Line-shape model for the modulated reflectance of multiple quantum wells

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In this paper we describe a model for the reflectance and modulated reflectance line shapes of quantum-well (QW) systems. In particular, we concentrate on electroreflectance (ER) of multiple quantum wells (MQW's). The work is an extension of our previous line-shape model for single quantum wells (SQW's), which is summarized briefly. In both cases we show that optical interference of light reflected from the well(s) and from the front surface of the sample has a dramatic effect upon the line shape. A simple analytical formula is derived for both the SQW and MQW cases that relates the ER to the QW dielectric function. Line shapes are calculated for a single exciton with different numbers of wells in the stack, which are then compared with an exact analysis using a transfer-matrix method. For SQW's the line shape is a mixture of the modulated real and imaginary parts of the QW dielectric function, which depends on the depth of the well in the sample. Therefore, in a MQW, each of the wells has a different contribution to the ER and the line shape is not just a scaled-up version of that for a SQW, as it is for absorption spectroscopies. As the stack thickness increases, the line shape becomes increasingly complicated, essentially due to the variation of the light penetration depth across the exciton profile. Our explicit analysis allows a clear understanding of the influence of each sample parameter on the line shape. It is shown that the center of the line shape is determined by the front of the MQW, while extra features appear in the wings as the number of wells increases, which originate from light reflected by the back of the stack. Finally, we discuss how our predictions compare with our recent measurements on $GaAs/Al_{0.3}Ga_{0.7}As$ MQW samples. This comparison demonstrates the importance of including interference effects, while also highlighting the limitations of the model.

I. INTRODUCTION

In modulated reflectance measurements, a periodic perturbation is applied to a sample and the reflectivity spectrum is measured at the frequency of the perturbation. In electroreflectance (ER), the electric field is modulated by applying a periodic bias,[1] while in photoreflectance (PR) the internal surface electric field of the sample is modulated using a mechanically chopped laser beam.[2] For a general review of modulation spectroscopy of bulk semiconductors, see Ref. 3.

Recently ER and PR spectroscopies have been popular methods for measuring quantum-well (QW) structures, and many of the recent studies are listed in Ref. 4. These take advantage of the enhanced sensitivity of this differential technique, to measure the transition energies of the systems under different conditions. However, identifying the transition energies is not as straightforward as for absorption spectroscopies, such as transmission, photocurrent, or photoluminescence excitation, where the peak can be measured. The line shape can be different for each transition and can also vary with an applied perturbation. This difficulty is essentially the result of the line shape depending on which parameters are modulated by the field, and is also a consequence of the fact that the reflectivity is determined by both the real and imaginary parts of the dielectric function. In previous publications we have derived a simple analytical line-shape formula for single quantum wells (SQW's),[5,6] which allows the transition energy as well as the linewidth and the transition strength to be extracted. In this paper we derive a similar formula for multiple-quantum-well (MQW) structures.

In bulk semiconductors, there is a single reflecting interface at the front of the sample. The ER can be expressed in terms of the modulated real ($\Delta\epsilon_r$) and imaginary ($\Delta\epsilon_i$) parts of the dielectric function by differentiation of Fresnel's relation for the reflectivity of this interface.[7] It can be shown that at the fundamental band edge, the coefficient of $\Delta\epsilon_i$ is zero, and so the ER is determined solely by the real part.[7] These arguments have been used to suggest that the PR of MQW systems can also be described by the real part alone, but we think this is mistaken, for reasons discussed below, and inconsistent with our ER measurements on both SQW's (Refs. 5 and 6) and MQW's.[1]

In QW systems the situation is entirely different from bulk. The sample necessarily contains a number of interfaces and the reflections from these will interfere. The reflectivity will consist of a large background due to the front surface of the sample, with much smaller features arising from the QW's. The QW layers are bounded by layers of similar refractive index and so have a much smaller reflectivity than the semiconductor-air interface at the front of the sample. The electric field modulates only the QW features, and so only these appear in the ER spectrum at the energy of the QW transitions. However,

the ray reflected from the front surface still has a great effect upon the line shape, through optical interference with the rays reflected from the QW's.

Our analysis of QW systems assumes each layer to have a homogeneous and isotropic refractive index. Hence the sample can be analyzed by considering its various interfaces to have reflectivities given by Fresnel's relation. We also assume the light to be coherent.

The reflectivity of layered systems can be modeled using an optical-transfer-matrix method.[8] This was used by Klipstein and Apsley[9] to calculate ER spectra for a SQW structure[9] and thereby demonstrate the sensitivity of the line shape to the thickness of material between the well and the front surface of the sample. More recently, Terzis *et al.*[10] have also used the transfer-matrix method to calculate reflectance spectra of MQW's. However, the optical-transfer-matrix method is a numerical technique, which consequently provides little insight into the importance of each parameter. A clearer understanding of the problem can be gained by considering only the most important interfaces in the structures. The reflection at the semiconductor-air interface is much stronger than any other in the sample, and consequently only the ray reflected from the front surface need be considered along with those from the QW's. By adopting such an approach, we have been able to derive explicit line-shape formulas for both the SQW and MQW cases. We have verified the validity of these expressions by comparison with the more accurate optical-transfer-matrix calculations.

For a SQW structure, only reflections from the front surface and the QW are important, and we have shown that this yields a simple analytical formula (see Sec. II B or Refs. 5, 6, and 11). This formula predicts the ER line shape to be a linear sum of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, the mixture being determined by the phase delay between the ray reflected from the front surface and the first ray from the QW. Indeed, similar effects have been seen in bulk semiconductors, where a nonuniformity of the field can result in reflection originating over a range of depths inside the sample, which mixes the contribution of $\Delta\epsilon_r$ and $\Delta\epsilon_i$.[12,13] However, the effect in bulk is much less dramatic than that seen in QW structures.

We have verified this formula experimentally by making measurements as a function of angle of incidence and sample temperature,[5] which change the optical phase delay as discussed in Sec. II B. Consequently, it was possible to produce dramatic changes in the line shape, which agreed with the evolution predicted by the line-shape formula. In a subsequent paper,[6] we made least-squares fits of the formula to spectra taken at different angles of incidence and sample temperature. This demonstrated that spectra can be accurately modeled and we were able to deduce the shape of the exciton profile at low and room temperature. Since then, we have performed similar experiments on a number of other SQW samples, having different layer thicknesses and optical linewidths and in every case observed the expected line-shape evolution.[11] More recently, there have been several other studies confirming the importance of optical interference on reflectivity spectra of QW structures, including calculated PR of SQW's;[14] experimental reflectivity of $GaAs/Al_xGa_{1-x}As$ SQW's;[15] PR measurements on $GaAs/Al_xGa_{1-x}As$ MQW's;[16] and reflectance measurements on $GaAs/Al_xGa_{1-x}As$ MQW's.[10]

Now we seek to extend this analysis to MQW systems, where the wells are identical and equally spaced. The barriers are assumed to be sufficiently thick to ensure that the electronic structure of each well can be considered the same as that of an isolated well. In such a structure, each of the wells will contribute a different line shape to the overall reflectivity, due to the different optical phase delay compared to the ray reflected from the front surface. We show that this, together with the effect of absorption in the stack, can lead to complicated line shapes, since the light penetration depth changes while scanning across the exciton profile. For moderately thick stacks, extra features appear in the wings of the line shape, which could be mistaken for extra transitions, but which in fact just derive from optical interference effects. Again, we have adopted an analytical approach that allows us to determine the essential elements determining the line shape. We find that the total MQW line shape can be approximated by a linear sum of two SQW line shapes, one deriving from the front and the other from the back of the stack. The term from the front dominates the center of the line shape, while the wings are determined by the term from the back. Hence the line shape will be most sensitive to the depths of the front and back of the MQW stack and the amount of absorption in the stack. We have verified our explicit analysis by making more exact calculations using the optical-transfer-matrix method.

In Sec. II we briefly outline our model for the SQW ER line shape, before extending this to MQW's, to derive an appropriate line-shape formula. In Sec. III the calculated line shape is plotted for an increasing number of wells in the stack. The nature of the line shape is discussed along with the effect of the various parameters. Finally, we summarize in Sec. IV the important features of our model and make comparisons with experimental spectra.

The calculation uses parameters similar to those measured for the E1H1 exciton of one of the $GaAs/Al_xGa_{1-x}As$ MQW's studied in Ref. 1. The well width was taken as 88 Å, while the barriers were taken to be 60 Å. The linewidth of this sample was found to be larger than for a SQW grown under identical conditions, due to a systematic variation of the well width and electric field across the MQW stack.[1] Since the present analysis assumes the wells in the MQW stack to be identical, the linewidth is taken to be that of the SQW of 3 meV [full width at half maximum (FWHM)]. The exciton profile is assumed to be Gaussian, which gives a good fit to the ER spectrum of the SQW sample at low temperature.[11] The integrated absorption strength of the exciton was taken as that measured for the MQW studied in Ref. 1, of 240 $cm^{-1}eV$, which yields a peak absorption of 76 000 cm^{-1}. The exciton energy was taken to be 1.55 eV. The field is assumed to be modulated between extreme values of 17 and 24 kV/cm, corresponding to a peak-to-peak modulation of 0.5 V around an offset of 0 V, in our MQW sample of Ref. 1.

II. OPTICAL INTERFERENCE IN QUANTUM-WELL SYSTEMS

The analysis proceeds by considering in turn different pairs of interfaces in the structure, so it is convenient to start with the amplitude reflectivity of an etalon formed by two interfaces of reflectivity r_1 and r_2,

$$r=(r_1+r_2e^{ig})/(1+r_1r_2e^{ig}) , \tag{1}$$

$$g=2k_{\text{vac}}Ln\cos\theta , \tag{2}$$

where g is the phase delay for light making a round trip between the interfaces, which are separated by a medium of refractive index n and thickness L. The angle between the direction of light propagation in this medium and the interface normal is θ, and k_{vac} is the k vector of the light in vacuum.

A. Reflectivity of an isolated well

Considering just an isolated QW, an etalon is formed between the two interfaces with the barrier material. If n_{QW} and n_b are the QW and barrier refractive index, respectively, then Fresnel's relation yields

$$-r_1=r_2=(n_{\text{QW}}-n_b)/(n_{\text{QW}}+n_b) . \tag{3}$$

Assuming that the refractive index of the QW is not much larger than that of the barrier material, by substitution into Eq. (1),

$$r_{\text{QW}}\approx i(\epsilon_{\text{QW}}-\epsilon_b)\sin(\tfrac{1}{2}g_{\text{QW}})\exp(i\tfrac{1}{2}g_{\text{QW}})/2n_b^2 , \tag{4}$$

where ϵ_{QW} (ϵ_b) is the well (barrier) dielectric function and g_{QW} is the phase delay for the round trip across the QW.

Equation (4) is valid only if $|(n_{\text{QW}}-n_b)/n_b|\ll 1$, or equivalently that $|r_1|\ll 1$. This approximation allows the well reflectivity (r_{QW}) to be written as a linear function of the QW dielectric function (ϵ_{QW}) and so provides a clearer insight into the form of the line shape. The peak absorption strength was measured to be about 76 000 cm^{-1} for a 88-Å $GaAs/Al_{0.3}Ga_{0.7}As$ MQW,[1] which yields a value of $\text{Im}[n_{\text{QW}}]\approx 0.5$, and $|r_1|\approx 0.07$, at the exciton peak. Therefore, it is a reasonable assumption to ignore the second-order terms in r_1, even at the energy where r_1 is largest. Further validation for this approximation is demonstrated in Ref. 11 by the close agreement of a plot of the QW reflectivity calculated using Eq. (4) and that produced by an exact relation.

Notice that the overall QW reflectivity will be smaller than that of its interfaces, because the reflection from the front interface almost exactly cancels with that from the back, assuming the well to be thin. [If g_{QW} is small in Eq. (4), then $r_{\text{QW}}\approx i\frac{1}{2}g_{\text{QW}}r_1$.] The well reflectivity can also be derived by regarding the QW as an infinitely thin, polarizable medium and applying Maxwell's boundary conditions.[9]

B. Electroreflectance of SQW structure

To calculate the overall reflectivity of a SQW sample, the combined reflectance of both of the QW interfaces (r_{QW}) is taken as that derived in Sec. II A. The ray reflected from the QW interferes with the strong reflection from the front surface of the sample. It is a very good approximation to ignore all the other interfaces in the structure, as the reflections from these are much weaker. Hence the SQW sample can be analyzed as an etalon formed between the quantum well and the front surface. It was pointed out in Sec. II A that the QW reflectivity is very small, and consequently only the first ray reflected from the well need be considered, yielding

$$R_{\text{SQW}}\approx R_f+2(1-R_f)\text{Re}[r_fr_{\text{QW}}\exp(ig_{\text{over}})] , \tag{5}$$

where g_{over} is the phase delay for a round trip across the overlayer between the top surface and the well. The front surface amplitude and intensity reflectivity are written as r_f (which is negative) and R_f, respectively.

The modulated intensity reflectivity can be found by substitution of Eq. (4) into Eq. (5),

$$\Delta R_{\text{SQW}}\approx A\ \text{Re}[i\Delta\epsilon\exp(ig)] , \tag{6}$$

where

$$A=(1-R_f)r_f\sin(\tfrac{1}{2}g_{\text{QW}})/n_b^2 \tag{7}$$

and

$$g=g_{\text{over}}+\tfrac{1}{2}g_{\text{QW}} . \tag{8}$$

$\Delta\epsilon$ ($=\Delta\epsilon_r+i\,\Delta\epsilon_i$) is the change in the QW dielectric function produced by the perturbation. The phase factor g, which we called the "line-shape phase" in Refs. 5 and 6, is the round-trip phase delay for propagation between the front surface and the midpoint of the QW. Equations (6)–(8) were found to be in close agreement with an exact calculation, which uses an optical-transfer-matrix method and includes all the interfaces inside the sample.[11] (For a more detailed discussion of this derivation, see Refs. 5, 6, and 11. References 6 and 11 include a comparison with the similar "third-derivative functional form" formula,[17] which is only appropriate for bulk semiconductors, but which has been applied by a number of workers to QW structures.)

For ER, the modulating electric field can change the energy (E_{ex}), the linewidth (Γ_{ex}), or the square of the overlap integral (V_{ex}) of a transition, so that in general[11,2]

$$\Delta\epsilon=(\partial\epsilon/\partial E_{\text{ex}})\Delta E_{\text{ex}}+(\partial\epsilon/\partial\Gamma_{\text{ex}})\Delta\Gamma_{\text{ex}}+(\partial\epsilon/\partial V_{\text{ex}})\Delta V_{\text{ex}} . \tag{9}$$

If F_1 and F_2 are the extreme fields, such that $F_2>F_1$, then the sign of the ER is defined here by $\Delta R=R(F_2)-R(F_1)$, and hence $\Delta\epsilon=\epsilon(F_2)-\epsilon(F_1)$, so that ΔE_{ex} and ΔV_{ex} will be negative and $\Delta\Gamma_{\text{ex}}$ will be positive for the lowest subband transitions. For the E1H1 and E1L1 exciton, the dominant effect of an electric field is to shift the transition to lower energy—the quantum confined Stark effect.[18] The ratio of the energy- and overlap-modulated line shapes, the first and third terms in Eq. (9), is given by[11]

$$\frac{\Delta\epsilon_E}{\Delta\epsilon_V}=\frac{V_{\rm ex}(F_1)+V_{\rm ex}(F_2)}{\Gamma_{\rm ex}}\frac{E_{\rm ex}(F_2)-E_{\rm ex}(F_1)}{V_{\rm ex}(F_2)-V_{\rm ex}(F_1)} . \quad (10)$$

For an 88-Å GaAs/$Al_{0.3}Ga_{0.7}As$ QW, with $\Gamma_{ex}=3$ meV (FWHM) and electric fields of $F_1=17$ kV/cm and $F_2=24$ kV/cm, $(\Delta\epsilon_E/\Delta\epsilon_V)\approx 16$. This ratio was determined using an exact envelope-function calculation, which is discussed in detail in Ref. 11. In fact, it is generally true that the energy-modulated term $(\Delta\epsilon_E)$ is larger than the overlap-modulated term $(\Delta\epsilon_V)$ for typical well widths, modulating fields, and linewidths.[11] Meanwhile, the linewidth-modulated term is also dominated by the energy-modulated line shape, since the increase in linewidth with field is usually much smaller than the Stark shift.[11]

It is therefore a good approximation to consider only changes in the exciton energy to determine the modulated dielectric function. Using small-bias modulations ensures that the modulating Stark shift ($\Delta E_{ex}=0.5$ meV, for the above well width and extreme fields) is small compared with the linewidth, and so the energy-modulated line shape is approximately given by the first differential of the QW dielectric function with respect to the photon energy (E), $\Delta\epsilon=-(\partial\epsilon/\partial E)\Delta E_{ex}$ (using $-\partial/\partial E=\partial/\partial E_{ex}$). Hence, from Eq. (6), the ER line shape for an exciton is just a linear sum of $\mathrm{Re}(\partial\epsilon/\partial E)$ and $\mathrm{Im}(\partial\epsilon/\partial E)$, depending on the value of the line-shape phase (g). These two functions have the simple shapes shown in Fig. 2 of Ref. 6, with either a negative peak or zero crossing at the exciton energy, respectively. Changing the line-shape phase will "rotate" the line shape between $\mathrm{Re}(\partial\epsilon/\partial E)$ and $\mathrm{Im}(\partial\epsilon/\partial E)$ and their inverses. Equations (6)–(8) are still valid for transitions above the QW band edge, but then the modulation of the dielectric function may not be dominated by the Stark shift, as discussed in Ref. 11. In this paper we wish to emphasize the mixing of $\Delta\epsilon_r$ and $\Delta\epsilon_i$ in the ER spectra of SQW's and MQW's caused by optical interference effects, and so the actual mechanism producing $\Delta\epsilon_r$ and $\Delta\epsilon_i$ is not important. Note that $\Delta\epsilon_r$ and $\Delta\epsilon_i$ will always have opposite symmetry, since they are related by the Kramers-Kronig transform.[3]

We have verified Eq. (6) experimentally, by varying the line-shape phase and observing the ER due to the E1H1 and E1L1 excitons cycle between line shapes resembling $\mathrm{Re}(\partial\epsilon/\partial E)$ and $\mathrm{Im}(\partial\epsilon/\partial E)$. A line-shape rotation was also observed for the E2H2 feature. The line-shape phase was varied in two ways, firstly by changing the angle of incidence, which affects the angle in the overlayer, and secondly by varying the temperature, which changes the transition energy.[5] In both cases we observed a close agreement with the behavior predicted by Eqs. (6)–(8) for the overlayer thickness of the sample. In a later publication,[6] we used least-squares fitting of Eq. (6) to spectra taken at different angles of incidence and temperature, to show that spectra can be very accurately modeled. We were able to deduce the shape of the exciton-broadening profile at different temperatures. Since then we have performed similar experiments on a number of other SQW samples, with different well and overlayer thicknesses, and in every case have observed evolution in accordance with Eq. (6).[11] We have also varied the line-shape phase by changing the offset electric field, hence producing a Stark shift of the exciton energy, and again observed a line-shape evolution in good agreement with our model.[11]

C. Electroreflectance of MQW structures

Now we extend our analysis of interference effects to cover the case of a number of regularly spaced wells in the sample. The analysis again considers only the rays reflected from the QW interfaces along with that from the front surface, as shown schematically in Fig. 1, as the other sample interfaces can be justifiably ignored. We again regard the reflectance of the two interfaces of any one QW to be given by a single expression r_{QW}, derived in Sec. II A. Summing the reflected rays and again ignoring the small double (and higher-order) reflections from the QW's yields the overall amplitude reflectivity of the structure to be given by

$$r_{\rm MQW}\approx r_f+(1-R_f)\exp(ig_{\rm over})\sum_{m=0}^{N} r_{\rm QW}\exp(img_{\rm per}) , \quad (11)$$

where g_{per} is the phase delay for the round trip between adjacent wells and the stack contains a total of $N+1$ wells. Optical absorption in the stack is also included in Eq. (11) through the imaginary part of g_{per} [i.e., $\mathrm{Im}(n_{QW})$].

The summation is trivial if the reflectivity (and hence dielectric function) is the same for each of the wells. In this case, solving for the intensity reflectivity yields

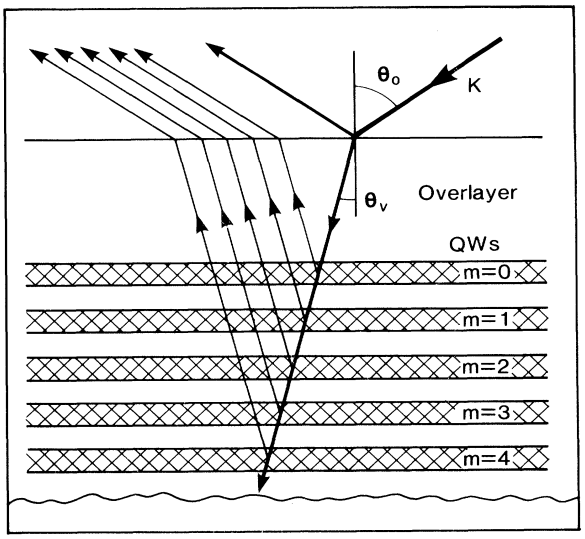

FIG. 1. Schematic of the most important reflected rays from a MQW sample.

$$R_{\mathrm{MQW}} \approx R_f + 2(1-R_f) \times \mathrm{Re}\left\{ r_f r_{\mathrm{QW}} \exp(i g_{\mathrm{over}}) \times \frac{1-\exp[i(N+1)g_{\mathrm{per}}]}{1-\exp(i g_{\mathrm{per}})} \right\} . \quad (12)$$

Equation (12) is similar to the result derived for a SQW [Eq. (5)], with an extra factor in the second term. Notice that this extra factor is similar to the amplitude distribution produced in the image plane of a diffraction grating.[19] The analogy is not surprising, as the diffraction pattern of a grating is the Fourier transform of the slit pattern in the object plane, while the ER is the transform of the distribution with depth of the quantum wells [see Eq. (11)]. Substituting for the well reflectivity from Eq. (6) yields

$$\Delta R_{\mathrm{MQW}} \approx A\ \mathrm{Re}\left[i \frac{\sin[\frac{1}{2}(N+1)g_{\mathrm{per}}]}{\sin(\frac{1}{2}g_{\mathrm{per}})} \Delta\epsilon \exp(ig) \right] , \quad (13)$$

where

$$g = g_{\mathrm{over}} + \tfrac{1}{2}N g_{\mathrm{per}} + \tfrac{1}{2} g_{\mathrm{QW}} \quad (14)$$

and A is again given by Eq. (7). Notice that the phase factor (g) in Eq. (14) is the line-shape phase determined at the midpoint of the MQW stack. Again, the two phase terms g_{QW} and g_{per} derive from small thicknesses inside the structure and so have only a slow variation with energy, which will not greatly affect the line shape. Meanwhile, the same cannot be said for the phase term ($\frac{1}{2}Ng_{\mathrm{per}}$), which derives from half of the total stack thickness. This term results in calculated MQW ER looking very different from that of a SQW, as is demonstrated in Sec. III, even if the same value is taken for the phase (g) in Eqs. (13) and (6).

III. MQW ER LINESHAPE

Figure 2 plots the calculated ER line shape of a single exciton for MQW's with between 1 and 200 wells in the stack. The line shape is calculated using Eq. (12), taking a well width of 88 Å and barriers of 60 Å. The exciton energy was taken as 1.55 eV, with an integrated absorption strength of 240 $\mathrm{cm}^{-1}\,\mathrm{eV}$, and the broadening profile was assumed to be Gaussian of 3 meV (FWHM). These parameters are those relevant to the E1H1 excitons of the samples studied in Ref. 1, as discussed in Sec. I. The modulated QW dielectric function was generated by a Stark shift of 0.5 meV, and the much less significant changes in the linewidth and overlap integral, as discussed in Sec. II A, were ignored. The overlayer thickness was taken as zero in these calculations, as its effect just rotates the entire line shape, in a similar manner as for SQW's,[5] while in this paper we wish to concentrate on effects due to the MQW alone.

The dashed lines in Fig. 2 plot another calculation of

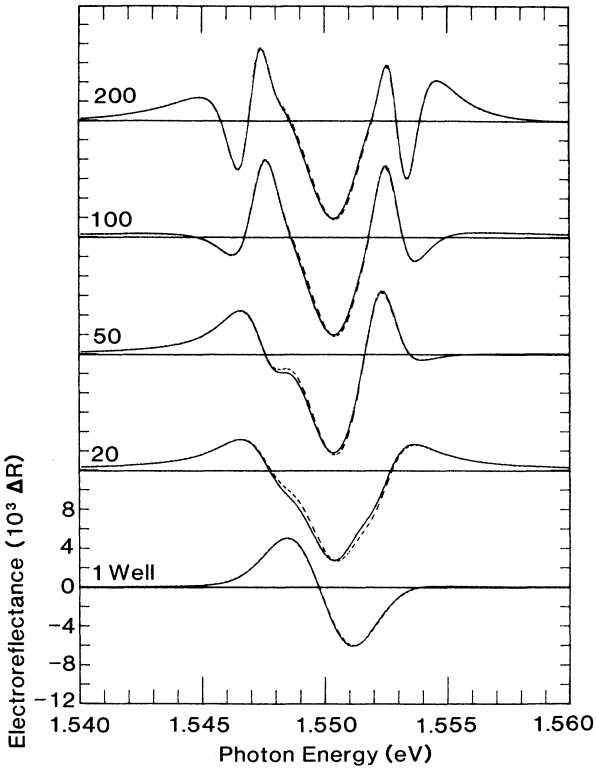

FIG. 2. ER line shape of a single exciton for different numbers of wells in the stack (indicated by each curve), calculated using either the explicit analysis detailed in text (solid lines) or an optical-transfer-matrix method (dashed lines). The sample thicknesses are as follows: wells, 88 Å; barriers, 60 Å; overlayer, 0 Å. The exciton is assumed to have a Gaussian profile with the following parameters: energy, 1.55 eV; linewidth, 3 meV (FWHM); integrated absorption strength, 240 $\mathrm{cm}^{-1}\,\mathrm{eV}$.

the ER line shape, which takes the same parameters, but this time uses the optical-transfer-matrix method.[8–10] The optical-transfer-matrix method is a numerical technique that takes account of all the reflected rays and so is more exact than the explicit analysis (solid lines) detailed earlier. It can be seen that the curves calculated using the exact and explicit analyses are almost indistinguishable and so justify our neglect of the second-order terms in r_{QW}. The explicit analysis has the considerable advantage of allowing a clearer understanding of the role of each sample parameter, as discussed below.

Figure 2 demonstrates that the ER signal does not strengthen significantly as the number of wells increases, as is the case for most other spectroscopies, such as photoluminescence excitation, photocurrent, absorption, or electrotransmission. This is explained by the reflectivity of each of the wells having a different shape, due to the variation of their depth (and hence their line-shape phase). The ER signal, therefore, does not increase in size when the total stack thickness is increased above an eighth of the wavelength of light in the material, which is typically about 300 Å for the GaAs QW band gap. Adding more wells changes the phase of the overall

reflectivity, but does not increase its strength.

In Fig. 2 the curve for the case of one well in the stack (i.e., $N=0$) is equivalent to that for a SQW with zero overlayer thickness, which is proportional to $\mathrm{Im}(\partial\epsilon/\partial E)$ [see Eq. (6)]. Notice also that the line-shape formula for MQW's [Eqs. (13)–(14)] reduces to that for a SQW [Eqs. (6)–(8)] for $N=0$, as would be expected. As the number of wells increases, the MQW line shapes in Fig. 2 become increasingly more complicated than those for a SQW, with extra features appearing in the wings of the line shape. Essentially, this is because the absorption depth changes as a function of energy across the exciton profile, thereby affecting the line-shape phase. For the absorption peak height used in these calculations, the penetration depth is only ≈ 0.25 μm at the central energy of the exciton, which is much less than the stack thicknesses typically employed in MQW samples. If the stack was nonabsorbing (a hypothetical situation), the MQW line shape would be similar to that of a SQW, with the line-shape phase of the midpoint of the stack [see Eqs. (13) and (14)].

The ER in the central region of the line shape, where the absorption is large, will originate from the front of the stack, while in the wings of the line shape the absorption coefficient falls toward zero and so the line-shape phase will be that for the middle of the stack. Concentrating firstly on the central region of the line shape, as the number of the wells is increased the line-shape phase changes very rapidly and the line shape rotates, until the stack thickness starts to exceed the light penetration depth, after which there is only a gradual change in the line shape. The exact shape around the center of the exciton feature is sensitive to the overlayer thickness and the penetration depth.

The origin of the structure in the wings of the line shape in Fig. 2 can be deduced by rewriting Eq. (13) as the sum of two terms:

$$\Delta R_{\mathrm{MQW}} \approx B\,\mathrm{Re}\{\,\Delta\epsilon \exp(i g_{\mathrm{over}} + i\tfrac{1}{2} g_{\mathrm{QW}} - i\tfrac{1}{2} g_{\mathrm{per}}) - \Delta\epsilon \exp[\,i g_{\mathrm{over}} + i(N+\tfrac{1}{2}) g_{\mathrm{per}} + i\tfrac{1}{2} g_{\mathrm{QW}}]\}\,, \tag{15}$$

where

$$B = -(\tfrac{1}{2}A)/\sin(\tfrac{1}{2} g_{\mathrm{per}})\,. \tag{16}$$

Notice that the first term in Eq. (15) has a line-shape phase corresponding to a depth close to the front of the MQW stack, while the second term has a phase close to the deepest part of the stack. This then is equivalent to regarding the MQW stack as a thick homogeneous layer of material, which extends from the depths of half a period shallower than the middle of the first QW to half a period deeper than the middle of the final QW. It is therefore the positions of the front and back of the stack that have the most influence on the line shape. The finite thicknesses of the well and barrier introduce a correction term [the sinusoidal term in Eq. (16)], but this has a slow energy dependence, which does not greatly affect the line shape. This can be thought of as due to the reflected rays from adjacent wells in the body of the stack tending to interfere destructively with one another, leaving only contributions from the edges of the stack.

The two terms in Eq. (15) are plotted separately in Fig. 3. The contribution from the front of the stack (dashed line) does not change as the number of wells increases and this term dominates the center of the ER profile. Clearly, the second term in Eq. (15) (solid line), from the back of the stack, is responsible for the features in the wings of the line shape in Fig. 2. With an increasing number of wells, the line-shape phase of the second term varies very rapidly, as the depth of the back of the stack increases. Note that adding just 300 Å to the stack thickness will rotate the features in the wings from a contribution resembling $\mathrm{Re}(\partial\epsilon/\partial E)$ to one like $\mathrm{Im}(\partial\epsilon/\partial E)$, or vice versa. Notice also that the contribution of the back falls to zero at the center of the line shape, as it is limited by absorption in the stack.

IV. SUMMARY OF MQW MODEL AND COMPARISON WITH EXPERIMENTAL SPECTRA

A model has been proposed for the ER line shape of a MQW, where the contributions of the individual wells are summed. Unlike spectroscopies that depend solely on the imaginary part of the QW dielectric function, the contributions from each well have different line shapes, due to

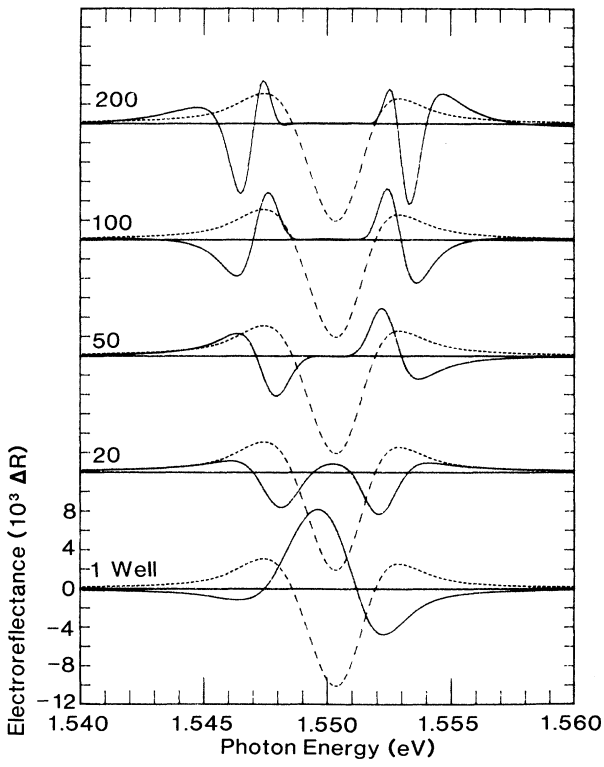

FIG. 3. ER line shapes, calculated with the same parameters as in Fig. 2, decomposed into the first (dashed line) and second (solid line) term of Eq. (15). (The first term does not vary with the number of wells.) These contributions correspond to ER from near the front and back of the MQW stack (see text).

their different mix of $\Delta\epsilon_r$ and $\Delta\epsilon_i$, depending upon the depth of the QW inside the sample. Consequently, the MQW ER line shape is more complicated in shape than that of a SQW, and is a little stronger.

The complicated line shape derives from the line-shape phase varying across the exciton profile as the optical penetration depth changes. At energies where the absorption is small, or for narrow MQW stacks, the line-shape phase is that for the midpoint of the stack. Usually though, absorption in the MQW stack is strong near the center of the exciton peaks and light will only penetrate a fraction of the stack.

The summation can be easily performed when the dielectric function of each QW is identical and this yields a simple line-shape formula. Inspection of this formula shows it to be comprised of two terms: one due to the front of the MQW stack and the other due to the back. The contribution of the back is limited by the absorption in the stack and so is only significant in the wings of the line shape. The total line shape therefore shows a large central feature, due to the ER from the front, which is flanked by two smaller features due to the back of the stack. The phases of these three features depend upon the overlayer and stack thicknesses and the absorption coefficient.

The calculations presented here are for a single Gaussian-broadened 1*s* bound exciton. Including the excited excitonic states and the band-band continuum will introduce extra absorption in the MQW stack, which will result in a weakening of the subsidiary features in Fig. 2 on the higher-energy side of the line shape. We have chosen to use the simplest possible representation of the QW dielectric function here, in order to highlight the effects due to optical interference alone. Line shapes qualitatively similar to those in Fig. 2 are produced by using a Lorentzian-broadened exciton profile, although the subsidiary features are then somewhat weaker. This is because the absorption coefficient changes less rapidly in the wings of the line shape for a Lorentzian-, rather than for a Gaussian-, broadened profile.

Finally, we discuss how our calculated line shapes agree with those measured experimentally. In previous papers[5,6,11] we have verified the ER of SQW samples to be in excellent agreement with our model, by varying g in Eq. (6) and observing the line shape rotate between $\Delta\epsilon_r$ and $\Delta\epsilon_i$. We therefore expect our analysis of the optical interference effects to yield a similar agreement for MQW samples. Despite this, we are unaware of a modulated reflectance spectrum in the literature which shows clear evidence for the subsidiary features predicted in Fig. 2 for thick MQW stacks.

The lack of experimental evidence for these subsidiary features may be due to slight imperfections in the measured MQW stacks, as a systematic variation in the exciton transition energy across the stack of just a few meV (comparable to the width of the features) will greatly suppress their strength. This change in the exciton transition energy across the stack could derive from a variation in the well width or the electric field due to the background ionized-impurity charge. We have made measurements on two $GaAs/Al_{0.3}Ga_{0.7}As$ MQW's (Ref. 1) which show a systematic variation in the transition energy across the stack due to (a) the linear increase in electric field produced by the unintentional background doping of the stack and (b) relatively abrupt monolayer increases in the well width at certain depths in the stack.

For such an imperfect MQW stack, each well cannot be assumed to have an identical dielectric function and the summation in Eq. (11) can no longer be performed easily, making Eq. (12) invalid. Analytical analysis is then more difficult, although a numerical solution using an optical-transfer-matrix approach[8–10] is certainly possible. Imperfections of the stack will have a large effect upon the subsidiary features, as the contribution from the back will be attenuated differently by the absorption in the stack. The model discussed in this paper is inappropriate for exact line-shape fitting of the experimental spectra we have measured.[1] However, the ideas discussed here are still valid. Features appear in the MQW ER reported in Ref. 1 due to the front and back of the stack, and their line shapes depend upon their depth in the sample and the absorption in the stack. The model presented here allowed a good qualitative explanation of the ER line shapes measured in our experimental study and their dependence on the angle of incidence and the applied bias, as discussed in Ref. 1. We expect a better quantitative agreement for MQW samples with better uniformity across the stack and indeed this would be an excellent test of sample quality. The subsidiary features apparent in Fig. 2 would be most pronounced in thick stacks, where the exciton lines are sharp and have a large peak absorption coefficient.

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