Sensitivity of the multiple-scattering speckle pattern to the motion of a single scatterer

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The influence of changing the position of a small number of scatterers on the speckle pattern produced by a disordered system is studied. The sensitivity of the speckle pattern and the total transmission to changes in the scatterers' configuration is calculated for two- and three-dimensional systems. For two-dimensional systems the speckle pattern, as well as the total transmission, are found to be extremely sensitive to displacement of a single scatterer.

Statistical properties of the speckle pattern (the angular intensity dependence) for transmitted and reflected waves from disordered systems have recently attracted much attention. As a result of interference effects, the speckle pattern, produced by multiple-scattering disordered systems,[1] shows a wide variety of interesting phenomena. The ensemble average of the reflected speckle pattern exhibits an enhanced backscattering peak.[2] Correlations and fluctuations of the angular intensity were studied,[3–6] revealing interesting effects such as the locking of the speckle pattern to the incident direction, known as the memory effect,[4,6] and long-range correlations between angular intensities in different directions.[3–5]

The intensity transmission coefficient $T(\mathbf{q}_a,\mathbf{q}_b)$ fluctuates rapidly as the incoming-wave vector $\mathbf{q}_a$ or the outgoing-wave vector $\mathbf{q}_b$ is changed, thus creating the behavior of the angular intensity known as the speckle pattern. The total transmission $T(\mathbf{q}_a)$ is given as a sum of transmission coefficients:

$$T(\mathbf{q}_a)=\sum_{\mathbf{q}_b} T(\mathbf{q}_a,\mathbf{q}_b) , \tag{1}$$

and the conductance of a system may then be calculated with use of a Landauer formula,[7]

$$G=\sum_{\mathbf{q}_a} T(\mathbf{q}_a) . \tag{2}$$

The conductance of a two-dimensional (2D) mesoscopic system is known to be very sensitive to the motion of a single impurity.[8,9] Even for three-dimensional systems, the sensitivity of the conductance to the motion of a single impurity is much more than expected from a classical (no interference effects taken into account) calculation. Based on the fact that the transmission coefficients and total transmission are more sensitive to a total change in the scatterer configuration than the conductance, it is reasonable to expect a similar enhanced sensitivity in the case of moving a single scatterer.

In this paper we shall present an analytical calculation of the sensitivity of the transmission coefficients, as well as the total transmission, to the movement of a single scatterer for two- and three-dimensional systems. The fraction of scatterers that should be moved in order for the transmission coefficients to be altered by the same amount as for changing the whole scatterer configuration will be determined.

We shall use a simple model which assumes the scattering potential of a single impurity to be short ranged and isotropic. The scattering potential of a system of N impurities located at positions $\mathbf{r}_1,\ldots,\mathbf{r}_N$ is given by

$$U(r)=\sum_{i=1}^{N} u\,\delta(\mathbf{r}-\mathbf{r}_i) , \tag{3}$$

where u is the scattering strength. For such a system the mean free path l will be related to the scattering strength and the number of impurities in the following way: For a two-dimensional system,

$$l_{2\mathrm{D}}=\frac{4k_0}{nu^2} , \tag{4a}$$

where n is the density of scatterers, and $k_0=2\pi/\lambda$. For a three-dimensional system,

$$l_{3\mathrm{D}}=\frac{4\pi}{nu^2} . \tag{4b}$$

A typical change in the transmission coefficient due to moving one scatterer by a distance, $\delta\mathbf{r}$, from its original position $\mathbf{r}$ may be defined as

$$\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle \equiv \tfrac{1}{2}\langle [T(\mathbf{q}_a,\mathbf{q}_b)-T_1(\mathbf{q}_a,\mathbf{q}_b)]^2\rangle , \tag{5}$$

where $T_1(\mathbf{q}_a,\mathbf{q}_b)$ is the transmission coefficient for an identical scatterer configuration as $T(\mathbf{q}_a,\mathbf{q}_b)$ except that one of the scatterers is moved by the distance $\delta\mathbf{r}$, and the angular brackets indicate an ensemble average over different configurations of scatterers. After subtracting and adding the averaged transmission coefficient $\langle T(\mathbf{q}_a,\mathbf{q}_b)\rangle$, we obtain

$$\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle = \langle \delta T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle - \langle \delta T(\mathbf{q}_a,\mathbf{q}_b)\delta T_1(\mathbf{q}_a,\mathbf{q}_b)\rangle . \tag{6}$$

The first term on the right-hand side of Eq. (6) is the usual transmission-coefficient fluctuation $\langle \delta T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle = \langle [T(\mathbf{q}_a,\mathbf{q}_b)-\langle T(\mathbf{q}_a,\mathbf{q}_b)\rangle]^2\rangle$, which is equal to $\langle T(\mathbf{q}_a,\mathbf{q}_b)\rangle^2$. It is possible to calculate $\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle$ by a diagrammatical technique similar to the one applied by Feng, Lee, and Stone[8] to calculate the conductance sensitivity to the motion of one single impurity. The displacement of one scatterer by a distance $\delta\mathbf{r}$ is described by the "diamond" vertex. For a three-dimensional system in momentum space, the diamond vertex is given by

$$V(\mathbf{p})=\frac{4\pi}{l\Omega}\mathrm{Re}(1-e^{i\mathbf{p}\cdot\delta\mathbf{r}}) , \tag{7}$$

where l is the mean free path and Ω is the volume. For two-dimensional systems the constant $4\pi/l$ should be replaced by $4k_0/l$. After inserting the diamond vertex randomly in the diagrams used to calculate $\langle \delta T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle$ and retaining only ones to the lowest order in Ω^{-1}, it remains to evaluate diagrams of the type shown in Fig. 1(a). We calculate the typical change in the transmission coefficient due to moving a scatterer by $\delta\mathbf{r}$ for a slab geometry of thickness L and a cross-sectional area A. Assuming $A \gg L^{D-1}$, where D is the dimension of the system, and strong scattering (i.e., $\Omega/l^D N\sim 1$), we obtain, for a two-dimensional system

$$\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle_{2\mathrm{D}}=\langle T(\mathbf{q}_a,\mathbf{q}_b)\rangle^2\frac{2}{3}\frac{L}{A}C_2(k_0\,\delta r) , \tag{8}$$

where $C_2(x)=1-J_0^2(x)$; $J_0(x)$ is a Bessel function of order 0. For a three-dimensional system, a typical change is

$$\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle_{3\mathrm{D}}=\langle T(\mathbf{q}_a,\mathbf{q}_b)\rangle^2\frac{lL}{A}C_3(k_0\,\delta r) , \tag{9}$$

where $C_3(x)=1-[\sin(x)/x]^2$. In the case of weak scattering, the result for the sensitivity of the speckle pattern for two- and three-dimensional systems should be multiplied by $\Omega/l^D N$.

From Eq. (8) one concludes that moving one scatterer per cross-sectional length L will completely change the speckle pattern for a two-dimensional slab. The physical reason for this extreme sensitivity of the speckle pattern is that for a two-dimensional slab a photon trajectory through the system has a finite probability of passing through any scatterer in an $L\times L$ section of the slab. Therefore, any change in the position of one of the scatterers will change the phases of all the trajectories passing through it, causing the speckle pattern to change significantly. Even for a three-dimensional system, the change of the speckle pattern due to moving a single scatterer is much larger than expected from a classical calculation. The usual classical argument would assume that the change in the speckle pattern would be proportional to N_L^{-1}, where N_L is the number of scatterers in a L^D volume of the slab. On the other hand, when taking into account multiple-scattering interference effects, we obtain the result given in Eq. (9), where the sensitivity of the speckle pattern is proportional to $N_L^{-1/3}$.

The change of the total transmission due to moving a scatterer by a distance $\delta\mathbf{r}$ may be defined in a similar way to the change of the speckle pattern,

$$\langle \delta_1 T^2(\mathbf{q}_a)\rangle=\langle \delta T^2(\mathbf{q}_a)\rangle-\langle \delta T(\mathbf{q}_a)\delta T_1(\mathbf{q}_a)\rangle , \tag{10}$$

where $\langle \delta T^2(\mathbf{q}_a)\rangle$ is the usual total transmission fluctuation, which, for a 2D slab, is proportional to[10] $(1/k_0 l)(L/A)\langle T(\mathbf{q}_a,\mathbf{q}_b)\rangle^2$ and which for a 3D slab is proportional to[3,5] $[1/(k_0 l)^2](lL/A)\langle T(\mathbf{q}_a)\rangle^2$.

The change in the total transmission due to the movement of a single scatterer is calculated by randomly inserting the diamond vertex into the usual diagram used to calculate $\langle \delta T^2(\mathbf{q}_a)\rangle$. After calculating diagrams of the type shown in Fig. 1(b), we obtain, for a two-dimensional system,

$$\langle \delta_1 T^2(\mathbf{q}_a)\rangle_{2\mathrm{D}}=\langle T(\mathbf{q}_a)\rangle^2 N_2\frac{1}{k_0 l}\left[\frac{L}{A}\right]^2 C_2(k_0\,\delta r) , \tag{11}$$

and, for the three-dimensional case,

$$\langle \delta_1 T^2(\mathbf{q}_a)\rangle_{3\mathrm{D}}=\langle T(\mathbf{q}_a)\rangle^2 N_3\frac{1}{(k_0 l)^2}\left[\frac{lL}{A}\right]^2 C_3(k_0\,\delta r) , \tag{12}$$

where the numerical constants are $N_2=32\pi^2$ and $N_3=2^5\pi^4/5$. In the case of weak scattering, the result for the sensitivity of the speckle pattern for two- and three-dimensional systems should be multiplied by $(\Omega/l^D N)^2$.

The total transmission sensitivity is reduced, compared with the speckle pattern sensitivity, by a factor of $(k_0 l)^{1-D}$, where D is the dimension of the system. For a three-dimensional system, the total transmission sensitivity is further reduced by a factor proportional to $N_L^{-2/3}$, as compared with the speckle-pattern sensitivity which is proportional to $N_L^{-1/3}$. A reduced sensitivity of the total transmission, compared with the sensitivity of the speckle pattern, is expected because the total transmission is the result of summation over different speckle patterns.

The sensitivity of the speckle pattern to changing the position of m scatterers in the sample $\langle \delta_m T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle$ or the total transmission sensitivity $\langle \delta_m T^2(\mathbf{q}_a)\rangle$ to changing the position of m scatterers in the sample may be defined in a similar way to the definitions for moving one scatterer given in Eqs. (6) and (10). In this case it is convenient to use an alternative approach based on the work of Altshuler and Spivak.[9] In this approach a pole α_p/l^2

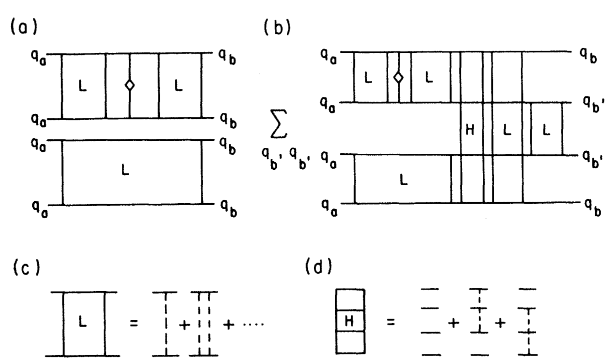

FIG. 1. (a) Typical Feynman diagram for the calculation of $\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle$. The diamond vertex is given in Eq. (5). (b) Typical Feynman diagram for the calculation of $\langle \delta_1 T^2(\mathbf{q}_a)\rangle$. (c) Ladder propagator. (d) Hikami vertex (see Ref. 11).

is inserted into the ladder propagator wherever the propagator appears in between two lines belonging to two different scatterer configurations in the calculation of $\langle \delta T(\mathbf{q}_a,\mathbf{q}_b)\delta T_m(\mathbf{q}_a,\mathbf{q}_b)\rangle$ or $\langle \delta T(\mathbf{q}_a)\delta T_m(\mathbf{q}_a)\rangle$. α_D is defined as $\alpha_2=(4/N)m\tilde{C}$ for a two-dimensional system and $\alpha_3=(3/N)m\tilde{C}$ for a three-dimensional system. $\tilde{C}$ is some kind of average over $C_D(k_0\delta r)$. As long as $\alpha_D \ll (l/L)^2$, we obtain the same results as for $\langle \delta_1 T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle$ and $\langle \delta_1 T^2(\mathbf{q}_a)\rangle$ multiplied by m, the number of scatterers moved. In the saturation limit $\alpha_D \gg (l/L)^2$, we obtain $\langle \delta_m T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle = \langle \delta T^2(\mathbf{q}_a,\mathbf{q}_b)\rangle^2$ and $\langle \delta_m T^2(\mathbf{q}_a)\rangle = \langle \delta T^2(\mathbf{q}_a)\rangle^2$. Thus by altering the position of more than A/L scatterers for the two-dimensional case, or more than A/lL scatterers for the three-dimensional case, the scatterer configuration may be considered as a new *unrelated* configuration.

In conclusion, we have presented an analytical calculation of the influence of changing the positions of a finite fraction of scatterers on the angular intensity transmission coefficients and on the total transmission. The results suggest that those changes, in particular for two-dimensional systems, are significant. An estimate of the number of scatterers that should be moved in order to obtain a configuration which may be considered an unrelated configuration was obtained.

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