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Resonances in the hopping probability between flexible quantum dots: The case of superlattices under parallel electric and magnetic fields

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The superlattice eigenstates in parallel electric and magnetic fields are spatially localized and the vertical transport is necessarily assisted. We show that the hopping probability along the growth axis displays huge oscillations whenever the optical phonon energy and Wannier-Stark and Landau spacings are commensurate.

A great amount of work has been recently devoted to the electronic transport along the growth axis of structures of reduced dimensionality. Resonant features in the vertical conductance of double barriers[1,2] and unbiased superlattices[3,4] have been observed, raising the problem of the nature of the transport in these systems. Coherent and sequential elastic processes must be simultaneously considered, whereas inelastic-optical-phonon contributions to the current are more easily distinguishable.

In a biased superlattice (SL) the electric field (F) destroys the interwell resonant coupling, and the vertical transport through the structure must be determined by sequential (assisted elastic and/or inelastic) processes. In fact, biased SL's are characterized[5] by an evenly spaced (by eFd, where d is the SL period) ladder of states, the Wannier-Stark ladder, and by oscillatory envelope functions along the field direction $\phi_p(z)$ which increasingly shrink around one SL period with increasing electric field (the pth state around the pth period). But, in spite of this Wannier-Stark localization, if the SL period is not so large (≤ 100 Å) a small but not negligible interwell coupling exists and a hoppinglike current can flow through the SL. Oscillations of assisted tunneling times in biased superlattices have already been predicted.[6-8] They are associated either with the oscillatory character of the Wannier-Stark functions with F,[7,8] or with the interwell electric-field-induced elastic[6] and inelastic ($eFd=\hbar\omega_{\mathrm{LO}}$) (Ref. 8) resonances. In these cases, and contrary to the bulk case where the current along the field direction is associated with delocalized crystal states, the vertical current is the result of a sequence of scatterings involving quasi-bidimensional (quasi-2D) states which are strongly localized along the field direction.

In this work we consider the possibility of a different class of vertical transport in biased superlattices, i.e., the one where the carrier flow along the growth axis takes place through quasi-0D states. A great effort has been recently devoted to the study of quantum-mechanical transport through quasi-1D and quasi-0D resonant states appearing in the constricted region separating two quasi-2D electron gases.[9] Striking resonances in the conductance of such systems have been theoretically and experimentally reported. However, the engineering of quantum dot structures, in spite of its fast development, does not yet provide good enough samples, and many physical aspects are blurred by imperfections introduced in the growth processes.

Quasi-0D states are made possible in a biased SL subjected to an additional magnetic field $\mathbf{B}\parallel\mathbf{F}$ (see Ref. 10). In this case the in-plane free motion (at $B=0$) concentrates into Landau orbits and the energy spectrum admits only discrete values. In the one-band approximation for the conduction states the tight-binding envelope functions and energies are given by $\Psi_{n,p}=\varphi_n(\rho)\phi_p(z)$ and $E_{n,p}=(n+1/2)\hbar\omega_c+peFd+E_0$, where ω_c is the cyclotron frequency; $n,|p|=0,1,2,\ldots$ and E_0 is the center of the SL miniband (at $F=B=0$). Here $\rho=(x,y)$ and the Landau orbits $\varphi_n(\rho)$ are described in the symmetric gauge $\mathbf{A}=\mathbf{B}\times\boldsymbol{\rho}/2$. These have a in-plane extent given by the magnetic length $\lambda_n^2=(2n+1)\hbar c/eB$, and so at high fields $\Psi_{n,p}$ strongly localizes both along the SL axis and in the layer plane. Thus, the SL eigenstates look like dots whose sizes are tunable by varying F and B. We stress that this flexibility is a considerable advantage over actual dots with lengths fixed by etching and patterning. We also emphasize that the flexible dots should be of much higher quality than the fabricated ones since their lateral dimensions are fixed by a magnetic field. Interesting features are expected when the electric (eFd) and magnetic ($\hbar\omega_c$) energies are commensurate.[11] For F and B values such that $eFd/\hbar\omega_c=m/q$ where $m,q=\pm1,\pm2,\ldots$ (and if no coupling exists between the F- and B-induced motions) each energy level becomes infinitely degenerate: the ground-state Landau orbit around the 0th well (energy $E_{0,0}$) is degenerate in energy with the kmth Landau orbit associated to the kqth Wannier-Stark level (and so centered around the kqth well in the SL), where $k=1,2,3,\ldots$. In this case one electron initially in the $n=p=0$ level can be elastically scattered and propagates along the growth axis. Notice that coherent processes are not allowed in this one-band approximation since the Landau index is not conserved ($\Delta n\neq0$) during the propagation. The transport is thus assisted. Recent magnetotunneling experiments[12] involving two barrier-separated quasi-2D electron gases have evidenced assisted elastic $\Delta n\neq0$ transitions. However, as we will show, inelastic scatterings are also very efficient for vertical processes in SL's near the optical-phonon resonances:

$$\Delta n\hbar\omega_c+\Delta peFd+\hbar\omega_{\mathrm{LO}}=0\,, \tag{1}$$

where $\hbar\omega_{\rm LO}$ is the energy of the emitted longitudinal-optical phonon.

In the following we consider the probabilities of some scattering processes which couple states centered in different periods of the SL and contribute to the assisted transport along its growth axis. The scattering-time calculations have been performed in the Born approximation:

$$\hbar/2\pi\tau_{0,0}=\sum_{n,p}|\langle\Psi_{n,p}|V_{\rm sc}|\Psi_{0,0}\rangle|^2\delta(E_{0,0}-E_{n,p}-E_{\rm sc})\,, \quad (2)$$

where the initial state has been chosen as the ground Landau orbit of the central Wannier-Stark level ($n=p=0$) and the summation is over all the final ($n\geq 0, p<0$) states. We have considered two possible scattering processes: elastic scatterings induced by randomly distributed (in the layer plane) ionized impurities at all the equivalent interfaces of the superlattice ($E_{\rm sc}=0$) and inelastic emission of dispersionless optical phonons ($E_{\rm sc}=\hbar\omega_{\rm LO}$). The scattering potentials $V_{\rm sc}$ are given[8] by the Coulomb interaction and the Fröhlich Hamiltonian, respectively. As expected from the translational symmetry of the SL, $\tau_{n,p}$ is p independent.

An inhomogeneous broadening of the levels has been phenomenologically introduced by replacing $\delta(E)$ by $\exp(-(E/\Gamma)^2)/\Gamma\sqrt{\pi}$ in Eq. (2), and results are presented for a 40-40 Å $\mathrm{Ga_{0.7}Al_{0.3}As}$-GaAs superlattice. In Fig. 1 we show the electric-field dependence of the elastic-scattering time $\tau^{(e)}$. Both for $B=0$ and 10 T the scattering-time increases with F as a result of the Wannier-Stark localization of the biased SL eigenstates. However, we note that at $B=10$ T this increase is modulated by huge oscillations, in contrast to the $B=0$ situation. These oscillations reflect the singular density of states of the biased SL at high B, and their magnitude depends on the broadening of Γ introduced. The valleys are easily associated with various possible resonances induced by the electric field, and are labeled by (n,p): $n\hbar\omega_c+peFd=0$. Nearest-neighbor contributions ($p=-1$) are by far the most important.

The inelastic emission of optical phonons is also very sensitive to the field values, as is clear from Fig. 2 where we show the variation of the decimal logarithm of $\tau^{(i)}$ with F for three B values: 0, 10, and 20 T. $T=0$ K and $\hbar\omega_{\rm LO}=36$ meV. Inelastic resonances occur for field values satisfying Eq. (1), and the principal one ($\Delta n=0$, $\Delta p=-1$) near $F_0=45$ kV/cm corresponds to the fastest traveling of carriers through the SL ($eFd=\hbar\omega_{\rm LO}$). At $B=0$ $\tau^{(i)}$ increases monotonically with increasing $F>F_0$, due to both the increasing localization of the Wannier-Stark states and the increasing in-plane momentum transfer ($\Delta\mathbf{k}_\perp$) during the scattering. On the other hand, increasing F in the presence of a large magnetic-field induces strong $\Delta n\neq 0$, $\Delta p=-1$ resonances, which are periodic in F (with period $\Delta F=\hbar\omega_c/ed$) and displays $\tau^{(i)}$ at resonances which increase with increasing Δn. These large oscillations at high electric fields are the salient feature of the $B\neq 0$ case compared to the $B=0$ situation. Indeed, when $F<F_0$ both the $B=0$ and finite B curves display a series of $\Delta p\leq -2$ resonances ($\Delta n=0$ if $B\neq 0$; $\Delta\mathbf{k}_\perp=0$ if $B=0$), eventually washed out in the broadened ($B\neq 0$) case. These are periodic in F^{-1} [see Eq. (1)] and display $\tau^{(i)}$ at resonances which increase with increasing $|\Delta p|$ (since they correspond to hoppings between increasingly distant dots). In this low-electric-field region elastic processes should dominate the transport, and less important oscillations are expected according to Fig. 1.

Both elastic and inelastic scatterings simultaneously occur and add their contributions. These can be very different according to the existence or not of energetic resonances induced by the fields, which are determined by the commensurabilities of eFd, $\hbar\omega_c$, and $\hbar\omega_{\rm LO}$. Figure 3 shows the B dependences of both $\tau^{(e)}$ and $\tau^{(i)}$, for four electric-field values. In Figs. 3(a) and 3(b) $eFd<\hbar\omega_{\rm LO}$ and optical-phonon emissions occur between second-neighbor dots. In Fig. 3(b) ($F=30$ kV/cm) the hopping is controlled by the elastic processes, which are mainly

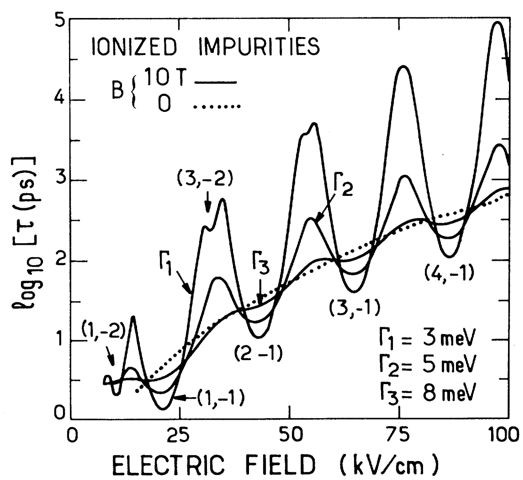

FIG. 1. Electric-field dependence of the decimal logarithm of the elastic scattering time. Ionized impurities; areal density: 10^{10} cm^{-2}; $B=0$, $\Gamma=0$ (dotted line); $B=10$ T: $\Gamma=3$, 5, and 8 meV (solid lines).

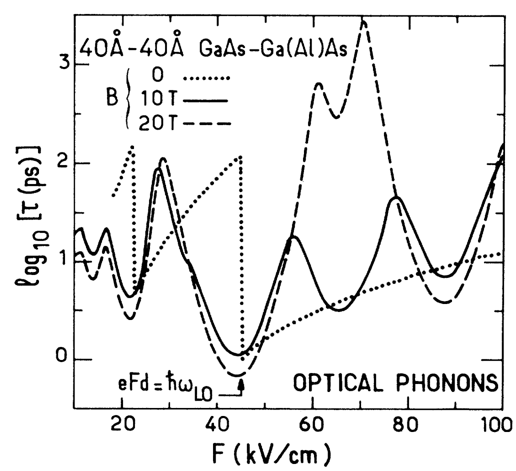

FIG. 2. Electric-field dependence of the decimal logarithm of the inelastic scattering time for three values of B: 0 T (dotted line, $\Gamma=0$); 10 T (full curve; $\Gamma=4$ meV); and 20 T (dashed; $\Gamma=4$ meV). $T=0$ K; $\hbar\omega_{\rm LO}=36$ meV. For clarity, $B=0$ results for $F<17.5$ kV/cm were omitted.

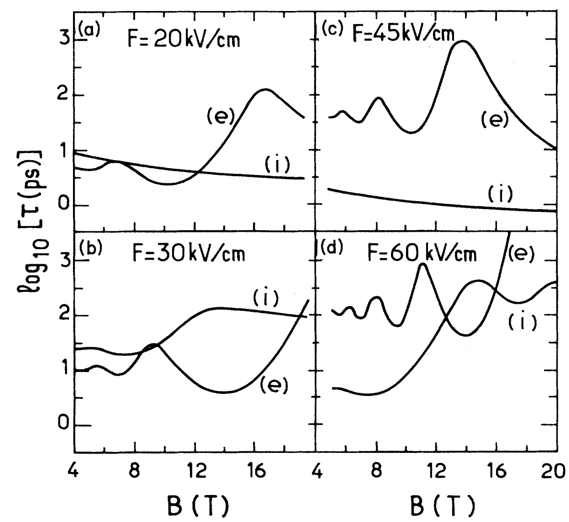

FIG. 3. Magnetic-field dependence of both elastic (e) and inelastic (i) scattering times (see Figs. 1 and 2 and text) for four F values: (a) 20 kV/cm; (b) 30 kV/cm; (c) 45 kV/cm, and (d) 60 kV/cm. $\Gamma=5$ meV.

nearest-neighbor dependent ($n=1, p=-1$ in the main valley at $B\approx 14$ T). On the other hand, in Fig. 3(a) $\hbar\omega_{\mathrm{LO}}=2eFd$ ($F=20$ kV/cm), and both the $p=-1$ elastic and $p=-2$ resonant inelastic contributions are of the same order of magnitude. In Fig. 3(c) $\hbar\omega_{\mathrm{LO}}=eFd$, and $\tau^{(e)}$ is at least one decade longer than $\tau^{(i)}\approx 1$ ps ($F=45$ kV/cm). In Fig. 3(d) $eFd>\hbar\omega_{\mathrm{LO}}$ and $\tau^{(i)}$ is smaller than $\tau^{(e)}$, except around the strong elastic resonance at $B\approx 14$ T ($F=60$ kV/cm, $n=2$, $p=-1$). In conclusion, both ionized impurities and optical-phonons play an important role in the hopping, and at high fields ($eFd>\hbar\omega_{\mathrm{LO}}$) an oscillatory transport limited by resonant processes takes place.

The picture of a vertical transport limited by interwell processes presupposes the existence of a fast intrawell relaxation mechanism. This is surely true when $\hbar\omega_c\approx\hbar\omega_{\mathrm{LO}}$, which corresponds to $B\approx 20$T for GaAs-based and $B\approx 12$ T for Ga(In)As-based materials. In that case, once interwell hopping occurs, a subsequent fast intrawell relaxation follows, and only the above calculated hopping times define the transport. However, we can also visualize more complex intrawell deexcitation paths for other specific B values. For example, let B satisfy $\hbar\omega_c\approx\hbar\omega_{\mathrm{LO}}/2$ (Fig. 2, $B=10$ T). In the high electric-field region ($eFd\gtrsim\hbar\omega_{\mathrm{LO}}$), phonon emissions dominate over elastic processes (Figs. 1 and 2). We should focus on the F regions around one interwell optical-phonon resonance, where an intrawell relaxation should follow a fast hopping process. For the $n=0$ interwell resonance ($F\approx 45$ kV/cm) the problem of a subsequent intrawell deexcitation is irrelevant. For the $n=2$ resonance ($F\approx 90$ kV/cm) a fast intrawell relaxation is warranted (since $2\hbar\omega_c\approx\hbar\omega_{\mathrm{LO}}$). Only near $F_1\approx 67$ kV/cm does the problem of intrawell relaxation appear; this corresponds to the $n=1, p=-1$ interwell transitions. Indeed, in this case no fast relaxation towards the $n=0, p=-1$ state is expected. However, a subsequent $\Delta n=1$ interwell resonant optical-phonon emission can occur, leaving the carrier in the final $n=2$, $p=-2$ level, from where a fast intrawell relaxation is possible, restarting the cycle. Thus, in both $\hbar\omega_c\approx\hbar\omega_{\mathrm{LO}}$ and $\hbar\omega_c\approx\hbar\omega_{\mathrm{LO}}/2$ cases a fast intrawell mechanism can be justified,[13] and so any oscillatory feature in the assisted transport is attributed to interdot (hopping) processes.

In conclusion, we believe that biased SL's in high magnetic fields are interesting for considering transport through states of reduced dimensionality. Compared to actual dots, biased superlattices represent much more accessible and "cleaner" heterostructures. More importantly they display a field-induced flexibility of their shapes and eigenstates which makes them attractive structures for a detailed understanding of transport in quantum dots.

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