Vortex lattice of highly anisotropic layered superconductors in strong, parallel magnetic fields

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The equations for the gauge-invariant phases obtained in the framework of the Lawrence-Doniach model are used to study the vortex lattice in layered superconductors in strong, parallel magnetic fields H. As H changes, a sequence of first-order phase transitions occurs between the lattices with different periods. The largest jumps in magnetization, of order H_{c1}, occur at fields of order $H_0=\phi_0/\gamma s^2$, where ϕ_0 is the flux quantum, s is the interlayer spacing, and γ is the anisotropy ($\gamma=\lambda_c/\lambda_{ab}$, where λ_c and λ_{ab} are the penetration depths for currents perpendicular and parallel to the layers, respectively). The asymptotic dependence of the magnetization M at high fields $H \gg H_0$ is $M \propto H^{-3}$.

I. INTRODUCTION

We consider the vortex lattice in superconductors with Josephson coupling of the layers[1] in the presence of a strong magnetic field parallel to the layers. For such an orientation, supercurrents should flow between the layers; these are limited by the Josephson character of interlayer coupling. In this case the properties of the vortex lattice may be quite different from those obtained in the framework of the three-dimensional (3D) anisotropic London or Ginzburg-Landau approach. We show that in strong magnetic fields of order $H_0=\phi_0/\gamma s^2$ we do have a new phenomenon: first-order phase transitions occur between the lattices with different periods $l=2ks$ in the direction perpendicular to the layers. Here $k=1,2,\ldots,s$ is the interlayer spacing, $\gamma=\lambda_c/\lambda_{ab}$ is the anisotropy ratio, λ_c is the penetration depth for currents along the c axis (perpendicular to the layers), and λ_{ab} is the penetration depth for currents in the ab plane (parallel to the layers). These jumps in the lattice structure are caused by the discrete layer structure of the crystal and the Josephson nature of layer coupling. We show also that the magnetization drops with field more rapidly than is predicted by the 3D anisotropic London theory.

In the following we use the mean-field approach, neglecting fluctuations. Thus the obtained results are valid at temperatures far below the vortex lattice melting temperature.[2–5] The structure of a single vortex parallel to the layers has been studied previously in the framework of such an approach.[6–8] It was shown that such a vortex has, instead of a normal core, a nonlinear Josephson core with major axis γs in the ab direction parallel to the layers and minor axis s in the c direction perpendicular to the layers. Outside the nonlinear core the vortex is described accurately by the anisotropic London theory. At magnetic fields of order $H_0=\phi_0/\gamma s^2$, however, the nonlinear Josephson cores of vortices overlap strongly, and the behavior of the vortex lattice is determined completely by the Josephson nature of the layer coupling.

The London 3D anisotropic theory is not appropriate in this situation. In the following we study the vortex lattice structure in the fields H of order of or larger than H_0.

II. BASIC EQUATIONS

We first present the equations that describe the Josephson behavior in layered superconductors.[9]

In the framework of the Lawrence-Doniach model[1] the free energy functional is

$$F=\frac{H_c^2 s}{4\pi}\sum_n\int d\mathbf{r}\left[\xi_\parallel^2(T)\left|\left[-i\frac{\partial}{\partial\mathbf{r}}+\frac{2\pi}{\phi_0}\mathbf{A}_\parallel(n)\right]\Psi_n\right|^2-|\Psi_n|^2+\frac{1}{2}|\Psi_n|^4+\rho\left[|\Psi_n|^2+|\Psi_{n+1}|^2-\Psi_n\Psi_{n+1}^*e^{-i\chi_{n,n+1}}-\Psi_n^*\Psi_{n+1}e^{i\chi_{n,n+1}}\right]\right]+\int d\mathbf{r}\int dz\frac{\mathbf{h}^2}{8\pi}\,,$$

$$A_\parallel=(A_x,A_y),\quad \chi_{n,n+1}=\frac{2\pi}{\phi_0}\int_{ns}^{(n+1)s}A_z dz,\quad \mathbf{h}=\operatorname{curl}\mathbf{A}\ . \tag{1}$$

Here $\Psi_n(\mathbf{r})=|\Psi_n(\mathbf{r})|\exp[i\Phi_n(\mathbf{r})]$ is the order parameter in layer n, $\mathbf{r}=(x,y)$ and the z axis is perpendicular to the layers, H_c is the bulk thermodynamic critical field, $\mathbf{A}_\parallel(n)$ is the parallel component of the vector potential at $z=ns$, and the parameter $\rho \ll 1$ characterizes the weak Josephson coupling between layers. The superconducting layers are assumed to have negligible thickness.

For a magnetic field parallel to the layers, the amplitude $|\Psi_n|$ of the order parameter can be assumed to be independent of x, y, and n. Then the free-energy functional with respect to the phases Φ_n is:

$$F\{\Phi_n\}=s\sum_n\int d\mathbf{r}\left[\frac{H_c^2}{4\pi}\xi_\parallel^2(T)\left[\frac{\partial\Phi_n}{\partial\mathbf{r}}+\frac{2\pi}{\phi_0}A_\parallel(n)\right]^2+\frac{\hbar j_0}{2es}[1-\cos(\Phi_n-\Phi_{n+1}-\chi_{n,n+1})]\right], \quad (2)$$

where $j_0=cs\rho H_c^2/\phi_0=c\phi_0/8\pi^2s\lambda_c^2$ is the Josephson interlayer critical current density.

We choose the orientation of magnetic field to be along the y axis. Then $A_y=0$; the phases Φ_n depend on n and x; and A_x and A_z depend on x and z. We introduce the gauge-invariant differences of phases between the neighboring layers n and $n+1$,

$$\varphi_{n,n+1}(x)=\Phi_n(x)-\Phi_{n+1}(x)-\frac{2\pi}{\phi_0}\int_{ns}^{(n+1)s}dz\,A_z(x,z)\,. \quad (3)$$

Using the functional (2) we obtain the following equations for $\varphi_{n,n+1}(x)$:

$$\frac{\partial^2\varphi_{n,n+1}}{\partial x^2}-\frac{1}{\lambda_J^2}(2\sin\varphi_{n,n+1}-\sin\varphi_{n+1,n+2}-\sin\varphi_{n-1,n})-\frac{1}{\lambda_c^2}\sin\varphi_{n,n+1}=0\,, \quad (4)$$

where $\lambda_J=\gamma s$. The periodic solutions $\varphi_{n,n+1}(x)$ in x and n determine the vortex lattice. They give the Josephson currents between the layers, and the currents along the layers $j_{xn}(x)=-c\phi_0(\partial\Phi_n/\partial x+2\pi A_x/\phi_0)/8\pi^2\lambda_{ab}^2$ can be found using the current continuity equation:

$$\frac{\partial j_{xn}}{\partial x}+\frac{j_0}{s}(\sin\varphi_{n,n+1}-\sin\varphi_{n-1,n})=0\,. \quad (5)$$

Thus the free energy can be calculated using (2) to determine the period of the vortex lattice at given external field H. For a periodic vortex array, the magnetic induction between layers n and $n+1$ is given by the relation:

$$B_y=\langle h_y\rangle=\frac{\phi_0}{2\pi s}\left\langle\frac{\partial\varphi_{n,n+1}}{\partial x}\right\rangle, \quad (6)$$

where the angular brackets mean the average value over the period of the vortex lattice along the x axis. The magnetization $\mathbf{M}$ can be found using the relation:

$$\mathbf{M}=\frac{\mathbf{B}}{4\pi}-\frac{\partial F_0}{\partial\mathbf{B}}\,, \quad (7)$$

where F_0 is the minimal value of F given by (1).

In the case of a dense vortex lattice with distances between the vortices along the x axis of order λ_J or smaller, the last term on the left-hand side of (4) can be omitted. Then the gauge-invariant phases $\varphi_n(x)$ for the layers n can be used to describe the system.

These can be determined as the solutions of the equations

$$\frac{\partial^2\varphi_n}{\partial x^2}-\frac{1}{\lambda_J^2}[\sin(\varphi_n-\varphi_{n+1})-\sin(\varphi_{n-1}-\varphi_n)]=0\,, \quad (8)$$

such that the phase differences $\varphi_{n,n+1}=\varphi_n-\varphi_{n+1}$ obey Eq. (4) (without the last term on the left-hand side). Comparing (5) with (8) we see that now the currents $j_{xn}(x)$ are given by the expression:

$$j_{xn}=-\frac{c\phi_0}{8\pi^2\lambda_{ab}^2}\left[\frac{\partial\varphi_n}{\partial x}-C_n\right], \quad (9)$$

where constants C_n are independent of x. We note that $\varphi_n(\mathbf{r})$ is not equal to

$$[\Phi_n(\mathbf{r})+(2\pi/\phi_0)\int d\mathbf{r}\,\mathbf{A}_\parallel(\mathbf{r})]\,,$$

but their derivatives with respect to x coincide. The free energy functional with respect to the gauge-invariant phases φ_n is

$$F\{\varphi_n\}=\frac{\phi_0^2s}{32\pi^3\lambda_{ab}^2}\sum_n\int d\mathbf{r}\left[\left[\frac{\partial\varphi_n}{\partial\mathbf{r}}-C_n\right]^2+\frac{2}{\lambda_J^2}[1-\cos(\varphi_n-\varphi_{n+1})]\right], \quad (10)$$

where the constants C_n should be determined by minimizing the free energy.

The obtained equations generalize the corresponding expressions for a single junction[10] to a multilayer system.

III. HIGH-FIELD BEHAVIOR

We now study the vortex lattice in very high fields, $H\gg H_0$. In this field range the centers of vortices are arranged in a triangular lattice as sketched in Fig. 1, where the period of the lattice in the c direction is $l=2s$ and along the x axis is a, where $a\ll\lambda_J$. The area of the unit cell is as, and $B=\phi_0/sa$ for this vortex lattice. (It can be shown that the rectangular lattice has a higher energy.) Due to the periodicity of the system we obtain

$$\begin{aligned}&\varphi_{2n,2n+1}(x)=2\pi x/a+f(x)\,,\\&\varphi_{2n+1,2n+2}(x)=2\pi x/a+\pi+f(x+a/2)\,,\\&\varphi_{2n}=C_{2n}x+b(x)\,,\\&\varphi_{2n+1}=C_{2n+1}x+b(x+a/2)\,,\end{aligned} \quad (11)$$

where $f(x+a)=f(x)$, $b(x+a)=b(x)$, and $\langle f(x)\rangle=\langle b(x)\rangle=0$. From the symmetry of the lattice we get $b(x+a/2)=-b(x)$. Taking into account the relations

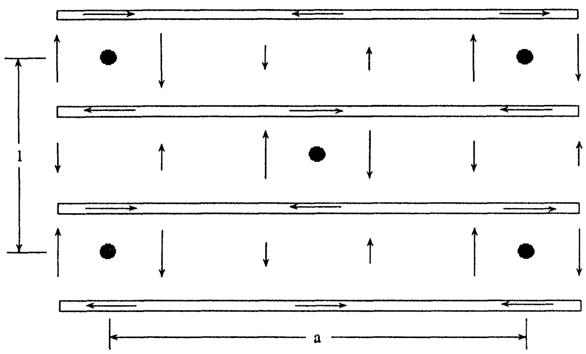

FIG. 1. The vortex lattice with period $l=2s$ in the c direction for a strong magnetic field parallel to the layers. Arrows show the direction of currents.

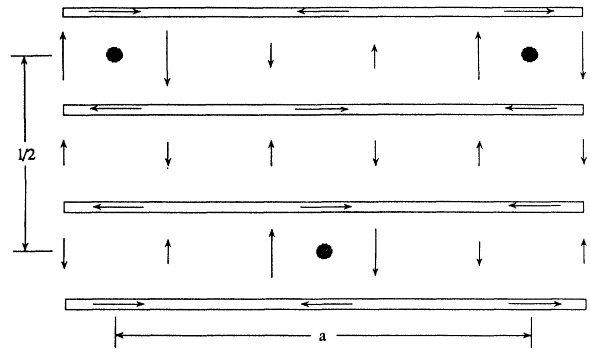

FIG. 2. The vortex lattice with period $l=4s$ in the c direction.

$$\varphi_{2n,2n+1}(x)=(C_{2n}-C_{2n+1})x+b(x)-b(x+a/2)\,,$$
$$\varphi_{2n+1,2n+2}(x)=(C_{2n+1}-C_{2n+2})x+b(x+a/2)-b(x)\,, \tag{12}$$

we see that $f(x)=-f(x+a/2)=2b(x)$. Using (4) and (11) we obtain the equation for $f(x)$:

$$\frac{\partial^2 f}{\partial x^2}=\frac{4}{\lambda_J^2}\sin\left[\frac{2\pi x}{a}\right]\cos f\,. \tag{13}$$

The free energy functional for one layer of this lattice is given by the expression

$$F=\frac{\phi_0^2 s}{32\pi^3\lambda_{ab}^2}\int d\mathbf{r}\left[\frac{1}{4}\left[\frac{\partial f}{\partial x}\right]^2+\frac{2}{\lambda_J^2}+\frac{2}{\lambda_J^2}\sin\left[\frac{2\pi x}{a}\right]\sin f\right]+\int d\mathbf{r}\frac{B^2 s}{8\pi}\,. \tag{14}$$

In the limit $a/\lambda_J \ll 1$ we can use perturbation theory to obtain $f(x)$. To second order in a/λ_J we obtain

$$f(x)=-\frac{a^2}{\pi^2\lambda_J^2}\sin\left[\frac{2\pi x}{a}\right]. \tag{15}$$

The free energy per unit volume at given a is

$$F_0(a)=\frac{\phi_0^2}{8\pi s^2 a^2}+\frac{\phi_0^2}{16\pi^3\lambda_{ab}^2\lambda_J^2}-\frac{\phi_0^2 a^2}{64\pi^5\lambda_{ab}^2\lambda_J^4}\,. \tag{16}$$

Using this expression and (7), we obtain for $H \gg H_0$ the asymptotic behavior of the magnetization:

$$M(H)=-\frac{\phi_0}{32\pi^5\lambda_{ab}\lambda_c}\left[\frac{H_0}{H}\right]^3. \tag{17}$$

Such a dependence is valid for high magnetic fields except those close to the upper critical field H_{c2}, which is determined by the paramagnetic effect for the parallel orientation.[11,12]

We note that in the London model the magnetization in the field range $H_{c1} \ll H \ll H_{c2}$ is given by the expression[13]

$$M(H)=-\frac{\phi_0}{4\pi\lambda_{ab}\lambda_c}\ln\left[\frac{\beta H_{c2}}{H}\right], \tag{18}$$

where the numerical parameter β is of the order unity. Note that in the regime where the Josephson cores overlap [Eq. (17)], the magnetization decreases with field more rapidly than that given in Eq. (18).

As the magnetic field decreases down to the values of order H_0, the period a of the lattice in the x direction grows, while the period l in the c direction remains the same. Then at some value H_1 of order H_0 the lattice with period $4s$ in the c direction starts to be more favorable than the lattice with $l=2s$. As H further diminishes, the transition to the lattice with period $l=6s$ occurs, and so on. The jumps in l are accompanied by the jumps in period a because of the relation $B=2\phi_0/la$. At fields H much smaller than H_0 we approach the London limit, for which the ratio a/l is $\gamma/\sqrt{3}$.[13]

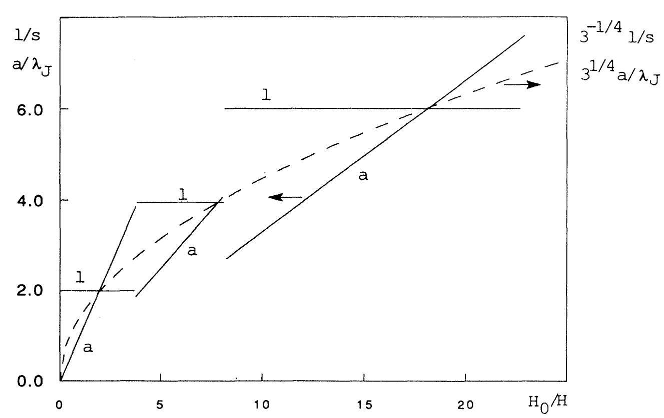

FIG. 3. The schematical dependence of the lattice periods l/s (lines l) and a/λ_J (lines a) on the inverse magnetic field parallel to the layers in Lawrence-Doniach model (solid lines, left scale) and $3^{-1/4}l/s$ and $3^{1/4}a/\lambda_J$ in the London model (dashed line, right scale).

The vortex lattice with $l=4s$ is shown in Fig. 2. In this lattice the magnetic induction is determined by the relation $B=\phi_0/2sa$ and the phases differences are described by two periodic functions $f_1(x), f_2(x)$:

$$\varphi_{2n,2n+1}(x)=2\pi x/a+f_1(x) ,$$
$$\varphi_{2n+1,2n+2}(x)=\varphi_{2n+3,2n+4}(x)=f_2(x) , \quad (19)$$
$$\varphi_{2n+2,2n+3}(x)=2\pi x/a+\pi+f_1(x+a/2) ,$$

where $\langle f_1(x)\rangle=\langle f_2(x)\rangle=0$. For phases inside the layers we obtain $\varphi_{2n}(x)=-\varphi_{2n+1}(x)=f_1(x)/2$ and $\varphi_{2n+2}(x)=-\varphi_{2n+3}(x)=f_2(x)/2$ taking into account the symmetry of the lattice. The equations for $u=(f_1+f_2)/2$ and $v=(f_1-f_2)/2$ are

$$\frac{\partial^2 u}{\partial x^2}=\frac{2}{\lambda_J^2}\left[\sin v \cos\left[\frac{2\pi x}{a}+u\right]+\sin u\right] ,$$
$$\frac{\partial^2 v}{\partial x^2}=\frac{2}{\lambda_J^2}\sin\left[\frac{2\pi x}{a}+u\right]\cos v . \quad (20)$$

The free energy functional (for one layer) with respect to the functions $u(x), v(x)$ is

$$F=\frac{\phi_0^2 s}{128\pi^3\lambda_{ab}^2}\int d\mathbf{r}\left\{\left[\frac{\partial u}{\partial x}\right]^2+\left[\frac{\partial v}{\partial x}\right]^2+\frac{4}{\lambda_J^2}\left[4-2\cos u+\sin\left[\frac{2\pi x}{a}+u\right]\sin v\right]\right\}+\int d\mathbf{r}\frac{B^2 s}{8\pi} . \quad (21)$$

Comparison of the free energy of the lattice $l=2s$ with the lattice $l=4s$ gives the magnetic field H_1 for the first transition.

Using the same procedure for lattices with periods $l=2sk$ and $a=\lambda_J H_0/kB$ along the z and z axes such that $B=\phi_0/ksa$, we can determine the critical fields H_k for the all sequence of transitions. The mean density of the free energy for the vortex lattice with period $l=2ks$ can be written in the form

$$F_k(B)=\frac{\phi_0^2}{16\pi^3\lambda_{ab}^2\lambda_J^2}\left[1-G_k\left[\frac{H_0}{kB}\right]\right]+\frac{B^2}{8\pi} , \quad (22)$$

where the functions $G_k(H_0/kB)$ are dimensionless. Now the magnetic induction B_k at the transition $l=2ks \to l=2(k+1)s$ is determined by the equation:

$$G_k(H_0/kB)=G_{k+1}(H_0/(k+1)B) . \quad (23)$$

According to (7), (22), and (23) in the range of magnetic field under interest the magnetic induction B differs from the magnetic field H by the term which is as small as s^2/λ_{ab}^2 and thus with the same accuracy the values H_k depend on the single parameter $H_0=\phi_0/\gamma s^2$, i.e., on γ. As a result the phase transitions in temperature at fixed parallel magnetic field are absent if anisotropy γ is independent of temperature. The most interesting are the first several transtions, because the jump in magnetization drops rapidly with growth of k. We note that in the London theory,[13] for which $a/l=\gamma/\sqrt{3}$, we have $l/s=\sqrt{3}a/\lambda_J=(H_0 2\sqrt{3}/H)^{1/2}$. So at least at large H_0/H, the value of k is given by the integer part of $(\sqrt{3}H_0/2H)^{1/2}$. Assuming that the same is valid for $k=1,2$ we can estimate $H_1\approx H_0/3$ and $H_2\approx H_0/8$ (accurate values of the critical fields and magnetization jumps can be found using equations presented above). The dependences of l/s and a/λ_J on H_0/H are shown schematically in Fig. 3.

We note that similar but weaker commensuration effects were observed for vortex lattice in modulated structures with periodically varying concentration of impurities[14] and they were analyzed in the standard London model.[15,16]

IV. DISCUSSION

In conclusion, we have obtained the magnetization [Eq. (17)] of highly anisotropic layered superconductors in strong, parallel magnetic fields, $H \gg H_0$, where $H_0=\phi_0/\gamma s^2$. A sequence of first-order phase transitions in a vortex lattice in a parallel magnetic fields is also found, the scale of the corresponding critical fields being of order H_0. The maximum jump of the magnetization (scale set by H_{c1}) occurs approximately at the field $H_0/3$. This field can be estimated as 3T in $Tl_2Sr_2CaCu_2O_8$ [$s\approx 10$Å and $\gamma\approx 200$ (Ref. 17)] and 12T in $Bi_2Sr_2CaCu_2O_8$ [$\gamma\approx 55$ (Ref. 18)].

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