summing over all the degenerate orbit centers. This is even more transparent in the expression for the finite temperature, one-particle Green function $G_0(\mathbf{r}_1,\mathbf{r}_2;i\omega_n)$ associated with Eq. (1). Writing G_0 in terms of (2), (3), and summing over the orbit center, we get[6]

$$G_0(\mathbf{r}_1,\mathbf{r}_2;i\omega_n)=\sum_{N=0}^{\infty}\int_{-\infty}^{\infty}\frac{dX}{2\pi l^2}\sum_{\alpha}\frac{\psi_{NX\alpha}(\mathbf{r}_1)\psi^*_{NX\alpha}(\mathbf{r}_2)}{i\omega_n-E_{N\alpha}} \tag{8}$$

$$=\exp[-i(x_2y_1-x_1y_2)/2l^2]\frac{1}{2\pi l^2}\sum_{N=0}^{\infty}\exp(-|\rho_{12}|^2/4l^2)L_N(|\rho_{12}|^2/2l^2)\sum_{\alpha}\frac{\xi_\alpha(z_1)\xi^*_\alpha(z_2)}{i\omega_n-E_{N\alpha}} \,. \tag{9}$$

Here ω_n is the usual Matsubara frequency. This form of the one-particle Green function displays the cylindrical symmetry explicitly and is found very useful in calculations.

The same result is obtained in the angular-momentum representation by using Eq. (4) and the following identity, which may be proved by using various generating functions, etc., found in the Gradshteyn-Ryzhik Tables[12]

$$\sum_{m=-N}^{\infty}\exp[-im(\phi_1-\phi_2)](\rho_1\rho_2/2l^2)^{-m}\exp[-(\rho_1^2+\rho_2^2)/4l^2]\frac{(N+m)!}{N!}L_{N+m}^{-m}(\rho_1^2/2l^2)L_{N+m}^{-m}(\rho_2^2/2l^2)$$

$$=\exp[-i(x_2y_1-x_1y_2)/2l^2]\exp(-|\rho_{12}|^2/4l^2)L_N(|\rho_{12}|^2/2l^2) \tag{10}$$

$$=\sum_{m=-N}^{\infty}[-im(\phi_1-\phi_2)](\rho_1\rho_2/2l^2)^m\exp[-(\rho_1^2+\rho_2^2)/4l^2]\frac{N!}{(N+m)!}L_N^m(\rho_1^2/2l^2)L_N^m(\rho_2^2/2l^2) \,. \tag{10a}$$

Equation (10a) follows in view of the relationship

$$L_N^m(x)=(-1)^m\frac{(N+m)!}{N!}x^{-m}L_{N+m}^{-m}(x) \qquad (N+m\geq 0) \tag{11}$$

Quite often, in the literature, a semiclassical approximation is employed by taking the Green function given on the right-hand side (rhs) of Eq. (9) in the form of a phase factor $\exp[-i(x_2y_1-x_1y_2)/2l^2]$ multiplied by the free-particle Green function in the absence of the magnetic field. This takes into account only the curvature of the path but not the quantization of the motion. If, in Eq. (9), the sum on the Landau quantum numbers N is changed to an integral, along with the relation

$$\exp(-|\rho_{12}|^2/4l^2)L_N(|\rho_{12}|^2/2l^2)J_0[(2N/l^2)^{1/2}|\rho_{12}|]$$

valid for large N and small magnetic fields, with the integral representation

$$J_0[k|\rho_{12}|]=\int_0^{2\pi}\frac{d\theta}{2\pi}\exp(ik|\rho_{12}|\cos\theta) \,,$$

and making the identification $k^2=2N/l^2$, we obtain the semiclassical result mentioned above.

The point of this demonstration is to make explicit the value of the orbit-center representation in computations whereas the angular-momentum representation brings out the geometric symmetry in the problem explicitly. In discussing localization problems, it appears that the orbit-center representation provides a physical picture in constructing useful approximations. The one-particle Green function is unique and, apart from a gauge-dependent phase factor, displays the inherent cylindrical symmetry only upon summing over the degeneracy index in either representation.

III. DISCUSSION OF THE SOLUTION OF THE SUPERCONDUCTING GAP EQUATION

We will now examine the exact solution of the linearized superconducting gap equation in the presence of a strong magnetic field in these two representations to gain some insight into the nature of pairing. In particular, the exact solution provides an opportunity to examine the nature of pairing at the level of representations of the single-particle states. By so doing, we can understand the nature of pairing correlations that must exist between different Landau states in either representation thus providing a guide in formulating the theory of the superconducting state in the presence of a strong magnetic field.

An exact solution[6] of the linearized equation for the gap function $\Delta(\mathbf{r}_1,\mathbf{r}_2)$ within the BCS-like model has the form

$$\Delta(\mathbf{r}_1,\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2)\Delta^0(z_1)\exp(-\rho_1^2/2l^2) . \qquad (12)$$

The gap function in the angular-momentum representation is

$$\Delta_{Nm\alpha,N'm'\alpha'}\equiv\int\int\psi_{Nm\alpha}(\mathbf{r}_1)\Delta(\mathbf{r}_1-\mathbf{r}_2)\psi_{N'm'\alpha'}(\mathbf{r}_2)d^3r_1d^3r_2$$
$$=\Delta^0_{\alpha\alpha'}\delta_{m+m',0}\left[\frac{\mu!\mu'!}{(\mu+|m|)!(\mu'+|m'|)!}\right]\int_0^\infty\frac{\rho\,d\rho}{l^2}\exp(-\rho^2/l^2)(\rho^2/2l^2)^{|m|}L_\mu^{|m|}(\rho^2/2l^2)L_{\mu'}^{|m|}(\rho^2/2l^2) .$$

Using Gradshteyn-Ryzhik (Ref. 12), p. 844, 7.414 formula 4), the integral over ρ can be evaluated and we finally obtain

$$\Delta_{Nm\alpha,N'm'\alpha'}=\Delta^0_{\alpha\alpha'}\delta_{m,-m'}\frac{(N+N')!}{2^{N+N'+1}}\times\frac{1}{[N!N'!(N+m)!(N'-m)!]^{1/2}} . \qquad (13)$$

Recalling Eq. (5), we have the condition on m in the forms

$$m=-m'=-N,-N+1,\ldots,-1,0,1,\ldots,\infty$$
$$=N',N'-1,\ldots,1,0,-1,\ldots,-\infty ,$$

which together lead to the selection rule

$$m=-N,-N+1,\ldots,-1,0,1,\ldots,N'-1,N' . \qquad (14)$$

In the orbit-center representation, the corresponding matrix element is either computed directly or by using Eq. (6) to obtain the expression

$$\Delta_{NX\alpha,N'X'\alpha'}=\frac{\Delta^0_{\alpha\alpha'}(-1)^N(\pi l^2)^{1/2}}{2^{N+N'}(N!N'!)^{1/2}}\exp[-(X^2+X'^2)/2l^2]\times H_{N+N'}[(X-X')/2^{1/2}l] . \qquad (15)$$

The implication of Eq. (14) is that, for a given pair of Landau levels (N,N'), the allowed pairing of angular momenta $(m,-m)$ are $(N+N'+1)$ values of m given above. Thus, only a subset of the allowed angular momenta form the pairing complex of states for a given pair of Landau levels considered. On the other hand, Eq. (15), which is a further simplification of the result reported in Ref. 6, shows that, in the orbit-center representation, all values of X,X' are allowed. Expressing

$$X^2+X'^2=(X+X')^2/2+(X-X')^2/2 ,$$

we observe that the result in Eq. (15) may be formally interpreted in terms of the center of mass and relative orbit centers. $\Delta_{NX\alpha,N'X'\alpha'}$ is thus seen to be proportional to the product of the zeroth Landau wave function in the center of mass coordinate and $(N+N')$th Landau wave function in the relative coordinate of the orbit centers. In the next section, the linearized gap equation for a BCS-type interaction is solved completely in the orbit-center representation. It is found that the above decomposition in terms of the center of mass and relative orbit-center coordinates occurs as a natural basis for the description of pairing phenomena in the presence of a magnetic field.

IV. SOLUTIONS OF THE LINEARIZED GAP EQUATION

In the last section we showed that an exact solution of the linearized gap equation in the BCS-like model exhibits the features that all Landau levels participate in the pairing [vis., Eq. (15)]. We will now generalize this result and obtain, in principle, all the solutions of the linearized gap equation in the orbit-center representation (for simplicity we focus here on two dimensions):

$$\Delta_{N_1N_2}(X_1,X_2)=\bar{V}\sum_{N_3,X_3,N_4,X_4}\langle N_1X_1N_2X_2|N_3X_3N_4X_4\rangle T_{N_3N_4}\Delta_{N_3N_4}(X_3,X_4) , \qquad (16)$$

where

$$T_{N_3N_4}=\frac{\tanh(\beta\varepsilon_{N_3}/2)+\tanh(\beta\varepsilon_{N_4}/2)}{2(\varepsilon_{N_3}+\varepsilon_{N_4})} \qquad (17)$$

has the usual cutoff in energy as in the BCS model. The matrix element can be expressed in a succinct form after using the generating functions, etc.,

$$\langle N_1X_1N_2X_2|N_3X_3N_4X_4\rangle=\frac{1}{2^{1/2}}2\pi l^2\delta(X_1+X_2-X_3-X_4)\sum_{P=0}^{\infty}\lambda^{(P)}_{N_1N_2}\lambda^{(P)}_{N_3N_4}\phi_{N_1+N_2-P}(Y/2^{1/2})\phi_{N_3+N_4-P}(Y'/2^{1/2}) \qquad (18)$$

with

$$\lambda_{NN'}^{(P)}=\left[\frac{N!N'!}{P!(N+N'-P)!}\right]^{1/2}\frac{(-1)^N}{2^{(N+N')/2}}\times\sum_{a=0}^{P}(-1)^a\begin{bmatrix}N+N'-P\\N'-a\end{bmatrix}\begin{bmatrix}P\\a\end{bmatrix} \quad (19)$$

and $Y=X_1-X_2$, $Y'=X_3-X_4$.

By going to the center of mass and relative coordinate system in the orbit-center space, since the ϕ's are orthonormal in y space, we see from Eq. (16) that (with $X_1+X_2=2X$)

$$\Delta_{N_1N_2}(X_1,X_2)=\Delta_{N_1N_2}(X,Y)=\sum_{P=0}^{N_1+N_2}\lambda_{N_1N_2}^{(P)}\phi_{N_1+N_2-P}\times(Y/2^{1/2})A^{(P)}(X) \quad (20)$$

with

$$A^{(P)}(X)=\frac{\overline{V}}{4\pi l^2}\frac{1}{2^{1/2}}\sum_{N_3N_4}\int dY^1\,\lambda_{N_3N_4}^{(P)}\phi_{N_3+N_4-P}\times(Y'/2^{1/2})T_{N_3N_4}\times\Delta_{N_3N_4}(X,Y')\ . \quad (21)$$

Self-consistency for nonzero $A^{(P)}(X)$ from (20) and (21) leads to, after using the orthonormality of the ϕ's, for each channel p:

$$1=\frac{\overline{V}}{4\pi l^2}\sum_{N_3N_4}[\lambda_{N_3N_4}^{(P)}]^2T_{N_3N_4}\ . \quad (22)$$

For $P=0$, using Eq. (19),

$$\lambda_{NN'}^{(0)}=\frac{(-1)^N}{2^{(N+N')/2}}\left[\frac{(N+N')!}{N!N'!}\right]^{1/2} \quad (23)$$

leading to the result obtained in Ref. 6 by a special choice of $\Delta(\mathbf{r})$ given in Eq. (12). Also, from Eqs. (21) and (24) we note that the corresponding solution is

$$\Delta_{N_1N_2}(X,Y)=\lambda_{N_1N_2}^{(0)}\phi_{N_1+N_2}(Y/2^{1/2})A^{(0)}(X)\ , \quad (24)$$

which is identical in form to Eq. (15).

We have thus not only recovered the old result, but also obtained all other solutions. Note that, from the structure of $\phi_{N_3+N_4-P}$, the sum in Eq. (21) stops after $P=N_3+N_4$. In fact, the kernel of Eq. (20) is a finite-rank kernel since the sum of P stops as soon as P equals the smaller of N_1+N_2 or N_3+N_4.

V. PLANE-WAVE MATRIX ELEMENTS

A class of matrix elements which occur in the discussions of quantum Hall effects and other physical phenomena are those of the plane wave $\exp(i\mathbf{Q}\cdot\mathbf{r})$. In the orbit-center representation, this result is well known[9] and we present it here for purposes of comparison:

$$\langle NX\alpha|\exp(i\mathbf{Q}\cdot\mathbf{r})|N'X'\alpha'\rangle=\langle\alpha|\exp(iQ_zz)|\alpha'\rangle 2\pi l\delta[(X-X')/l-lQ_y]\exp(iQ_xX)J_{N,N'}(Q_x,Q_y)\ . \quad (25)$$

In the angular-momentum representation, using the transformation Eq. (7), and Eq. (16) above, we obtain

$$\langle Nm\alpha|\exp(i\mathbf{Q}\cdot\mathbf{r})|N'm'\alpha'\rangle=\langle\alpha|\exp(iQ_zz)|\alpha'\rangle(-1)^{(m-|m|+m'-|m'1/2)+N+N'}J_{N,N'}(Q_x,Q_y)J^*_{N+m,N'+m'}(-Q_x,-Q_y)\ . \quad (26)$$

Here

$$J_{N,N'}(Q_x,Q_y)=\int_{-\infty}^{\infty}dx\ \phi_N(x/l-Q_yl)\exp(iQ_xx)\phi_{N'}(x/l) \quad (27)$$

$$=\exp(iQ_xQ_yl^2/2)\left[\frac{N!}{N'!}\right]^{1/2}\left[\frac{Q_yl+iQ_xl}{2^{1/2}}\right]^{N'-N}\times L_N^{N'-N}\{[(Q_xl)^2+(Q_yl)^2]/2\}\exp\{-[(Q_xl)^2+(Q_yl)^2]/4\} \quad (28)$$

valid for all N,N'. The customary form of this expression for $N>N'$ can be obtained form Eq. (11).

These expressions exhibit interesting features associated with the representations. From Eq. (25), the orbit centers (X,X') must obey the relation $X'-X=l^2Q_y$ because otherwise the matrix element would be zero. From Eq. (26) we note that, on the other hand, there are no additional conditions on the angular momenta (m,m'). Also, from Eq. (28), we may note that, in the orbit-center representation, the matrix element falls off as $\exp(-l^2\mathbf{Q}_\perp^2/2)$, whereas in the angular-momentum representation, it falls off as $\exp(-l^2\mathbf{Q}_\perp^2)$, where $\mathbf{Q}_\perp=(Q_x,Q_y)$ for fixed (N,N'). The faster falloff is a consequence of two-dimensional confinement in the angular-momentum basis whereas it is one-dimensional in nature in the orbit-center basis. In terms of orbit-center position, from (25), we note that the matrix element falls off exponentially in $(X-X')^2$ on a scale of the square of the Larmor radius. In contrast, the angular-momentum dependence resides in the factor $J^*_{N+m,N'+m'}(-Q_x,-Q_y)$, in a complicated fashion, being buried in the Laguerre polynomial [see Eq. (28)].

VI. CONCLUDING REMARKS

The result contained in Eqs. (13)–(15) implies that, in constructing the equation for the superconducting state below T_c, the Cooper pairing among the Landau level states must involve off-diagonal terms. The diagonal approximations usually made are insufficient and will not lead to the same expression for T_c obtained by means of the solution[6] given by Eq. (12). In contrast, the use of the orbit-center representation leads to the result that $\Delta_{NX\alpha,N'X'\alpha'}$ is nonzero for all pairs of Landau levels (N,N') but falls off as the square of the orbit-center positions on a scale of the Larmor radius modulated by polynomials of distance between the centers also on a scale of the Larmor radius. In Sec. IV, all the solutions of the linearized gap equation are found. The construction of the theory of the superconducting state in the presence of a strong quantizing magnetic field thus calls for a different analysis of the problem beyond what has been done. This problem has now been solved in the form of vortex solutions to the Gorkov equations, which we will present in another paper.[10,11]

It should be noted that the discussion of the matrix elements of the plane wave in Sec. V is in contrast with the discussion of the representations of the gap functions in Sec. III in that the phases of the wave functions in Eqs. (2) and (4) play different roles in the two instances, leading to different features noted in the respective sections. In particular, the selection rule on m for the gap, Eq. (13), should be noted, in contrast to the selection rule $X'-X=l^2Q_y$ for the plane-wave matrix element, Eq. (25). Also the gap exhibits a dependence on $(X+X')$ in Eq. (15) in addition to the dependence on $(X-X')$ because a translation in the orbit center entails a concomitant gauge transformation to restore the covariance of Eq. (1).

The additional features concerning the matrix elements of the plane wave as well as the Cooper-pairing phenomena are thus seen to be direct consequences of the two representations described here. Even though the representations are formally equivalent in the transformation-theory sense, the consequences of their choice lead to significant insight into the many interesting phenomena that occur in strong magnetic fields.

ACKNOWLEDGMENTS

J. C. R. acknowledges partial financial support from NRC and A. K. R. from ONR. A. K. R. thanks Professor P. Kumar and Professor T. V. Ramkrishnan for several remarks on various aspects of the subject discussed here.

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