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Voltage quantization by ballistic vortices in two-dimensional superconductors

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The voltage generated by moving ballistic vortices with a mass m_v in a two-dimensional superconducting ring is quantized, and this quantization depends on the amount of charge enclosed by the ring. The quantization of the voltage is the dual to flux quantization in a superconductor, and is a manifestation of the Aharonov-Casher effect. The quantization is obtained by applying the Bohr-Sommerfeld criterion to the canonical momentum of the ballistic vortices. The results of this quantization condition can also be used to understand the persistent voltage predicted by van Wees for an array of Josephson junctions.

Aharonov and Casher have predicted that a neutral particle with a magnetic moment will exhibit a force-free interference effect when its path encloses a charged wire.[1,2] This Aharonov-Casher (AC) effect is the dual of the Aharonov-Bohm effect describing the force-free interaction between an electrical charge whose path encloses magnetic flux. Aharonov and Casher commented that as a result of this effect, the introduction of electric charge into a multiple connected region of a superfluid comprised of magnetic moments causes circulation of the moments.[2] In discussing the nonlocality of the AC effect, Reznik and Aharonov further explored the macroscopic quantum example of a vortex (a magnetic fluxon) in a superconductor.[3] Recently, van Wees has suggested that the charge vector potential can also produce a persistent voltage in a ring made up of arrays of Josephson junctions.[4] In this paper, we find the generalized electric dual of the fluxoid quantization condition in a superconductor by quantizing the canonical momentum for a vortex that moves ballistically in a two-dimensional (2D) superconducting system.

Fluxoid quantization in a superconductor is a manifestation of the macroscopic quantum nature of the superconducting state and gives rise to vortices in a type-II superconductor. The ballistic motion of these vortices in a constant applied magnetic field $\mathbf{B}$ and current density $\mathbf{J}$ is described by a Lorentz-like equation of motion. The Hamiltonian of this equation of motion can be written in terms of vector and scalar potentials related to the charge in the superconducting system. Quantizing the corresponding canonical momentum leads to the quantization of the electrical dual of the fluxoid (DOF), the total charge resulting from the sum of the applied charge and induced effective charge created by the motion of the vortices. In other words, the vortices (which are themselves a reflection of quantization on a macroscopic scale) lead to a further quantization condition. For this criterion, as well as the AC effect, to hold, it is important that the force on the vortex be Galilean invariant.[1-3] To ensure this, the vortices must not interact with the lattice; that is, they must be able to move ballistically without dissipation. Because ballistic vortices are more realizable in two-dimensional superconductors and in arrays of Josephson junctions,[5-7] we will restrict our attention to such 2D systems. (Although Aharonov and Casher[2] have shown, in analogy to a general field-theoretical observation of 't Hooft,[8] that a coherent vortex state cannot coexist with the condensed state of the electrons in a superconductor, we believe that such a coexistence is possible in a finite sized system. This issue will be addressed at the end of the paper.)

In standard models of fluxoid motion,[9-11] the force per unit length on a vortex produced by the driving current is given by $\mathbf{f}_1 = \mathbf{J}\times\mathbf{\Phi}_0$, where $\mathbf{\Phi}_0$ is the vorticity of the vortex and has a magnitude Φ_0 (one flux quantum) and a direction along the line segment that traces out the core of the vortex. (In our 2D superconductors, $\mathbf{\Phi}_0$ is orthogonal to the plane of the system.) There is an additional driving term,[10,11] sometimes referred to as the Magnus force, given by $\mathbf{f}_2 = -n_{3D}q\mathbf{v}\times\mathbf{\Phi}_0$ where $\mathbf{v}$ is the velocity of the vortex. Because we are considering the case of ballistic vortices, we neglect any drag forces. If d is the thickness of the 2D superconductor, the total force on the vortex $\mathbf{F}$ is thus given by $\mathbf{F} = (\mathbf{f}_1+\mathbf{f}_2)d = \Phi_0(\mathbf{K}\times\hat{\mathbf{z}} - nq\mathbf{v}\times\hat{\mathbf{z}})$. Here, $n = n_{3D}d$ is the 2D density of superconducting electrons, $\hat{\mathbf{z}}$ is a unit vector orthogonal to the plane of the 2D system, and $\mathbf{K} = \mathbf{J}d$ is the surface current. If the vortex retains its identity during its motion, as is usually assumed, the electric-field created by the moving vortex can be incorporated into the dynamics by letting the vortex have a mass m_v.[5-7] For a 2D superconductor this mass is $\Phi_0^2\epsilon_0 d/(4\pi\xi^2)$ where ξ is the coherence length of the superconductor.[7] For a square 2D superconducting array of identical Josephson junctions with a normal-state resistance R, capacitance C, and periodicity p, the mass is $\Phi_0^2 C/2p^2$.[5-7] Consequently, the equation of motion for a ballistic vortex becomes

$$m_v(d\mathbf{v}/dt) = \Phi_0[\mathbf{K}\times\hat{\mathbf{z}} + \mathbf{v}\times(-nq\hat{\mathbf{z}})]\,. \qquad (1)$$

The force given in Eq. (1) is Galilean invariant, since $\mathbf{K} = nq\mathbf{v}_s$, where $\mathbf{v}_s$ is the velocity of the superconducting electrons. When the normal core of the vortex imparts momentum to the lattice, drag forces result which are responsible for flux-flow resistivity and the Hall angle in a superconductor.[9-11] These drag forces destroy the Galilean invariance of the force which is necessary for the effects considered here. As a result, we discuss only the cases where the drag terms are negligible so that the vortex will move with the superfluid velocity $\mathbf{v}_s$ when there is no net force on the vortex. The drag terms are negligible if the Hall angle θ_H satisfies $\tan\theta_H \gg 1$. For a continuous 2D superconductor this criterion is equivalent to the condition $\Delta \gg \mathcal{E}_F$, where Δ is the gap energy and $\mathcal{E}_F$ is the Fermi energy.[9-11] This condition is not satisfied in any conventional superconductor.

In contrast, consider a superconducting array of Josephson junctions, where the criterion demands that $\Delta \ll \mathcal{E}_c(R_n/R_e)$. Here $\mathcal{E}_c$ is the confinement energy of an electron in the unit cell of the array given by $\hbar^2/(mp^2)$, R_n is the normal-state resistance of the tunnel junctions, and R_e is the equivalent resistance of the junction.[12] The criterion can be satisfied in a superconducting array because the gap is on the order of a few meV, the periodicity is on the order of ten microns, and R_e can be a shunt resistor which can be orders-of-magnitude smaller than R_n.[13] Even if this criterion is satisfied, we must also demand that the vortices remain ballistic during the course of the experiment. In terms of length scales, this means that the size of the system must be smaller than the mean free path l_v for the vortices. To estimate l_v, we note that the vortex in the array cannot move faster than one unit cell in a time $\hbar/\Delta$ without creating many quasiparticles. Moreover, if a vortex is subjected to an impulse of force, it will decay with the characteristic RC time constant of a junction.[7] This means that $l_v = p\Delta RC/\hbar$. Since arrays can be made such that l_v is hundreds of times larger than p, it is possible to encounter physical situations where the motion of the vortices is indeed governed by Eq. (1). Similarly, the ballistic motion can also be observed if the time scale of the experiment is much shorter than the decay time of a vortex (the RC time constant).

Equation (1) can be rewritten as a Lorentz-like equation of motion:

$$m_v(d\mathbf{v}/dt) = \Phi_0(\mathbf{e} + \mathbf{v} \times \mathbf{b}) . \quad (2)$$

The quantity analogous to electric charge is the flux quantum of the vortex, Φ_0. The analogous electric and magnetic fields are $\mathbf{e}$ and $\mathbf{b}$, respectively. Both $\mathbf{e}$ and $\mathbf{b}$ are constant fields and are defined as

$$\mathbf{e} \equiv \mathbf{K} \times \hat{\mathbf{z}} \quad (3)$$

and

$$\mathbf{b} \equiv -nq\hat{\mathbf{z}} . \quad (4)$$

Hence, the motion of the vortex is analogous to the motion of a charged particle in a constant externally applied electric and magnetic fields. This analogy can be further extended by noting that $\mathbf{b}$ is solenoidal since this reflects the fact that the charge density in the 2D superconductor does not vary in the z direction. Likewise, the statement $\nabla \times \mathbf{e} = -\partial\mathbf{b}/\partial t$ simply reflects charged conservation of the superconducting electrons.[14] Consequently, these vortex fields can be expressed in terms of vector and scalar potentials: $\mathbf{b} = \nabla \times \mathbf{a}$ and $\mathbf{e} = -\nabla\phi - \partial\mathbf{a}/\partial t$.

We now express the equation of motion in terms of these potentials. The Lagrangian consistent with Eq. (2) is $\mathcal{L} = m_v v^2/2 - \Phi_0\phi + \Phi_0 \mathbf{v} \cdot \mathbf{a}$. The canonical momentum $\mathbf{p}$ for the vortices is thus given by

$$\mathbf{p} = \partial\mathcal{L}/\partial\mathbf{v} = m_v\mathbf{v} + \Phi_0\mathbf{a} , \quad (5)$$

which, in analogy to charged particles, is the sum of both the kinematic and the field momenta. In terms of the canonical variables $\mathbf{r}$ and $\mathbf{p}$, the Hamiltonian is $\mathcal{H} = (\mathbf{p} - \Phi_0\mathbf{a})^2/(2m_v) + \Phi_0\phi$. We note that this Hamiltonian differs in two ways from that found by van Wees for 2D arrays. First, van Wees does not have a scalar potential term because $\mathbf{b}$ was set to zero, thus making a transverse gauge appropriate.[15] The second significant difference between the two Hamiltonians is that van Wees' has an extra term $E_p(x,y)$ which describes the periodic potential caused by the underlying 2D array of junctions. In the continuous 2D superconductor considered here this term vanishes. Nevertheless, E_p can influence the observability of ballistic vortices in real arrays if it causes significant pinning. Here we assume that an array can be made with sufficiently low pinning.

It has been shown[5,6] that the transition from classical mechanics to quantum mechanics for the vortex in the presence of an applied current density with no Magnus force ($\mathbf{b} = 0$) is the same as that for an electron. We assume that this transition is the same even in the presence of the Magnus force. Although we could now define a wave function for the vortices, we restrict ourselves to quantizing the canonical momentum using the Bohr-Sommerfeld criterion, $\oint_C \mathbf{p} \cdot d\mathbf{l} = nh$, along a closed path C in the 2D system. Using the canonical momentum given in Eq. (5), we find the quantization condition yields

$$2en = -Q + \frac{1}{\Phi_0}\oint_C m_v \mathbf{v} \cdot d\mathbf{l} , \quad (6)$$

where the charge of the superconducting electrons enclosed by the contour, Q, is defined by the relation

$$\oint_C \mathbf{a} \cdot d\mathbf{l} = -Q , \quad (7)$$

and the expressions $q = -2e$ and $\Phi_0 = h/2e$ have been used. Consequently, the quantization condition is analogous to fluxoid quantization in a superconductor: it is not the charge but rather the DOF (the sum of the charge and an electrical induction contributed by the motion of a vortex) that is quantized.

To demonstrate the quantization condition, consider a single vortex in the Corbino geometry, shown in Fig. 1, which is similar to that examined in Refs. 2 and 4. The 2D superconducting disk is bounded on the inside and outside by a 3D superconductor that will repel the vortex and therefore confine it to the 2D region. We will assume that the confinement ensures that the vortex moves only in the azimuthal direction along the fixed radius r_a. If the vor-

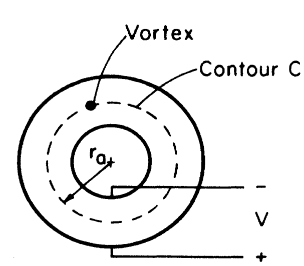

FIG. 1. A 2D superconducting disk contains a single vortex whose motion is restricted to the closed contour C of radius r_a. As a result of the vortex motion, a voltage V appears across the disk guaranteeing DOF quantization.

tex moves with a constant velocity v_θ, a voltage V is generated across the inner and outer radii of the ring given by the rate of change of flux across the path:

$$V=\Phi_0 v_\theta/(2\pi r_a) \tag{8}$$

(for the case of multiple vortices the result scales with the total number of flux quanta present). This voltage manifests itself in two ways. First, it creates an induced electrical charge proportional to the electrical capacitance of the system, $C_{\rm sys}$. As a result, in order to be self-consistent Q must be expressed as $Q=Q_{\rm ext}-C_{\rm sys}V$, where $Q_{\rm ext}$ is the externally applied charge. In addition, the voltage is also associated with the charge created by the electric induction. Indeed, one may define a *kinetic capacitance* $C_{\rm kin}$ such that

$$C_{\rm kin}V\equiv\frac{1}{\Phi_0}\oint_C \mathbf{m}_v\mathbf{v}\cdot d\mathbf{l}\,. \tag{9}$$

For the single vortex in the Corbino geometry $C_{\rm kin}=m_v(2\pi r_a/\Phi_0)^2$. Notice that the kinetic capacitance associated with the motion of flux carriers is completely analogous to the conventional kinetic inductance that is associated with the motion of charged carriers in a superconductor. The quantized DOF is therefore composed of three charge components (the externally applied charge, the electrically induced charge, and the kinetically generated charge) and therefore the self-consistent expression for the voltage across the 2D system is

$$V=(2en+Q_{\rm ext})/(C_{\rm sys}+C_{\rm kin})\,. \tag{10}$$

We thus find that the voltage across the system can only assume certain quantized values for a given applied charge. As a result, the quantization of the superconducting system's energy is manifested in a series of displaced parabolas:

$$E=\tfrac{1}{2}(C_{\rm sys}+C_{\rm kin})V^2=\frac{(2en+Q_{\rm ext})^2}{2(C_{\rm sys}+C_{\rm kin})}\,. \tag{11}$$

The consequences of Eq. (6) may be illustrated in the following manner. Suppose the 2D system is cooled through its superconducting transition while $Q_{\rm ext}=0$. Because the system seeks to minimize its energy as well as maintain DOF quantization, it will choose to be in the lowest quantum state ($n=0$). Hence, there will be no net motion of the vortices and $V=0$. If we now introduce an insulating rod carrying the charge $Q_{\rm ext}$ into the multiply connected region, the vortices will move in such a manner to maintain the $n=0$ condition (thereby shielding the applied charge) since there is now no dynamical way for the superconducting system to change levels. In other words, a constant voltage $V=Q_{\rm ext}/(C_{\rm sys}+C_{\rm kin})$ will be induced across the superconducting structure from the motion of the vortices. In the limit $C_{\rm kin}\gg C_{\rm sys}$, the proportionality constant between the induced voltage and external charge when the system is in the ground state is $C_{\rm kin}$ rather than $C_{\rm sys}$ as would be expected classically. In the other extreme where $C_{\rm kin}\ll C_{\rm sys}$, the charge Q would remain zero for all values of $Q_{\rm ext}$ subsequently applied (this is analogous to the flux remaining zero in a multiply connected superconductor regardless of the applied field).

Furthermore, if by some process, the system did not cool into its ground state but instead to the level $n=n'$, the voltage across the system would be nonzero even if no charge is externally introduced. Indeed, if an external charge were now applied, the resulting V would maintain the DOF level n'. As a result, the measured voltage would be different than the expected classical result where V would strictly result from induced charge effects.

Other nontrivial effects of DOF quantization can be demonstrated by the following thought experiment. Suppose we modify our previous experiment by cooling the 2D system through its superconducting transition while the rod of charge $Q_{\rm ext}$ is inserted in the structure.[16] In this case, the vortices will move in such a way so that $|2en+Q_{\rm ext}|$, and hence the energy, is minimized. We characterize this level of the system with the value n'. If the charged rod is now removed, the vortices will alter their velocity to maintain the same value of n' since there is again no dynamical way for the superconducting system to change levels. Therefore, when the rod is removed and $Q_{\rm ext}=0$, V will change correspondingly so that $(C_{\rm sys}+C_{\rm kin})V=2en'$. The resulting voltage across the structure is thus quantized in steps of

$$\delta V=2e/(C_{\rm sys}+C_{\rm kin}) \tag{12}$$

as illustrated in Fig. 2.

In the limit $C_{\rm sys}\gg C_{\rm kin}$, this thought experiment is the analog of the classic demonstration of flux quantization,[17,18] and DOF quantization becomes charge quantization. It should be stressed that this effective-charge quantization is *not* a result of the quantized unit of charge on an image electron; rather, it results from the velocity of the vortices and for the case of a single vortex $v_\theta=2\pi r_a(2en'+Q_{\rm ext})/(\Phi_0 C_{\rm sys})$. Moreover, because the induced charge is persistent, remaining indefinitely after the applied charge is removed, the superconducting system behaves as if it has an infinitely large RC time constant. In other words, because the induced charge cannot relax, the superconducting system in this configuration appears to have an infinite resistance.

In the opposite extreme where $C_{\rm sys}\ll C_{\rm kin}$, the electrically induced charge is negligible. If a single vortex is formed to maintain DOF quantization, it moves with a speed $v_\theta=\Phi_0(2en'+Q_{\rm ext})/(2\pi r_a m_v)$ and, as might be expected in this dynamically dominated limit, the energy of

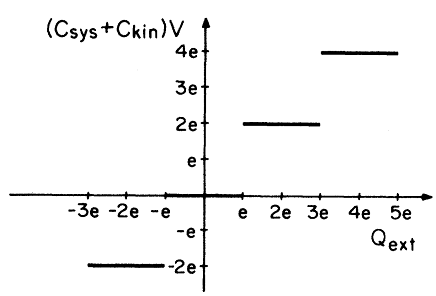

FIG. 2. The final induced voltage V left in the 2D superconducting system shown in Fig. 1 when a charge $Q_{\rm ext}$ is applied before the system is in the superconducting state and then removed after the system has cooled.

the system is given by the kinetic energy of the moving vortex.[19] The quantized voltage levels in this limit are spaced $\delta V = (e\Phi_0^2)/(2\pi^2 m_v r_a^2)$, which, for a typical array of Josephson junctions with $C = 10^{-15}$ F, can be of the order of a few microvolts.

The periodicity of the underlying lattice in an array of Josephson junctions can have a more profound effect on the ground-state energy levels of the system. The energy levels in an array are periodic when $Q_{\rm ext}$ is changed by $2e$. This causes the energy levels to open up a gap where they cross, thus forming energy bands. In such a periodic structure, van Wees has shown that a persistent current will flow in the ground state of the system.[4] The maximum value of the persistent velocity and voltage V' occurs when $Q_{\rm ext} = e$ and $n = 0$. For the limit $C_{\rm kin} \gg C_{\rm sys}$, we find $V' = ep^2/(2C\pi^2 r_a^2)$ which follows from Eq. (8) and agrees with Ref. 4.

Finally, we wish to comment on how the results presented here are affected by Aharonov and Casher's observation that a coherent state of the vortices will destroy the superconducting state. It may at first appear that the effects described in this paper hold only for isolated (independent) vortices since a coherent quantum-mechanical state of vortices is precluded as discussed in Ref. 2. However, the general observation made by Aharonov and Casher depends on the fact that the vector potential of the vortices decays away exponentially at large distances rather than as $1/r$ as in the analogous Aharonov-Bohm effect. If, however, the size of the superconducting array is smaller than the effective penetration depth the vector potential will fall off as $1/r$. Since the effective penetration depth for an actual two-dimensional system can be on the order of centimeters, we suggest that a coherent state of vortices can indeed form in a *finite-sized* sample. This practical issue is similar to that used to argue for the occurrence of the Kosterlitz-Thouless phase transition in superconductors,[20] a transition which was first thought impossible because of the exponential decay of the fields. Exactly how the finite size of the sample affects the observation of Aharonov and Casher, and of the more general field-theoretical observation of 't Hooft,[8] remains to be worked out in detail.

In summary, we have shown that by introducing analogous scalar and vector potentials for the motion of ballistic vortices, a quantization condition for the charge in a superconductor is found. This DOF quantization is the dual of the usual fluxoid quantization in a superconductor and is a manifestation of the Aharonov-Casher effect. The quantization condition leads to quantized voltages due to the motion of ballistic vortices when the system is cooled while subject to an applied external charge. Indeed, the behavior of the 2D superconducting systems considered can be envisioned in terms of the motion of virtual magnetic monopoles. Furthermore, the voltage quantization should also be seen for vortices undergoing diffusive transport as long as the dimensions of the superconductor are smaller than the corresponding phase-coherence length of the vortices.

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[13]Care must be taken not to let R_n exceed a few MΩ so that the array remains in the classical regime. In other words, the Josephson-coupling energy should always exceed the charging energy for our analysis to hold.

[14]Although they are not necessary for the analysis here, the other two analogous Maxwell's equations can be found and allow for the development of London-like equations for the vortices.

[15]Another set of potentials can be used instead since gauge invariance demands only that $\mathbf{a}' = \mathbf{a} + \nabla\chi$ and $\phi' = \phi - \partial\chi/\partial t$ for any irrotational scalar function $\chi(\mathbf{r},t)$. For the Corbino geometry in Fig. 1, van Wees uses $\mathbf{a}' = It/(2\pi r) i_\theta$ and $\phi' = 0$ where θ is the polar angle. By choosing $\chi = I\theta t/(2\pi)$, we obtain $\mathbf{a} = 0$ and $\phi = -I\theta/(2\pi)$. Hence, the scalar potential alone can describe this problem, in contradiction to the statement made in Ref. 4.

[16]In the thought experiment, we assume that vortices will form during the cooling process before the critical temperature is reached. This assumption allows us to see the effects of ballistic vortices in the superconducting state, though it may be difficult to realize in practice.

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