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Relaxation processes due to collisions involving conduction electrons and magnon solitons in magnetic semiconductors

M. V. Satarić
Faculty of Technical Sciences, Novi Sad, Serbia, Yugoslavia

J. A. Tuszyński
Department of Physics, The University of Alberta, Edmonton, Alberta, Canada, T6G 2J1
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The statistics of collisions of conduction electrons with magnon solitons in a ferromagnetic semiconductor with an easy axis of anisotropy is considered by taking into account the exchange interaction of the Vonsovsky type. The relaxation time of such collisions is calculated in the framework of the nonequilibrium-statistical-operator technique. Possible manifestations of the discussed mechanism in scattering experiments are discussed.

I. INTRODUCTION

In recent years "quasi-one-dimensional" (QOD) magnets have turned out to be suitable systems for theoretical investigations of nonlinear modes, and it was also possible to relate these results to experiments and to computer simulations.[1–4] At low temperatures a large variety of QOD nonlinear magnetic systems can be mapped onto the well-known sine-Gordon model.[2,5,6] For example, the dynamics of easy-plane ferromagnetic and antiferromagnetic chains in an external field breaking the easy-plane symmetry can be approximated by the sine-Gordon equation. Moreover, it has been shown,[7,8] that QOD compressible magnetic chains can lead to the existence of solitons.

Of special interest to our work is the possibility of the formation of solitonlike excitations in QOD ferromagnets with "easy-axis" anisotropy (EAA). Such a possibility is indicated[9] under the assumption that the magnetic anisotropy energy is small compared to the exchange energy. A similar problem, but using a different approach, has been examined recently by Aksenov and Žakula[10] and involves magnetoelastic coupling. Our treatment is consistent with the method given in Ref. 9.

It is well known[11] that the existence of localized spin moments that couple to the conduction electrons has important consequences on the electrical conductivity of the corresponding crystals. The magnetic ions act as scattering centers so that at sufficiently low temperatures the scattering they cause will be the primary source of electrical resistance.

For example, in magnetic alloys it has been known[11] that instead of dropping monotonically, the electric resistivity has a rather shallow minimum occurring at low temperatures (~ 10 K) that depends weakly on the concentration of dissolved magnetic ions.

Vonsovsky[12] appears to have been the first researcher to recognize that an additional contribution to electrical resistivity could occur in ferromagnets as a result of the exchange interaction between conduction electrons and the localized magnetic ions, often called the *s-d* or *s-f* interaction.

It seems that crystals with EAA can support the existence of solitons and that, on the other hand, possess conducting features that may be found among magnetic semiconductors (MS). In a very interesting book dedicated to MS, Nagaev[13] discussed the conducting properties of several classes of MS. For instance, it follows from his book that the EuO crystal possesses properties which allow it to be mapped approximately onto the EAA chain. The EuO crystals attract a great deal of attention from researchers in the field on account of the simplicity of their crystallographic structure (cubic, of the NaCl type) and due to the absence of an orbital angular momentum of the electrons in the partially filled f shells of the Eu^{2+} ions. The ground state of those f shells is the $^8S_{7/2}$ with $L=0$, $S=\frac{7}{2}$. It should be added that, according to Ref. 14, the magnetic dipole-dipole energy of ferromagnetically ordered spins in a cubic lattice is zero. Those are the reasons for the crystallographic anisotropy of such crystals to be relatively very small (the anisotropy field for EuO is equal to 0.02 T while the effective exchange field is of the order of 10^2 T) and for the crystals to be almost ideal Heisenberg magnets. In accordance with Ref. 9 and with our analysis, it may be inferred that EuO can be a good candidate for the appearance of solitonlike bound states of magnons. On the other hand, from Refs. 13 and 15, it follows that the conduction electron-magnon interaction in MS can be quite adequately described by the interaction of Vonsovsky type.

All the aforementioned arguments have led us to examine an appropriate microscopic model describing MS from the point of view of collisions involving conduction electrons and magnon solitons. By using the nonequilibrium-statistical-operator technique (NSOT),[16] we have derived equations describing the relaxation of conduction electrons involving both the free magnon gas and the ideal gas of magnetic solitons. Particular attention has been given to relations between the relaxation time and the strength of an external magnetic field.

II. THE HAMILTONIAN OF THE SYSTEM

For the description of MS we use the model first proposed by Vonsovsky.[12] In his model the electrons in localized magnetic d or f shells interact with one another via the Heisenberg nearest-neighbor exchange mechanism while an entirely distinct subsystem exists which is composed of quasifree electrons in Bloch states of the conduction band (s). Since in this model the localized d and f electrons can be analyzed using a virtually identical treatment, we shall use the symbol "l" (localized) for both.

Let us construct the s-l model Hamiltonian due to Vonsovsky:

$$H_v = H_s + H_l + H_{sl} \ . \tag{2.1}$$

The first term on the right-hand side represents the noninteracting conduction electrons described by the operators $a_{k\sigma}$, $a^\dagger_{k\sigma}$ which destroy and create an electron with the wave vector k and with an up (↑) and down (↓) spin projection ($\sigma = \pm\frac{1}{2}$), respectively:

$$H_s = \sum_{k,\sigma} E_{k\sigma} a^\dagger_{k\sigma} a_{k\sigma} \ . \tag{2.2}$$

The normalized energy of electronic Bloch states, including the Zeeman energy due to an external field and the shift caused by the s-l interaction, can be written in the form

$$E_{k\sigma} = \frac{\hbar^2 k^2}{2m^*} + \sigma g_s m_B h - S W_{kk} \delta_{\sigma\downarrow} \ , \tag{2.3}$$

where m^* is the effective mass of a conduction electron; g_s is the Landé factor; m_B is the Bohr magneton; h is an external magnetic field directed along the z axis; S is the spin of a single magnetic ion; W_{kk} is the interaction energy of Vonsovsky type, which is on the order of $W_{kk} \sim 10^{-3}$ eV.

The second term on the right-hand side of Eq. (2.1) represents the interionic magnetic interaction which is of direct or indirect exchange type,[13] i.e., it is a function solely dependent on the angle the atomic spins make with each other:

$$H_l = -g_s m_B h \sum_n S^z_n - \frac{J_0}{4} \sum_n (S^+_n S^-_{n+1} + S^-_n S^+_{n+1}) - \frac{J^z_0}{2} \sum_n S^z_n S^z_{n+1} \ , \tag{2.4}$$

where the spin ladder operators are $S^\pm_n = S^x_n \pm i S^y_n$ and the spin projection operator in the direction of the external field is S^z_n. These spin operators satisfy the usual commutation relations

$$[S^+_i, S^-_j] = 2\delta_{ij} S^z_i \quad [S^z_i, S^+_j] = \pm\delta_{ij} S^\pm_i \ . \tag{2.5}$$

The energy parameters introduced above are the exchange interaction constant $J_0 = \sum_n J(n-n')$ and the anisotropic exchange interaction constant $J^z_0 = \sum_n J^z_0(n-n')$. The last term on the right-hand side of Eq. (2.1) describes the exchange coupling between the conduction electrons and the spins of magnetic ions. In the single-band approximation this can be expressed in the direct space representation, as follows:

$$H_{s\text{-}l} = -\sum_{n,\sigma,\sigma'} W_{n\sigma\sigma'} (\mathbf{S}_n \cdot \mathbf{s})_{\sigma\sigma'} a^\dagger_{n\sigma'} a_{n\sigma} \ , \tag{2.6}$$

where the exchange energy $W_{n\sigma\sigma'}$ is of short-range type, $\mathbf{s}_{\sigma\sigma'}$ are Pauli matrices, and $(a^\dagger_{n\sigma'}, a_{n\sigma})$ are Fourier transforms of the operators introduced in Eq. (2.3). Note that a constant part of this exchange energy is always included in Eq. (2.3) as its third term.

Subsequently, one may take into account only the s electron's interaction with the atom it occupies, since in this case the exchange integral $W(0)$ is dominant being of zeroth order in the overlap of the Bloch function with the wave function of the l shell. The s-l interaction influences the optical, magnetic, and transport properties of MS and rare-earth metals.

III. THE BOUND STATE OF $\mathcal{N}$ MAGNONS AND A SOLITONLIKE SOLUTION

The formation of bound states of two, three, four, and five magnons in a QOD ferromagnet with EAA was experimentally observed for the first time some 20 years ago.[17,18] A theoretical explanation of this effect is given in Ref. 19. The basic discussion about the necessary conditions for the formation of a magnon bound state in one- and three-dimensional ferromagnets with EAA is given in Ref. 9. The starting point in this discussion is the assumption about smallness of anisotropy, i.e., the relative magnitude of the energy $(J^z_0 - J_0)$ in comparison with the exchange energy J_0. The main results obtained in Ref. 9 in terms of a classical approach will be reproduced in the present paper using a quantum-mechanical method formulated in the space of coherent states.

We start from Hamiltonian (2.4). The linear chain, which models the magnetic system under consideration, is taken to be along the z axis. In order to obtain clusterized solitonlike bound states, we consider a chain with a large number of chain sites ($N \gg 1$) separated by an equilibrium distance R_0. Holstein-Primakoff (HP) representation allows us to go over from the spin operators to the magnon annihilation and creation operators $(B_n, B^\dagger_n)$, as follows:

$$S^z_n = S - B^\dagger_n B_n \ ,$$
$$S^+_n = (2S)^{1/2} \left[1 - \frac{1}{4S} B^\dagger_n B_n \right] B_n \ , \tag{3.1}$$
$$S^-_n = (2S)^{1/2} B^\dagger_n \left[1 - \frac{1}{4S} B^\dagger_n B_n \right] .$$

After performing the transformations (3.1) and discarding small terms (in orders higher than the fourth), the model Hamiltonian (2.4) becomes

$$H_l = E_0 + \Delta \sum_n B_n^\dagger B_n - J_0 S \sum_n B_n^\dagger (B_{n+1} + B_{n-1}) - J_0^z \sum_n B_n^\dagger B_{n-1}^\dagger B_{n-1} B_n + \frac{J_0}{4} \sum_n \{[(B_n^\dagger)^2 + (B_{n-1}^\dagger)^2] B_n B_{n-1} + B_{n-1}^\dagger B_n^\dagger (B_{n-1}^2 + B_n^2)\} , \tag{3.2}$$

where the new energy parameters are

$$E_0 = -NS^2 J_0^z - g_s m_B h N S , \quad \Delta = g_s \mu_B h + 2SJ_0^z . \tag{3.3}$$

For the sake of simplicity we can now use the normal mode expansion of magnon excitations,

$$B_n = N^{-1/2} \sum_q B_q \exp(iqR_0 n) . \tag{3.4}$$

When the number of spin deviations is not large, the Hamiltonian of the magnon gas in the k representation is given by the approximate expression

$$H_l = \sum_q \varepsilon_q B_q^\dagger B_q - N^{-1}(J_0^z - J_0) \sum_{q,q_1,q_2} B_{q+q_2}^\dagger B_{q_1-q_2}^\dagger B_q B_{q_1} , \tag{3.5}$$

where the insignificant constant term Δ has been omitted. The energy of a one-magnon state ε_q is defined in terms of the corresponding dispersion relation

$$\varepsilon_q = g_s m_B h + 2SJ_0^z - 2J_0 S \cos(qR_0) . \tag{3.6}$$

The second term on the right-hand side of Eq. (3.5) represents the magnon-magnon interaction, which is of exchange character. But in the long-wave limit $(qR_0 \to 0)$, the magnon-magnon interaction remains nonzero only as a result of a nonzero spin-wave collision amplitude. Under these circumstances small attractive forces between the magnons begin to appear.

In order to define the properties of the solitonlike clusterized bound state of $\mathcal{N}$ magnons, we consider the product wave function of Glauber's coherent states,

$$|\psi_{\rm sol}(t)\rangle = \pi_q |\beta_q\rangle . \tag{3.7}$$

The product state $|\psi_{\rm sol}(t)\rangle$ may be defined by the property that the equality

$$B_q |\beta_q\rangle = \beta_q(t) |\beta_q\rangle \tag{3.8}$$

is valid for all of B_q.

The expectation value of the Hamiltonian (3.5) in the state (3.7) is therefore a scalar function $\langle H \rangle(\beta_q(t), \beta_q^*(t))$ of all amplitudes $\beta_q(t)$ and their complex conjugates

$$\langle H \rangle = \langle \psi_{\rm sol}(t) | H_l | \psi_{\rm sol}(t) \rangle = \sum_q \varepsilon_q |\beta_q(t)|^2 - N^{-1}(J_0^z - J_0) \times \sum_{q,q_1,q_2} \beta_{q+q_2}^*(t) \beta_{q_1+q_2}^*(t) \beta_{q_1}(t) \beta_q(t) . \tag{3.9}$$

The amplitude $\beta_q(t)$ is usually treated as a generalized coordinate with the corresponding generalized momentum $i\hbar\beta_q^*(t)$. The equation of motion for our generalized dynamical variable is taken to be a classical Hamiltonian equation in which the expectation value of the quantum Hamiltonian (3.9) appears as the Hamilton function

$$i\hbar\dot\beta_q(t) = \partial\langle H \rangle / \partial\beta_q^*(t) .$$

Henceforth, overdots will be used throughout this paper to denote differentiation with respect to time. Hence, we can set up the relationships between coherent amplitudes:

$$i\hbar\dot\beta_q(t) = \varepsilon_q \beta_q(t) - 2N^{-1}(J_0^z - J_0) \sum_{q_1,q_2} \beta_{q_1-q_2}^*(t) \beta_{q-q_2}(t) \beta_{q_1}(t) . \tag{3.10}$$

Now performing the inverse Fourier transform,

$$\beta(z,t) = N^{-1/2} \sum_q \beta_q(t) \exp(iqz), \quad z = nR_0 \tag{3.11}$$

and going over to the continuum approximation $\cos(qR_0) \simeq 1 - \frac{1}{2}q^2R_0^2$, which leads to

$$\varepsilon_q \simeq g_s m_\beta h + 2S(J_0^z - J_0) - J_0 S q^2 R_0^2 ,$$
$$N^{-1/2} \sum_q \beta_q(t) q^2 \exp(iqz) = -\frac{\partial^2}{\partial z^2}\beta(z,t) , \tag{3.12}$$
$$N^{-3/2} \sum_{q,q_1,q_2} \beta_{q_1-q_2}^*(t) \beta_{q-q_2}(t) \beta_{q_1}(t) \exp(iqz) = |\beta(z,t)|^2 \beta(z,t) ,$$

we finally obtain the well-known Schrödinger equation with cubic nonlinearity (NLSE)

$$i\hbar\dot\beta(z,t) = [g_s m_B h + 2S(J_0^z - J_0)]\beta(z,t) - J_0 S R_0^2 \frac{\partial^2}{\partial z^2}\beta(z,t) - 2(J_0^z - J_0)|\beta(z,t)|^2 \beta(z,t) . \tag{3.13}$$

According to Ref. 9, if the condition of small anisotropy energy is fulfilled and if the chain supports a population of a large number of magnons, the possibility of clusterization of magnons into a stable bound state appears to be significant. Assuming that a fixed number of magnons $\mathcal{N}$ is involved in clusterization, we solved the NLSE including the normalization condition $\int_{-\infty}^{\infty} |\beta(z,t)|^2 dz = \mathcal{N}$. Moreover, the solution of the NLSE is required to be in the traveling-wave form. The corresponding envelope of the clusterized "magnon drop" moves along the chain with velocity v in the form of a bell-shaped soliton:

$$\beta(z,t)=\left[\mathcal{N}\frac{\mu}{2}\right]^{1/2}\frac{\exp\{i[k_s(z-z_0)-\omega_s t\}}{\cosh[(\mu/R_0)(z-z_0-vt)]} . \tag{3.14}$$

The corresponding parameters are explicitly given as follows: z_0 is the coordinate of the center of the cluster, the solitonic quasi-wave number k_s is given by

$$k_s=\frac{\hbar v}{2J_0SR_0^2}\mathcal{N} , \tag{3.15}$$

the inverse solitonic width $\mu=L^{-1}$ is defined as

$$\mu=\frac{J_0^z-J_0}{2SJ_0}\mathcal{N} . \tag{3.16}$$

The application of continuum approximation requires that the solitonic width be significantly greater than unity ($L>1$) which imposes the inequality

$$1<\mathcal{N}\ll 2SJ_0(J_0^z-J_0)^{-1} . \tag{3.17}$$

This is in agreement with the condition for smallness of anisotropy. It can be found that the energy of a solitonlike cluster has the approximate value of

$$E_{\rm sol}(v)=\hbar\omega_s=E_0+\tfrac{1}{2}M^*v^2 , \tag{3.18}$$

where the static energy of the cluster is expressed by

$$E_0=\varepsilon_0\mathcal{N}\left[1-\frac{(J_0^z-J_0)^2\mathcal{N}^2}{12J_0S\varepsilon_0}\right] \text{ and } \varepsilon_0=\varepsilon_q\ (q=0) . \tag{3.19}$$

This provides a clear physical insight. First, the only quantity that depends on the magnitude of the external magnetic field h is the carrier wave frequency ω_s. This, of course, affects the soliton energy as given in Eqs. (3.18) and (3.19). This is very much in agreement with the results obtained by Ivanov and Kosevich.[9] Second, the magnitude of the negative binding potential energy of magnons increases with an increasing number of magnons. This, of course, is limited by the conditions (3.17). The effective mass of the cluster is simply the sum of the effective masses of the participating magnons,

$$M^*=\frac{\hbar^2}{2J_0SR_0^2}\mathcal{N} . \tag{3.20}$$

The Fourier component of the solitonlike solution (3.14) is calculated as

$$\beta_q(t)=\frac{1}{R_0}\int_{-\infty}^{\infty}dz\,\beta(z,t)\exp(-iqz) . \tag{3.21}$$

After performing the corresponding integration, we readily obtain

$$\beta_q(t)=\left[\frac{\mathcal{N}}{2\mu}\right]^{1/2}\frac{\pi\exp\{i[(k_sv-kv-\omega_s)t-qz_0]\}}{\left[\cosh\left[\frac{\pi}{2}\frac{R_0}{\mu}(k_s-q)\right]\right]} . \tag{3.22}$$

According to Ref. 20, the density of magnons with the wave vector q involved in a solitonlike cluster can be determined as follows:

$$\frac{d\mathcal{N}}{dq}=\bar{N}_q^s=|\beta_q(t)|^2 \tag{3.23}$$

or explicitly

$$\bar{N}_q^s=\mathcal{N}\frac{\pi^2}{2\mu}\text{sech}^2\left[\frac{\pi R_0}{2\mu}(k_s-q)\right] . \tag{3.24}$$

This expression will be of basic importance to the application of NSOT in Sec. V.

It is interesting to see the consequences that follow from the application of the HP representation (3.1) and the inclusion of the constraint

$$1<\mathcal{N}\ll 2SJ_0(J_0^z-J_0)^{-1} . \tag{3.25}$$

It is well known that the HP transformation to boson operators satisfies all the basic spin commutation relations, but it becomes incorrect whenever $\langle B_n^\dagger B_n\rangle$ exceeds $2S$. This simply follows from the appearance of the square root in the exact transformation relations. However, in the case of the EuO crystal the value of spin is $S=\frac{7}{2}$ so that for $\mathcal{N}\le 7$ the application of the HP expansion is justified. Moreover, it has been shown[15] that, by using the Dyson-Maleev[21] transformation, Hamiltonian (2.4) gives the equivalent form (3.5) including attractive magnon-magnon interactions. We wish to stress here that the quantum treatment of a very closely related easy-axis Hamiltonian has been earlier given by Bonfim and De Moura.[22] Furthermore, a very comprehensive survey dedicated to the applicability of the Heisenberg model to the theory of solitons has been written by Schneider and Stoll.[23]

Finally, a few words about the possibility of an experimental generation of high magnon populations. In 1960 Schlömann *et al.*[24] predicated the phenomenon of "parallel pumping," i.e., the parametric excitation of magnons by an alternating magnetic field with its polarization parallel to the direction of magnetization. At the present time, parallel excitation is one of the main methods of generating magnons in ferromagnets. A recent survey of investigations dedicated to the emergence of solitons in parametrically excited magnons systems has been given by L'vov.[25] The other possibility of magnon creation is the coherent amplification of magnons by a beam of charged particles. When a charged particle moves through a ferromagnet with a sufficiently high velocity (conduction electrons), it excites both intrinsic electromagnetic waves and spin waves (magnons).[26]

IV. THE RELAXATION RATE OF CONDUCTION ELECTRONS DUE TO THE SCATTERING PROCESSES BETWEEN CONDUCTION ELECTRONS AND SOLITONS

As is well known, MS display much stronger anomalies in the temperature dependence of the conductivity than ferromagnetic metals. In particular, for most of them the specific resistivity exhibits a sharp peak at a temperature of the order of the Curie point T_c. In the presence of a sufficiently large external magnetic field, the peak disappears, indicating an exceptionally strong negative magnetoresistance. All these anomalies are due to the strong coupling between mobile electrons (i.e., conduction and donor electrons) and localized magnetic moments of atoms, which is caused by the exchange interaction (2.6) between them. The electron levels in the band shift upwards with a diminishing degree of the ferromagnetic order, and vice versa. On the other hand, the degree of the local magnetic ordering depends on the local electron density since mobile electrons support the ferromagnetic ordering. An analysis carried out by Nagaev[27] shows that at low temperatures the charge carriers are in states of the spin-polaron type, i.e., with an electron spin everywhere directed along the local magnetic moment.

In oxygen-deficient samples of EuO there occurs a metal-semiconductor transition for which the high-temperature phase is nonmetallic.[28,29] This transition takes place far below the Curie temperature $T_c = 69.3$ K and is called the Mott transition. Our treatment here is confined to the metallic side of the transition. In the next stage we intend to consider the nonmetallic region where the concentration of carriers depends in a very complicated manner on temperature and the magnetic field.[29]

The relaxation processes involving conduction electrons in MS will be considered here in the framework of the NSOT. In the case where the thermodynamic system is under the influence of external fields, Kubo's method takes precedence, while in the case where the separation of mechanical and thermal perturbation is complicated, the NSOT is more convenient.

Let us begin by separating our system described by the Hamiltonian of Vonsovsky type, Eq. (2.1), into two interacting subsystems[16]

$$H_v = H_1 + H_2 , \tag{4.1}$$

where

$$H_1 = H_s, \quad H_2 = H_l + H_{s\text{-}l} . \tag{4.1a}$$

The basic assumption is that in a nonequilibrium state, where the magnon population is generated by an external field, the subsystems are at slightly different temperatures:[26,30]

$$\beta_1 = (k_B T_1)^{-1} \neq \beta_2 = (k_B T_2)^{-1} . \tag{4.1b}$$

From Akhiezer *et al.*,[26] in magnon systems for which the condition

$$\Theta_c \left[\mu_B \frac{M_0}{\Theta_c} \right]^{4/7} \ll T \ll \Theta_c, \quad \Theta_c = k_B T_c \tag{4.1c}$$

is fulfilled, where M_0 is the maximum magnetization per ion, the exchange magnon-magnon scattering is stronger than the magnon-phonon and the magnon-conduction electron interactions. Thus, the spin temperature may differ from both the temperature of the crystal lattice and that of the gas of conduction electrons. The corresponding magnetic moment also differs from the equilibrium magnetic moment of the crystal. Equalization of the spin and conduction electron temperatures may take place due to interactions capable of modifying the magnetic moment of the system, i.e., the magnetic dipole and the spin-orbit interactions. For the temperature region (4.1c) and for the EuO crystal where the orbital quantum number is $L = 0$, the dipole-dipole interaction is negligible compared to the exchange interaction. Therefore, the relaxation of the magnetic moment and equalization of the magnon and conduction electron temperatures occurs slowly in comparison with the process of establishing the quasi-equilibrium Bose distribution for magnons with a given value of the magnetic moment. Then, the energy current between the two subsystems is defined by

$$\dot{H}_1(t) = -\dot{H}_2(t) = (i\hbar)^{-1}[H_1, H_v] . \tag{4.2}$$

The *s-l* exchange interaction Hamiltonian (2.6) can be transformed into q space so that it explicitly becomes

$$H_{s\text{-}l} = -\left[\frac{2S}{N}\right]^{1/2} \sum_{k_1,k_2,q} W_{k_1k_2}\delta(k_2 - k_1 + q)(a^\dagger_{k_1\uparrow}a_{k_2\downarrow}B_q + a^\dagger_{k_2\downarrow}a_{k_1\uparrow}B^\dagger_q)$$
$$+(2N)^{-1} \sum_{\substack{k_1,k_2,\\ q_1,q_2}} W_{k_1k_2}\delta(k_2 - k_1 + q_2 - q_1)(a^\dagger_{k_1\downarrow}a_{k_2\uparrow} + a^\dagger_{k_1\uparrow}a_{k_2\downarrow})B^\dagger_{q_2}B_{q_1} . \tag{4.3}$$

On the basis of Eq. (4.3) the energy current (4.2) has the explicit form

$$\dot{H}_1(t) = -\frac{1}{i\hbar}\left[\frac{2S}{N}\right]^{1/2} \sum_{k,k',q} W_{kk'}(E_{k\uparrow} - E_{k'\downarrow})\delta(q - k + k')(B_q a^\dagger_{k\uparrow}a_{k'\downarrow} - a^\dagger_{k'\downarrow}a_{k\uparrow}B^\dagger_q)$$
$$+\frac{1}{i\hbar}\frac{1}{N} \sum_{\substack{k,k',\sigma\\ q,q'}} W_{kk'}(-1)^{\sigma+1/2}(E_{k\sigma} - E_{k'\sigma})\delta(q - q' - k + k')B^\dagger_q B_{q'} a^\dagger_{k\sigma}a_{k'\sigma} . \tag{4.4}$$

We now formulate the density matrix of NSOT as follows:

$$\rho = Z^{-1}\exp(-\beta_1 H_s - \beta_2 H_l + \delta M) , \quad (4.5)$$

where Z is the statistical sum, while the small perturbing term which is responsible for the interaction between subsystems is

$$\delta M = (\beta_2 - \beta_1)\int_{-\infty}^{0} e^{\tilde{\varepsilon} t}\dot{H}_1(t)dt, \quad \tilde{\varepsilon} \ll 1 . \quad (4.6)$$

Here $\tilde{\varepsilon}$ is a small parameter including the time adiabatically. The standard procedure[31] gives

$$\rho = \left[1 + \int_0^1 d\tau[\exp(-M\tau)\delta M \exp(M\tau)]\right]\rho_e , \quad (4.7)$$

where M is proportional to the diagonal part of Hamiltonian (2.1) including thermodynamical forces,

$$M = \sum_q \gamma_q B_q^\dagger B_q + \sum_{k\sigma} \gamma_{k\sigma} a_{k\sigma}^\dagger a_{k\sigma} ,$$
$$\gamma_{k\sigma} = \beta_1(E_{k\sigma} - \mu_1) , \quad (4.7a)$$
$$\gamma_q = \beta_2(\varepsilon_q - \mu_2) .$$

τ is a dimensionless parameter and μ_1 is the Fermi level of the conduction band, while μ_2 is the chemical potential of the magnon subsystem. Finally, the equilibrium statistical operator (ESO) has the usual form

$$\rho_e = e^{-M}/(\mathrm{Tr} e^{-M}) . \quad (4.7b)$$

The next step is to find the mean value of the energy current (4.4) taking the average with respect to NSO (4.7):

$$\langle \dot{H}_1 \rangle = \mathrm{Tr}(\rho \dot{H}_1) = (\beta_1 - \beta_1)L_{12} , \quad (4.8)$$

where L_{12} represents the kinetic coefficient of the process,

$$L_{12} = \int_{-\infty}^{0} dt\, e^{\tilde{\varepsilon} t}\int_0^1 d\tau \langle \dot{H}_1(0)e^{-M\tau}\dot{H}_1(t)e^{M\tau}\rangle_e . \quad (4.9)$$

Here we took into account that the mean value of the energy current in equilibrium is zero because $\mathrm{Tr}[\rho_e \dot{H}_1(t)] = 0$. The process of averaging with respect to ESO is denoted by $\langle\ \rangle_e$. On the basis of Eqs. (4.4), (4.7a), and (4.8), the expression (4.9) may be transformed after integration with respect to τ into the form

$$L_{12} = \int_{-\infty}^{0} dt\, e^{\tilde{\varepsilon} t}\sum_{k,k',q}\Phi_{kk'q}[(1 - e^{\Gamma_{kk'q}})\langle F_{kk'q}\dot{H}_1(t)\rangle_e - (1 - e^{-\Gamma_{kk'q}})\langle F_{kk'q}^\dagger \dot{H}_1(t)\rangle_e] , \quad (4.10)$$

where we have introduced the following set of symbols:

$$\Phi_{kk'q} = \frac{1}{i\hbar}\left[\frac{2S}{N}\right]^{1/2}\frac{W_{kk'}(E_{k\uparrow} - E_{k'\downarrow}\delta(q + k' - k)}{\Gamma_{kk'q}} ,$$
$$\Gamma_{kk'q} = \beta_1(E_{k\uparrow} - E_{k'\downarrow} - \mu_1) - \beta_2(\varepsilon_q - \mu_2) , \quad (4.11)$$
$$F_{kk'q} = a_{k\uparrow}^\dagger a_{k'\downarrow}B_q .$$

In the next step we take into account the standard relations

$$e^{\Gamma_{kk'q}}\langle F_{kk'q}\dot{H}_1(t)\rangle_e = \langle \dot{H}_1(t)F_{kk'q}\rangle_e ,$$
$$e^{-\Gamma_{kk'q}}\langle F_{kk'q}^\dagger \dot{H}_1(t)\rangle_e = \langle \dot{H}_1(t)F_{kk'q}^\dagger\rangle_e , \quad (4.12)$$

so that we can use a boson retarded Green's function as follows:

$$L_{12} = \int_{-\infty}^{0} dt\, e^{\tilde{\varepsilon} t} \times \sum_{k,k',q}\Phi_{kk'q}\frac{1}{i\hbar} \times [\langle\langle F_{kk'q}(0)|\dot{H}_1(t)\rangle\rangle + \langle\langle F_{kk'q}^\dagger(0)|\dot{H}_1(t)\rangle\rangle] . \quad (4.13)$$

Now, substituting Eq. (4.4) into (4.13), we turn to decoupling the corresponding correlators and after integration with respect to time, the kinetic coefficient is found as

$$L_{12} = \frac{2S}{\hbar N}\sum_{k,k'}\left[2\pi\delta(\varepsilon_q - E_{k\uparrow} + E_{k'\downarrow}) \times \frac{W_{kk'}^2(E_{k\uparrow} - E_{k'\downarrow})^2}{\Gamma_{kk'q}} \times [(1 + \bar{N}_q)\bar{n}_{k\uparrow}(1 - \bar{n}_{k'\downarrow}) - \bar{n}_{k'\downarrow}\bar{N}_q(1 - \bar{n}_{k\uparrow})]\right] , \quad (4.14)$$

where the statistical mean values are

$$\bar{n}_{k\sigma} = (e^{\beta_1 E_{k\sigma}} + 1)^{-1} , \quad (4.15a)$$

$$\bar{N}_q = (e^{\beta_2\varepsilon_q} - 1)^{-1} , \quad (4.15b)$$

for Fermi conducting electrons and for Bose magnons, respectively.

V. THE SOLITONIC CASE

If the magnon population taken together with the condition of small EAA enables the appearance of clusterized solitonlike bound states, the density of free magnons $\bar{N}_q$ given by Eq. (4.15b) can be replaced by the density of bound magnons involved in the magnon cluster $\bar{N}_q^s$ given by Eq. (3.24). Bearing in mind the prevailing conditions and taking into account the weak dependence of $W_{kk'}$ on quasimomentum ($W_{kk'} \approx W \sim 10^{-21}$ J) using Eq. (4.14), we obtain the formula for the kinetic coefficient

$$L_{12}^{\text{sol}}=\frac{2\pi^3SW^2\mathcal{N}}{\mu\hbar N}\times\sum_{k,k'}\left\{\frac{\varepsilon_{k-k'}^2}{\Gamma_{k,k',k-k'}}\delta(\varepsilon_{k-k'}-E_{k\uparrow}+E_{k'\downarrow})\times\frac{(\bar{n}_{k\uparrow}-\bar{n}_{k'\downarrow})}{\cosh^2[(\pi R_0/2\mu)(k_s-k+k')]}\right\} . \tag{5.1}$$

In order to estimate the contribution of the solitonic mechanism to the kinetic coefficient, it is most convenient to replace the function in the denominator by its maximum value given by the relation $k-k'=k_s$. Under these circumstances the summation can be replaced by integration as follows:

$$\frac{1}{N}\sum_k\to\frac{R_0}{2\pi}\int_{-\pi/R_0}^{\pi/R_0}dk . \tag{5.2}$$

Due to the fact that MS show metallic properties in the temperature range of 0 to a few tens of kelvins, it is reasonable to deal with the low-temperature regime where the inequalities $E_{k\uparrow}\beta_1\gg1$ and $E_{k-k'\downarrow}\beta_1\gg1$ are fulfilled. In such a regime, and performing the required integration, Eq. (5.1) becomes

$$L_{12}^{\text{sol}}=\frac{\pi^2SW^2R_0\varepsilon_{k_s}\mathcal{N}}{\mu\hbar^2\Gamma_{k_s}}\left[\frac{m^*}{2\pi\beta_1}\right]^{1/2}\text{erf}\left[\frac{\pi^2\hbar^2\beta_1}{8R_0^2m^*}\right]\times(e^{-(1/2)g_sm_Bh\beta_1}-e^{\beta_1(2SW+(1/2)g_sm_Bh)})\times e^{-\hbar^2k_s^2\beta_1/2m^*} , \tag{5.3}$$

where

$$\Gamma_{k_s}=\beta_1(\varepsilon_{k_s}-\mu_1)-\beta_2(\varepsilon_{k_s}-\mu_2) . \tag{5.3a}$$

Now, we can numerically estimate the value of the parameter $\Gamma_{k_s}(\beta_1,\beta_2,h)$. Bearing in mind that $J_0\sim10^{-19}$ J and $(J_0^z-J_0)J_0^{-1}\ll1$ and that the term

$$\frac{\hbar^2k_s^2}{2m^*}=\frac{\hbar^4v^2\mathcal{N}^2}{4J_0^2S^2R_0^4m^*}$$

has the value of the order 10^{-23} J, we see that for all values of the field $h<100T$ the approximation

$$\Gamma_{k_s}(\beta_1,\beta_2,h)\approx\beta_2\mu_2-\beta_1\mu_1 \tag{5.3b}$$

is valid on the basis of typical values of chemical potentials $\mu_1\sim\mu_2\sim10^{-19}$ J. If we assume that the temperatures of the two subsystems differ slightly so that one can write $T_2=T_1+\Delta T$, where $\Delta T\ll T_1$, we may transform Eq. (5.3b) into the form

$$\beta_2\mu_2-\beta_1\mu_1\approx-\frac{\mu_1\Delta T}{k_BT_1^2} . \tag{5.3c}$$

The latter expression for the kinetic coefficient (5.3) shows that the scattering process depends rather significantly on the solitonic velocity through the quasiwave number k_s [see Eq. (3.15)]. So far we have examined solitonlike clusters as magnon bound states. We should keep in mind that solitons show up in neutron scattering experiments as a gas of noninteracting quasiparticles moving along the ferromagnetic chain with velocities v. Therefore, here we can apply the framework of classical statistical mechanics concluding that at sufficiently low temperature the concept of an "ideal" gas of solitons (IGS) is still valid.[32] Subsequently, we can apply the procedure of averaging the kinetic coefficient over all allowed velocities of IGS by using the Maxwell distribution as follows:

$$\bar{L}_{12}^{\text{sol}}=\left[\frac{2M^*\beta_2}{\pi}\right]^{1/2}\int_0^\infty dv\,L_{12}^{\text{sol}}\exp\left[-\frac{M^*v^2}{2}\beta_2\right] . \tag{5.4}$$

On the basis of Eqs. (5.3), (5.4), and (5.3c), we obtain the kinetic coefficient's dependence on an external magnetic field and temperature in the form

$$\bar{L}_{12}^{\text{sol}}=-A(\beta_1,\Delta T)[B(\beta_2)+Ch]\times\{\exp(-\tfrac{1}{2}g_sm_Bh\beta_1)-\exp[\beta_1(2SW+\tfrac{1}{2}g_sm_Bh)]\} , \tag{5.5}$$

where the following symbols have been introduced:

$$-A(\beta_1,\Delta T)=-\frac{\pi^2SW^2R_0\mathcal{N}k_BT_1^2}{\hbar^2\mu\mu_1\Delta T}\left[\frac{m^*M^*\beta_2}{\pi^2\beta_1}\right]^{1/2}\times\text{erf}\left[\frac{\pi^2\hbar^2\beta_1}{8m^*R_0^2}\right] , \tag{5.6a}$$

$$B(\beta_2)=\frac{\hbar^4\mathcal{N}^2}{8J_0^2S^2R_0^4m^*M^*\beta_2}-SW , \tag{5.6b}$$

$$C=\tfrac{1}{2}g_sm_B . \tag{5.6c}$$

Let us consider the two limiting cases: first, without the external magnetic field and, second, in the presence of a strong field, respectively. This yields

$$(\bar{L}_{12}^{\text{sol}})_{h=0}=A(\beta_1,\beta_2)B(\beta_2)\exp(2SW\beta_1) , \tag{5.7a}$$

$$(\bar{L}_{12}^{\text{sol}})_{h>1T}=A(\beta_1,\beta_2)[B(\beta_2)+Ch]\times\exp[\beta_1(2SW+\tfrac{1}{2}g_sm_Bh)] . \tag{5.7b}$$

The main relaxation time for collisions of conducting electrons with IGS is defined by the simple equality

$$\tau=\frac{\langle H_1^2\rangle}{\bar{L}_{12}^{\text{sol}}} . \tag{5.8}$$

Obviously, it is now necessary to find a statistical mean value of the square of the energy of the electron's subsystem. Keeping only linear terms with respect to the conduction electron population ($\bar{n}_{k\sigma}$) gives

$$\langle H_1^2\rangle\cong\sum_k\left[\frac{\hbar^2k^2}{2m^*}+\tfrac{1}{2}g_sm_Bh\right]^2\bar{n}_{k\uparrow}+\sum_k\left[\frac{\hbar^2k^2}{2m^*}-2SW-\tfrac{1}{2}g_sm_Bh\right]^2\bar{n}_{k\downarrow} ; \tag{5.9}$$

then using the expression (4.15a) for the electron's population and replacing the summation by integration, one gets

$$\langle H_1^2\rangle=[f_1(\beta_1)+\tfrac{1}{2}f_2(\beta_1)g_s m_B h+\tfrac{1}{4}f_3(\beta_1)g_s^2 m_B^2 h^2]e^{-(1/2)g_s m_B h\beta_1}$$
$$+[f_1(\beta_1)+\tfrac{1}{2}f_2(\beta_1)(2SW+\tfrac{1}{2}g_s m_B h)+f_3(\beta_1)(2SW+\tfrac{1}{2}g_s m_B h)^2]e^{\beta_1[2SW+(1/2)g_s m_B h]}\ , \tag{5.10}$$

where the new set of symbols is used as follows:

$$f_1(\beta_1)=\frac{3}{8\sqrt{\pi}}\frac{R_0}{\beta_1^2}\left[\frac{2m^*}{\beta_1\hbar^2}\right]^{1/2}, \tag{5.11a}$$

$$f_2(\beta_1)=\frac{1}{4\sqrt{\pi}}\frac{R_0}{\beta_1}\left[\frac{2m^*}{\beta_1\hbar^2}\right]^{1/2}, \tag{5.11b}$$

$$f_3(\beta_1)=\frac{1}{8\sqrt{\pi}}R_0\left[\frac{2m^*}{\beta_1\hbar^2}\right]^{1/2}. \tag{5.11c}$$

We again focus our attention on the two limiting cases: $h=0$ and $h>1T$, so that we have, respectively,

$$\langle H_1^2\rangle_{h=0}=f_1(\beta_1)+[f_2(\beta_1)SW+f_3(\beta_1)4S^2W^2]e^{\beta_1 2SW} \tag{5.12a}$$

and

$$\langle H_1^2\rangle_{h>1T}=[f_1(\beta_1)+\tfrac{1}{2}f_2(\beta_1)(2SW+\tfrac{1}{2}g_s m_B h)+f_3(\beta_1)(2SW+\tfrac{1}{2}g_s m_B h)^2]e^{\beta_1(2SW+(1/2)g_s m_B h)}\ . \tag{5.12b}$$

Finally, the relaxation times of the solitonic mechanism for these two limiting cases are

$$(\bar{\tau})_{h=0}^{\text{sol}}=\frac{f_1(\beta_1)+[f_2(\beta_1)SW+f_3(\beta_1)4S^2W^2]\exp(2SW\beta_1)}{A(\beta_1,\Delta T)B(\beta_2)\exp(2SW\beta_1)}\ . \tag{5.13a}$$

For the actual data,

$$2SW=7\times10^{-21}\ \text{J}$$

and for $1\ \text{K}<T<20\ \text{K}$ so that $\beta_1^{-1}\sim10^{-22}$ J, the exponential terms are dominant and the expression (5.13a) becomes

$$(\bar{\tau})_{h=0}^{\text{sol}}=\frac{f_2(\beta_1)SW+4S^2W^2f_3(\beta_1)}{A(\beta_1,\Delta T)B(\beta_2)}\ . \tag{5.13b}$$

On the other hand, for large values of the external magnetic field the relaxation time has the value

$$(\bar{\tau})_{h>1T}^{\text{sol}}\approx\frac{a(\beta_1)+b(\beta_1)h+c(\beta_1)h^2}{m(\beta_1,\beta_2,\Delta T)+n(\beta_1,\Delta T)h}\ , \tag{5.14}$$

where we have used the symbols

$$a(\beta_1)=f_1(\beta_1)+SWf_2(\beta_1)+4S^2W^2f_3(\beta_1)\ ,$$
$$b(\beta_1)=\tfrac{1}{4}g_s m_B f_2(\beta_1)+2SWg_s m_B f_3(\beta_1)\ , \tag{5.14a}$$
$$c(\beta_1)=\tfrac{1}{4}g_s^2 m_B^2 f_3(\beta_1)\ ,$$
$$m(\beta_1,\beta_2,\Delta T)=A(\beta_1,\Delta T)B(\beta_2)\ , \tag{5.14b}$$
$$n(\beta_1,\Delta T)=A(\beta_1\Delta T)C\ .$$

Thus, the relaxation time shows a shallow minimum for

$$h_0=\frac{1}{2}\left[\frac{na-mb}{mc}\right] \tag{5.14c}$$

and becomes a linear function of h for extremely high values of the magnetic field. If we wish to make a semiquantitative estimation of the relaxation time, we can use the following set of parameters: $R_0\sim10^{-10}$ m, $m^*\sim10^{-30}$ kg, $T_1\sim T_2\sim10$ K, $\Delta T\sim1$ K, $S=\frac{7}{2}$, $W\sim10^{-21}$ J, $M^*\sim10^{-30}$ kg, $J_0\sim10^{-19}$ J, $\mathcal{N}\sim5$, $\mu_1\sim\mu_2\sim10^{-19}$ J, and $\mu\sim10^{-3}$. On the basis of the definitions (5.6) and (5.11), the most important constants become

$$A\sim2\times10^{-12}\ \text{J},\quad B\sim2\times10^{-23}\ \text{J},\quad C=2\times10^{-23}\ \frac{\text{J}}{\text{T}}\ ,$$
$$f_1\sim10^{-46}\ \text{J}^2,\quad f_2\sim3\times10^{-24}\ \text{J},\quad f_3\sim1.2\times10^{-2}\ . \tag{5.14d}$$

From Eqs. (5.13b) and (5.14d) we approximately obtain

$$(\bar{\tau})_{h=0}^{\text{sol}}\sim1.5\times10^{-8}\ \text{s} \tag{5.15}$$

and for $h>0$, using Eqs. (5.14) and (5.14d), we have

$$(\bar{\tau})_{h>0}^{\text{sol}}\approx\left[\frac{6+3h^2+h}{4(1+h)}\right]\times10^{-8}\ \text{s}\ , \tag{5.16}$$

which yields that for $h\simeq0.7T$ the minimum relaxation time has the value

$$(\bar{\tau})_{\min}^{\text{sol}}\sim1.10^{-8}\ \text{s}\ . \tag{5.17}$$

We now recall that the experimental evidence for the existence of solitons in magnetic systems is often based on neutron scattering experiments. On the other hand, for TMMC, which is a typical QOD ferromagnet with easy-plane anisotropy, the solitonic relaxation rate in the presence of magnetic impurities has been investigated experimentally.[33–35] Increasing the magnetic field, the ratio $(\bar{\tau})^{\text{sol}}/h^2$ has been found to be first sharply reduced down to a certain minimum value, and then it grows to infinity. The aforementioned theoretical attempt[10] which provided a description of such a relaxation phenomenon has been compared to experimental results[33,34] in spite of the fact that[10] it treats the QOD ferromagnets with easy-axis anisotropy. We, therefore, expect that our treatment when applied to some more appropriate materials as MS will provide a more accurate representation of these real physical systems.

VI. THE MAGNON CONTRIBUTION

Neutron scattering results for a typical ferromagnetic chain $CsNiF_3$ in a magnetic field are currently interpreted in terms of the IGS model.[3] However, a recent experiment[35] as well as numerical calculations indicate that there are deviations within such a simple model, namely, more rigorous treatments take into account the presence of free magnons as well as collisions between magnons and solitons. Keeping this fact in mind, we now estimate the relaxation of conduction electrons due to the presence of the "ideal gas" of magnons (IGM) without taking into account magnon-soliton collisions.

Substituting the distribution functions (4.15b) into (4.14) and linearizing with respect to $\beta_1-\beta_2$, it is clear that we have

$$L_{12}^{\text{mag}}=\frac{2SW^2}{\hbar N}(\beta_1-\beta_2)\times\sum_{k,k'}\frac{\varepsilon_{k-k'}^3}{\Gamma_{k,k',k-k'}}2\pi\delta(\varepsilon_{k-k'}-E_{k\uparrow}+E_{k'\downarrow})\times(1+\bar{N}_{k-k'})\bar{n}_{k\uparrow}(1-\bar{n}_{k'\downarrow}) . \quad (6.1)$$

Now, it must be noted that for the restriction $\beta_1^{-1}\ll\mu_1$ to be satisfied, the Fermi function $\bar{n}_{k-k'\downarrow}$ must be very small for $E_{k'\downarrow}$ significantly in excess of μ_1 and the complementary function $(1-\bar{n}_{k'\downarrow})$ must be very small except in the neighborhood of $E_{k'\downarrow}=\mu_1$. Consequently, it can be replaced as follows:

$$\bar{n}_{k'\downarrow}(1-\bar{n}_{k'\downarrow})=\beta_1^{-1}\delta(E_{k'\downarrow}-\mu_1) , \quad (6.2)$$

so that we now have

$$L_{12}^{\text{mag}}=\frac{2SW^2}{\hbar}\left[1-\frac{\beta_2}{\beta_1}\right]\left[\frac{R_0}{2\pi}\right]^2\times\int\int dk\,dk'\frac{\varepsilon_{k-k'}^3}{\Gamma_{k,k',k-k'}}2\pi\times\delta(\varepsilon_{k-k'}-E_{k\uparrow}+E_{k'\downarrow})\times\frac{\bar{n}_{k\uparrow}}{\bar{n}_{k'\downarrow}}\delta(E_{k'\downarrow}-\mu_1)(1+\bar{N}_{k-k'}) . \quad (6.3)$$

If there is no field $(h=0)$ integration gives

$$(L_{12}^{\text{mag}})_{h=0}=\left[1-\frac{\beta_2}{\beta_1}\right]\frac{SW^2R_0^2m^*\xi^{5/2}}{2\pi^2\hbar^3Q(\beta_1,\beta_2)}(\mu_1+2SW)^{-1/2}\times(1+\bar{N}_\xi)\frac{\bar{n}_{\xi-\mu_1}}{\bar{n}_{\mu_1}} , \quad (6.4)$$

where

$$Q(\beta_1,\beta_2)=\beta_1(\xi-\mu_1)-\beta_2(\xi-\mu_2)\cong\beta_2\mu_2-\beta_1\mu_1 ,$$

because $\xi=2S(J_0^z-J_0)\ll\mu_j$.

In the next step the (low-temperature regime) we obtain

$$(L_{12}^{\text{mag}})_{h=0}=\left[1-\frac{\beta_2}{\beta_1}\right]\frac{SW^2R_0^2m^*\xi^{5/2}}{2\pi^2\hbar^3Q(\beta_1,\beta_2)}(\mu_1+2SW)^{-1/2}\times e^{\mu_1\beta_1}(1+e^{-\xi\beta_2}) . \quad (6.5)$$

If there is a large external field, we find

$$(L_{12}^{\text{mag}})_{h>1T}=\left[1-\frac{\beta_2}{\beta_1}\right]\frac{SW^2R_0^2m^*\varepsilon^3(h)}{2\pi^2\hbar^3Q(\beta_1,\beta_2)}\times\{[p(h)+\xi][p(h)+\mu_1+2SW]\}^{-1/2}\times e^{-p(h)\beta_1} , \quad (6.6)$$

where $p(h)=\frac{1}{2}g_s m_B h$ and

$$\varepsilon(h)=g_s m_B h+2S(J_0^z-J_0) ,$$

or in a more compact form

$$(L_{12}^{\text{mag}})_{h>1T}=\Omega(\beta_1,\beta_2,h)\exp[-p(h)\beta_1] , \quad (6.7)$$

where

$$\Omega(\beta_1,\beta_2,h)=\left[1-\frac{\beta_2}{\beta_1}\right]\frac{SW^2R_0^2m^*\varepsilon^3(h)}{2\pi^2\hbar^3Q(\beta_1,\beta_2)}\times\{p(h)[p(h)+\mu_1+2SW]\}^{-1/2} . \quad (6.8)$$

Finally, using Eqs. (5.12b) and (6.7), the IGM contribution to the relaxation time is found as

$$(\tau_{\text{mag}})_{h>1T}=\frac{f_1(\beta_1)+\frac{1}{2}f_2(\beta_1)r(h)+f_3(\beta_1)r^2(h)}{\Omega(\beta_1,\beta_2,h)}\times e^{\beta_1[r(h)+p(h)]} , \quad (6.9)$$

where $r(h)=2SW+\frac{1}{2}g_s m_B h$.

VII. CONCLUSIONS

In this paper the conditions for the formation of solitonlike magnon clusters in QOD ferromagnets with EAA have been investigated in detail. According to the long-wavelength approximation used here, the inverse width of solitonic localization satisfies the inequality $\mu\ll1$. In this case, from Eq. (3.17) and from the demand for a large number of bound magnons, it immediately follows that

$$1<\mathcal{N}\ll\frac{2SJ_0}{J_0^z-J_0} , \quad (7.1)$$

which is a necessary condition for the formation of a solitonlike clusterized bound state of $\mathcal{N}$ magnons. From our point of view the best candidates to exhibit such nonlinear phenomena are some of the rare-earth compounds. Moreover, those compounds possess the properties of MS so that they can be successfully described using

Vonsovsky's model. In this sense the main aim of our paper has been the statistical treatment of relaxation processes of conduction electrons involving both IGS and IGM in MS compounds. The present calculations are given in the framework of NSOT and the relaxation times are found as functions of external field.

The solitonic mechanism of relaxation shows great selectivity—namely, the conduction electron spin's inversion processes play a crucial role in the energy exchange. The maximum value of the energy exchange is achieved under the condition $k-k'=k_s$. In the case of typical ferromagnets one has

$$\omega=\frac{E_{k\uparrow}-E_{k\downarrow}}{\hbar}=\frac{\hbar}{m^*}k_F(k-k')\simeq\tfrac{1}{2}10^{13}\ \mathrm{s}^{-1}\ , \qquad (7.2)$$

where k_F is the wave number corresponding to the Fermi level μ_1. Using the formula (3.15) and inserting the set of physical data, $m^*\sim10^{-30}$ kg, $\mu_1\sim1$ eV, $R_0\sim10^{-10}$ m, $J_0\sim10^{-20}$ J, $S\sim\frac{7}{2}$ and $N\sim10$, the solitonic velocity which plays the dominant role in the relaxation process is estimated to be $v\sim100$ m/s.

The present theoretical approach is analogous to that of Ref. 10, but here the mechanism of collisions is due to the Vonsovsky interaction which, to some extent, gives a different physical picture. An analysis of the relation (5.14) readily shows that the solitonic relaxation time has one shallow minimum for $h>0$ while for high values of h it becomes a linear function. We may explain this change in the character of the h dependence as a result of the fact that the Zeeman energy of the external field $g_s m_B h$ becomes comparable to the energy of the conduction-electron—magnon interaction SW.

On the other hand, it is evident from Eq. (6.9) that the relaxation time corresponding to the process of conduction-electron–IGM collisions increases exponentially with the external field. For sufficiently high values of the field the relaxation time becomes much greater than the relaxation time describing the conduction-electron–IGS collision. Consequently, the resistivity ρ of the MS subjected to a strong magnetic field is, according to the definition

$$\rho=\frac{m^*}{e^2\bar{n}\bar{\tau}} \qquad (7.3)$$

predominantly affected by the conduction-electron–IGS collisions. Here, e is the charge of an electron and $\bar{n}$ is the concentration of free electrons in the conduction band.

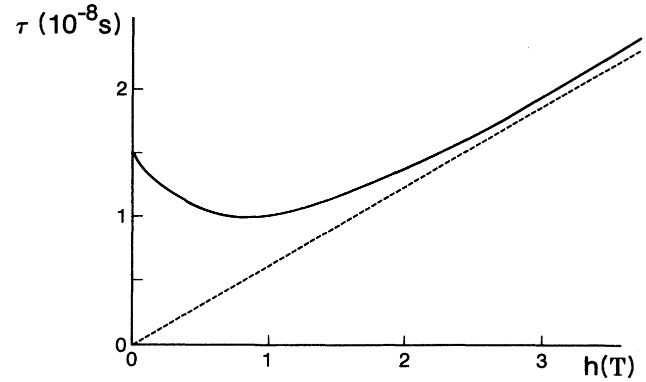

FIG. 1. The field dependence of the relaxation time corresponding to the conduction electron IGS at $T_1=10$ K and $\Delta T\sim1$ K.

A comparison of Fig. 1 with experimental data can be used as a test for the possible existence of solitons in MS. Our estimates are given for temperatures in the region of about 10 K which is in accordance with the experimental region of the unusual behavior of conductivity for MS.[28]

We believe that the method presented can be of especially great importance since substantial difficulties of neutronographic studies for MS stem from their high neutron absorption.[13] We expect that very valuable information can be extracted from the measurements of electric conductivity at low temperatures as a function of the external magnetic field. The method of parametric resonance can be used as a way of generating coherent magnons leading to soliton formation when it is aided by an alternating magnetic field[25] at radio frequencies. In a real crystal the electrical resistance is caused by scattering of conduction electrons by phonons, magnons, and solitons. So, simultaneous influences of all these mechanisms have to be included in order to provide a complete description of the resistivity's dependence on temperature and magnetic field.

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