Resonant tunneling through a symmetric triple-barrier structure

James Leo and G. A. Toombs
Department of Physics, University of Nottingham, Nottingham NG7 2RD, United Kingdom
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We consider tunneling of electrons through a triple-barrier structure. We focus our attention on the transmission coefficient T when the incident energy of the electrons is resonant with one of the quasibound energy levels of the structure. Although it is widely recognized that for a symmetric double-barrier structure T goes to unity at resonance, we shall show that this is not always the case for a symmetric triple-barrier structure in which the external barriers are half the width of the internal barrier.

The physics of resonant tunneling through a double-barrier structure has been fairly extensively studied in the last decade or so. This immense interest has been the result of growth techniques like molecular-beam epitaxy that have made it possible to construct, layer by layer, very thin wafers of semiconductor material. One of the most commonly used semiconductors is GaAs, which widens its band gap when the gallium is replaced by aluminum and thus forms barriers to the motion of electrons (holes) in the conduction (valence) band.

The double-barrier structure formed by two wafers of $Al_xGa_{1-x}As$ sandwiched between three sections of GaAs is one of the simplest devices that exhibit the full quantum-mechanical wave nature of electrons.[1] At resonance the energy of the electrons is such that the electron waves interfere constructively thereby significantly enhancing the probability of transmission. This results in a peak in the voltage-current (V-I) characteristics of the device followed by a region of negative differential resistance (NDR). If the device is symmetric, i.e., the barriers are of the same height and width, then the probability of transmission goes to unity, independent of the thickness or height of the barriers. Of course, the width of the resonance is very sensitive to the transmission properties of the individual barriers. Since the first observation of resonant tunneling in a double-barrier structure by Chang, Esaki, and Tsu[2] there has been much experimental activity in trying to improve the peak-to-valley ratio[3] and of observing NDR at room temperature.[4,5] For a review of the theory of resonant tunneling see the article by Toombs and Sheard.[6]

In this paper we shall be considering resonant tunneling through the "next most complicated structure," i.e., the triple-barrier structure. One may indeed be justified in asking why this structure should be studied. Is it not just a simple extension of the double-barrier structure? The fact is that this structure exhibits very unusual behavior which would never be observed in a double-barrier structure as it results from the coupling between the quasibound levels in the quantum wells. In a previous publication by Payne[7] a triple-barrier structure was also studied theoretically. However, he was mainly concerned with the current (calculated using the sequential model), which showed an unexpected increase as the outer barriers were widened. This result is related to the work presented here. In another publication by Nakagawa *et al.*[8] a triple-barrier structure was studied experimentally. The structure they used had external barriers that were exactly half the width of the internal barrier. We will show in this paper that if the external barriers were any thinner (or the internal barrier was any thicker) then the peak current through the device would be decreased.

Consider a symmetric triple-barrier structure where the central barrier's width is infinite and the outer barriers' widths are finite and furthermore, for simplicity let there exist only one quasibound state in each quantum well. As the structure is symmetric the levels will be degenerate, but as the central barrier is reduced in thickness the levels will couple forming an antibonding and bonding state. If the transmission coefficient T (defined as the ratio of the transmitted current density to the incident current density) of this structure is probed by firing in electrons at different incident energies, then one would expect to find two resonances whose splitting is dependent on the transmission coefficient of the central barrier. Furthermore, as the structure is symmetric, one would also expect the transmission coefficient to be unity at the two resonances. In this paper it is proved that this is not always the case and that the central barrier has to be thinner than a critical width (which is dependent on the width of the outer barriers) before two resonances with $T=1$ can be observed.

We shall now derive an expression for the transmission coefficient of a one-dimensional triple-barrier structure using the transfer-matrix formalism.[6] The transfer matrix M of the structure relates the amplitudes of the incoming and outgoing plane waves [$a(x)$ and $b(x)$] on the left of the barrier structure to the incoming and outgoing plane waves [$c(x)$ and $d(x)$] on the right through the following expression:

$$\begin{bmatrix} a(x) \\ b(x) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} c(x) \\ d(x) \end{bmatrix} . \quad (1)$$

Although it is possible to derive the elements of M, in practice it can be extremely complicated if there are several barriers and wells so we simplify things by writing

$$M = \Pi_n^N m_n , \quad (2)$$

where m_n is the transfer matrix for the nth segment (barrier, well, etc.) of the total structure and is given by

$$m_n=\begin{bmatrix} 1/t_n & r_n^*/t_n^* \\ r_n/t_n & 1/t_n^* \end{bmatrix} , \tag{3}$$

where t_n and r_n are the transmission and reflection amplitudes, respectively. Note that for the wells between the barriers, the transfer matrix takes on the following very simple form:

$$m_i=\begin{bmatrix} e^{-ikw} & 0 \\ 0 & e^{ikw} \end{bmatrix} , \tag{4}$$

where k is the wave vector of the electrons in the wells and w is the width of the wells. By multiplying the transfer matrices together, we can find the following form for the transmission amplitude of the triple-barrier structure in terms of the transmission and reflection amplitudes of the individual barriers:

$$t=\frac{t_e^2 t_i e^{2ikw}}{1-2r_e r_i e^{2ikw}+r_e^2(r_i/r_i^*)e^{4ikw}} , \tag{5}$$

where the i and e subscripts label the internal barrier and identical external barriers, respectively. Before taking the modulus squared of Eq. (5) to obtain the transmission coefficient, we define the phases ϕ and α from the definitions,

$$r_e=\sqrt{R_e}e^{-i\phi} , \tag{6}$$

$$r_i=\sqrt{R_i}e^{-i\alpha} . \tag{7}$$

The transmission coefficient is now given by

$$T=\frac{T_e^2 T_i}{1+4R_eR_i+R_e^2+2R_e\cos2\theta-4(1+R_e)\sqrt{R_e}\sqrt{R_i}\cos\theta} , \tag{8}$$

where $\theta=2kw-\phi-\alpha$ and the R's and T's are the reflection and transmission coefficients, respectively.

The resonance positions can be found from the maxima of T or equivalently from the minima of the denominator in Eq. (8). If we assume that the R's and T's are slowly varying functions of energies and therefore can be assumed to be energy independent, then we are left with only one variable, namely θ. To find the minima of the denominator we differentiate it with respect to θ and set this to zero. This gives the condition that at resonance,

$$\cos\theta=\left[\frac{R_i}{R_e}\right]^{1/2}\frac{1+R_e}{2} . \tag{9}$$

Furthermore, if we substitute this condition into Eq. (8), we find that the transmission coefficient at resonance does indeed equal unity. We expect there to exist two resonances with $T=1$ formed from the splitting of the two original degenerate quasibound well states. The positions of the two resonances are found by writing the cosine as a sine to form the following equation:

$$\sin(\theta/2)=\pm\left[\frac{1}{2}-\left[\frac{R_i}{R_e}\right]^{1/2}\frac{1+R_e}{4}\right]^{1/2} , \tag{10}$$

which gives the two positions in terms of θ as

$$\theta_1=+2\sin^{-1}\left[\frac{1}{2}-\left[\frac{R_i}{R_e}\right]^{1/2}\frac{1+R_e}{4}\right]^{1/2} , \tag{11}$$

and

$$\theta_2=-2\sin^{-1}\left[\frac{1}{2}-\left[\frac{R_1}{R_e}\right]^{1/2}\frac{1+R_e}{4}\right]^{1/2} . \tag{12}$$

The energy splitting between the resonances can now be found very straightforwardly and is given by

$$\Delta E=\frac{\hbar^2k_1^2}{2m^*}-\frac{\hbar^2k_2^2}{2m^*}$$
$$=\frac{\hbar^2}{m^*w^2}(\alpha+\phi)\sin^{-1}\left[\frac{1}{2}-\left[\frac{R_i}{R_e}\right]^{1/2}\frac{1+R_e}{4}\right]^{1/2} , \tag{13}$$

where we have assumed that $\alpha_1=\alpha_2=\alpha$ and $\phi_1=\phi_2=\phi$ which is a fair assumption especially when the splitting is small. Note that for thick barriers $\alpha\simeq\phi\simeq\pi$.

Up until now we have not proved anything very unusual. In fact, all the results would be expected from what we know about a double-barrier structure. However, the existence of two resonances with $T=1$ depends on the condition given by Eq. (9) being satisfied. The interesting point is that if the right-hand side is greater than one because $R_i \gg R_e$, then no θ can be found which would give a $T=1$. Therefore, we now have the very strange situation that even for a symmetric structure $T<1$ at resonance which is in complete contradiction to the double-barrier case. Using Eq. (9) we can explicitly find the condition that would give $T<1$ in terms of the barrier widths. We first rewrite Eq. (9) in terms of transmission coefficients ($R=1-T$), then after some simple manipulations we get that for $T<1$,

$$\frac{T_e^2}{T_i}>(2-T_e)^2 . \tag{14}$$

By ensuring that current and particle density is conserved across each boundary, the following form for the transmission coefficient of a single-barrier can be obtained:

$$T_n=\frac{16\kappa^2k^2}{(\kappa^2+k^2)(e^{2\kappa b}+e^{-2\kappa b})+(k+i\kappa)^4+(k-i\kappa)^4} , \tag{15}$$

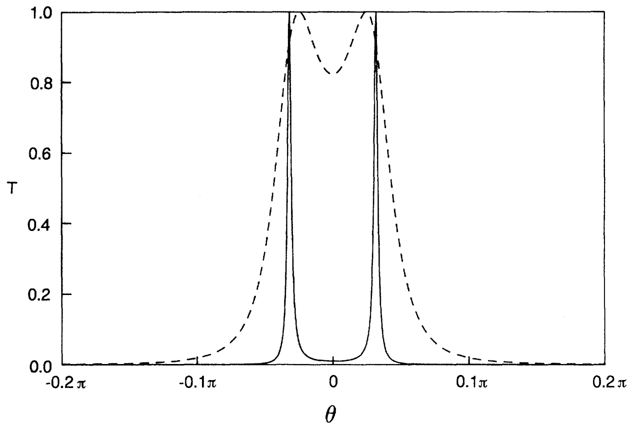

FIG. 1. The transmission coefficient of a triple-barrier structure plotted as a function of θ for two values of T_e. For the solid curve $T_e=0.01$ and for the dashed one $T_e=0.12$. For both curves $T_i=0.01$. Notice that the two resonances move together as T_e is increased showing that the coupling between the two wells can be affected by the external barriers.

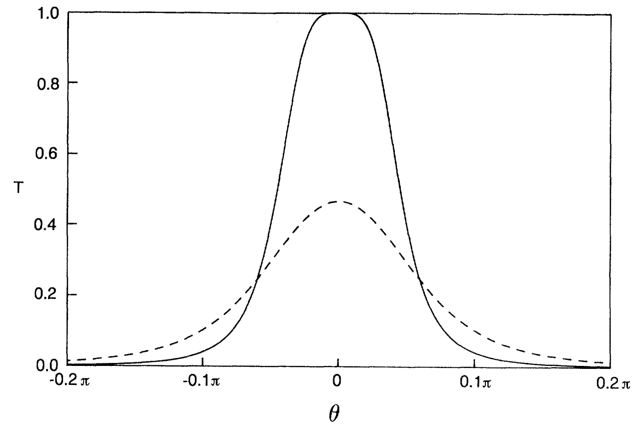

FIG. 2. As Fig. 1 but now for the solid curve $T_e=0.18$ and for the dashed one $T_e=0.4$. For the solid curve the critical value of the external barrier widths has been reached and only one resonance with a transmission coefficient of unity exists. As T_e is further increased, T reduces and will eventually tend to T_i.

where $i\kappa$ is the wave vector in the barrier and b is the width of the barrier. For typical III-V semiconductor barriers, the right-hand side of Eq. (15) will be dominated by the exponential term (this would then give a similar form for T as that given by the WKB approximation). Therefore, if we only consider this term and substitute the resulting form for T_e and T_i in to Eq. (14), we get that when

$$b < \frac{l}{2} - \frac{1}{\kappa} \ln(2-T_e) \tag{16}$$

then $T<1$ where b is the width of the external barriers and l is the width of the internal barrier. We have now derived a critical width for the external barriers to be before the transmission coefficient of the entire device can be equal to one. In fact Eq. (16) can be simplified even further to $b<l/2$ as the second term on the right-hand side is much smaller than either b or l.

Consider now what happens to the splitting when the width of the external barriers is decreased down to the critical width and beyond. As b tends to $l/2$ the right-hand side of Eq. (9) tends to one, which results in the argument of the sine term in Eq. (13) tending to zero. In Fig. 1 we have plotted the transmission coefficient from Eq. (8) as a function of θ for $T_e=0.01$ (solid line) and $T_e=0.12$ (dashed line), T_i is fixed at 0.01. The two resonances where $T=1$ can clearly be seen to move closer when T_e is increased (there is also a large amount of broadening). The minimum between the resonances is positioned at $\sin\theta=0$ which also happens to be the condition for resonance of a double-barrier structure. Our results can thus be interpreted as resonant tunneling through a double-barrier structure with another barrier in the well. There is another minimum in Eq. (8) when $\cos\theta=0$ which is away from the resonances. At this minimum $T \simeq T_e^2 T_i$. When b equals the critical value, the right-hand side of Eq. (9) equals one and the splitting ΔE equals zero. In other words, above this critical barrier width two resonances with $T=1$ exist and below it there is only one resonance with $T<1$. In Fig. 2 we have plotted T as a function of θ for $T_e=0.18$ (solid line) and $T_e=0.4$ (dashed line). The solid line is the critical point where only one resonance exists but where T still goes to unity. As T_e is increased the transmission coefficient decreases and tends to a value of T_i. The surprising thing about this is that we have affected the coupling between two wells without altering the height or width of the central barrier.

In conclusion, we have shown that resonant tunneling through a triple-barrier structure includes complications which do not exist for a double-barrier structure. This leads to a relationship between the widths of the external barriers and the internal barrier which shows under what conditions a transmission coefficient of unity will be observed.

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