Two-dimensional vortices in a stack of thin superconducting films: A model for high-temperature superconducting multilayers

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(Received 16 August 1990)

The structure of vortices within an infinite stack of thin superconducting layers is considered and examined in detail in the limit of zero interlayer Josephson coupling. The basic building block for the description of three-dimensional (3D) vortex lines is shown to be the 2D pancake vortex, which is a vortex located in only one of the layers; the other layers contain no vortices, but have an important effect in screening the magnetic field generated by currents in the first layer. It is shown that 3D vortex lines can be built up by superposing the contributions of stacks of 2D pancake vortices. Thermal excitation is shown to break up a single 3D vortex line at a temperature corresponding to the Kosterlitz-Thouless temperature of a single superconducting layer. The effect of thermally induced decoupling of the 2D vortex solids in different layers, corresponding to melting only in the direction perpendicular to the layers, is also considered. It is shown that Josephson coupling can be neglected in the high-temperature superconductors only under very stringent conditions. Although these conditions evidently are not met in $Bi_2Sr_2CaCu_2O_8$ and $Tl_2Ba_2Ca_1Cu_2O_8$, they should be satisfied in superconducting-insulating multilayer systems, such as $YBa_2Cu_3O_{7-\delta}/PrBa_2Cu_3O_{7-\delta}$.

I. INTRODUCTION

Since the discovery of the high-temperature copper-oxide superconductors,[1] much attention has been given to the anisotropy that arises because of the easier conduction parallel to the CuO_2 layers. To describe the anisotropy quantitatively, it has proved useful to make use of the London[2,3] and Ginzburg-Landau[4] theories, extended with a phenomenological anisotropic effective mass tensor.[5–18] In the reference frame aligned with the principal axes, this mass tensor is diagonal, and the diagonal elements m_i ($i=1,2,3=a,b,c$) are normalized such that $m_1m_2m_3=1$. The penetration depths $\lambda_1=\lambda\sqrt{m_i}$ describe the exponential decay of components of the supercurrent pointing along the principal directions, and the corresponding coherence lengths $\xi_i=\xi/\sqrt{m_i}$ characterize the spatial variation of the order parameter along these directions.

A common feature of the high-temperature superconductors is that ξ_3 ($=\xi_c$), the coherence length for spatial variation along the c direction (perpendicular to the CuO_2 layers), is much smaller than in conventional superconductors. When a magnetic field parallel to the layers produces the mixed state, the vortices (according to anisotropic mass-tensor theory) have cores of elliptical cross sections with caliper dimensions roughly $2\xi_c$ along the c direction. Although ξ_c is much larger than the spacing between CuO_2 layers near T_c, the value of ξ_c extrapolated to $T=0$ K is usually less than the unit-cell lattice parameter c and is often less than the spacing between adjacent CuO_2 layers. This suggests that, at low temperatures, the vortex structure is not well described by the anisotropic mass-tensor theory and that a model incorporating the discreteness of the CuO_2 layers is required.[19,20] The purpose of this paper is to explore some of the consequences of such a model in the limit of extreme anisotropy.

A good theoretical starting point is the Lawrence-Doniach model,[18–20] which describes Josephson-coupled superconducting layers of thickness d and stacking periodicity length s. It is assumed for simplicity that each superconducting layer is isotropic with intrinsic bulk penetration depth λ_s. Close to the transition temperature, where all the coherence lengths are much larger than s, such a model leads to a continuum theory equivalent to the anisotropic Ginzburg-Landau theory for a uniaxial superconductor with diagonal mass-tensor elements $m_a=m_b=m_\parallel$ and $m_c=m_\perp$. The corresponding penetration depths and coherence lengths parallel ($\parallel$) and perpendicular ($\perp$) to the layers are $\lambda_a=\lambda_b=\lambda_\parallel=\lambda\sqrt{m_\parallel}$, $\lambda_c=\lambda_\perp=\lambda\sqrt{m_\perp}$, $\xi_a=\xi_b=\xi_\parallel=\xi/\sqrt{m_\parallel}$, and $\xi_c=\xi_\perp=\xi/\sqrt{m_\perp}$. The decay length for currents flowing parallel to the layers is[18] $\lambda_\parallel=\lambda_s(s/d)^{1/2}$. The Lawrence-Doniach model yields the following expression for the decay length of the Josephson supercurrents flowing perpendicular to the layers in the limit of weak Josephson coupling:[18]

$$\lambda_\perp=(c\phi_0/8\pi^2 sJ_0)^{1/2}\gg\lambda_\parallel\ , \tag{1}$$

where J_0 is the maximum Josephson supercurrent density and $\phi_0=hc/2e$ is the superconducting flux quantum. The corresponding dimensionless effective masses can be expressed as

$$m_\parallel=(\lambda_\parallel/\lambda_\perp)^{2/3}\ll 1$$

and

$$m_\perp=m_\parallel^{-2}=(\lambda_\perp/\lambda_\parallel)^{4/3}\gg 1\ .$$

The ratio of $\xi_\perp$, the coherence length perpendicular to the layers, to $\xi_\parallel$, parallel to the layers, is therefore

$$\xi_\perp/\xi_\parallel=(m_\parallel/m_\perp)^{1/2}=\lambda_\parallel/\lambda_\perp \ll 1 \ . \tag{2}$$

This equation tells us that the condition of very weak interlayer Josephson coupling J_0 leads simultaneously to very large $\lambda_\perp$ and very small $\xi_\perp$.

In an anisotropic continuum theory based upon either the London or the Ginzburg-Landau theory, three-dimensional (3D) vortex lines passing through a stack of superconducting layers at a nonvanishing angle relative to the principal axes have a surprisingly complex structure,[13,14] owing to the tendency of the currents to be confined to the easy current directions, parallel to the layers. This result suggests that, in a theory accounting for atomic-level discreteness, one can picture a vortex line passing through a stack of weakly Josephson-coupled layers as a set of 2D pancake vortices connected by Josephson vortices, rather like beads on a necklace. The individual 2D pancake vortices have nearly circular current patterns essentially confined to individual superconducting layers. On the other hand, the Josephson vortices, whose axes thread through the Josephson junctions between superconducting layers, stretch from the center of a 2D pancake vortex in one superconducting layer to the center of the 2D pancake vortex in the next layer. Highly elliptical current patterns circulate about the axis of each Josephson vortex, with a large decay length $\lambda_\perp$ for the weak Josephson current density flowing perpendicular to the layers and a much smaller penetration depth $\lambda_\parallel$ for the stronger current density returning parallel to the layers.

To give some substance to this way of picturing vortices in highly anisotropic superconductors, I consider in this paper the nature of 2D pancake vortices in a stack of superconducting layers in the extreme limit of *zero* Josephson-coupling strength; this therefore corresponds to the Lawrence-Doniach model in the limit for which $J_0\rightarrow 0$, $\lambda_\perp\rightarrow\infty$, and $\xi_\perp\rightarrow 0$.

In Sec. II, to provide a reference point, I briefly review Pearl's[21,22] solution for the magnetic field generated by a 2D pancake vortex in an isolated thin superconducting layer. In Sec. III, I extend an approach used earlier for studying magnetically coupled vortices in the dc superconducting transformer[23–26] (two superconducting layers separated by an insulating layer) to examine the magnetic field generated by a 2D pancake vortex in the central superconducting layer of an infinite stack. The other superconducting layers, which do not contain vortices, serve only to screen the magnetic field generated by the vortex in the central layer. The resulting solution gives us the building block (or Green's function) needed to build up, by superposition, the magnetic field distribution produced by an arbitrary (perhaps wavy) vortex line represented by a string of 2D pancake vortices at various positions in different layers. In other words, we now have a method for finding the net field at some point by doing a vector addition of the contributions from all nearby 2D pancake vortices.

In Sec. IV, I first show that the magnetic field and current distribution produced by a straight stack of 2D pancake vortices aligned along the normal to the layers reduces (after conversion of the sum over contributions to an integral) to that of the usual London model. I also calculate the energy per unit length of the stack when the line of centers is tilted relative to the normal. In Sec. V, I first discuss how a single stack of 2D pancake vortices can be broken up by thermal agitation, and then I consider thermally induced decoupling of the 2D vortex solids in different layers. Finally, I present a brief summary in Sec. VI.

II. 2D PANCAKE VORTEX IN AN ISOLATED SUPERCONDUCTING THIN FILM

Because we shall be using a very similar method of solution for a more complicated situation, let us first review the solution for a vortex in a single, isolated, thin superconducting layer.[21–22]. The basic equations describing the currents in the superconductor and the magnetic fields throughout all space are Maxwell's equations plus the equation for the sheet current density $\mathbf{K}$,

$$\mathbf{K}=-n_{2\mathrm{D}}^{*}e^{*}\mathbf{v}_s \ , \tag{3}$$

where $n_{2\mathrm{D}}^{*}$ is the sheet density of Cooper pairs (number per unit area), $-e^{*}=-2e$ is the pair charge,

$$\mathbf{v}_s=(e^{*}/m^{*}c)[\mathbf{a}+(\phi_0/2\pi)\nabla\gamma]$$

is the pair supervelocity, $m^{*}=2m$ is the pair mass, $\mathbf{a}$ is the vector potential ($\mathbf{b}=\nabla\times\mathbf{a}$), and γ is the phase of the order parameter. This equation also can be written as

$$\mathbf{K}=-(c/2\pi\Lambda)[\mathbf{a}+(\phi_0/2\pi)\nabla\gamma] \ , \tag{4}$$

where Λ is the 2D thin-film screening length, defined by

$$\Lambda^{-1}=2\pi n_{2\mathrm{D}}^{*}e^{*2}/m^{*}c^{2} \ . \tag{5}$$

Alternatively, we can regard the superconducting layer as a thin film of thickness $d\ll\lambda_s$ of material with bulk penetration depth λ_s defined via

$$\lambda_s^{-2}=4\pi n_{3\mathrm{D}}^{*}e^{*2}/m^{*}c^{2} \ , \tag{6}$$

such that $n_{2\mathrm{D}}^{*}=n_{3\mathrm{D}}^{*}d$ and $\Lambda=2\lambda_s^2/d$. (Readers are warned that some papers in the literature define a Λ without the factor of 2.)

When the superconducting layer contains a pancake vortex with one fluxoid $\boldsymbol{\phi}_0=\hat{\mathbf{z}}\phi_0$ ($\phi_0=hc/2e$) at the origin, the Pearl solution[21,22] is obtained in cylindrical coordinates, $\rho=(x^2+y^2)^{1/2}$, $\phi=\tan^{-1}(y/x)$, and z (unit vectors $\hat{\boldsymbol{\rho}}=\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi$, $\hat{\boldsymbol{\phi}}=\hat{\mathbf{y}}\cos\phi-\hat{\mathbf{x}}\sin\phi$, and $\hat{\mathbf{z}}$) from the general solution (with mirror symmetry about the plane $z=0$ and cylindrical symmetry about the z axis) of the equation $\nabla\times\mathbf{b}=\mathbf{0}$, where $\mathbf{b}=\nabla\times\mathbf{a}$ and $\mathbf{a}=\hat{\boldsymbol{\phi}}a_\phi(\rho,z)$:

$$a_\phi(\rho,z)=\int_0^\infty dq \ A_0(q)J_1(q\rho)e^{-q|z|} \ . \tag{7}$$

From the fluxoid quantization condition [Eq. (4)] with $\gamma=-\phi$ and the discontinuity in the magnetic field's radial component across the film

$$K_\phi(\rho)=(c/4\pi)[b_\rho(\rho,0^+)-b_\rho(\rho,0^-)] \ , \tag{8}$$

we have

$$\int_0^\infty dq\ A_0(q)(1+q\Lambda)J_1(q\rho)=\phi_0/2\pi\rho\ . \tag{9}$$

Taking the Hankel transform using[27]

$$\int_0^\infty d\rho\,\rho J_1(q\rho)J_1(q'\rho)=q^{-1}\delta(q-q')\ , \tag{10}$$

we obtain

$$A_0(q)=(\phi_0/2\pi)(1+q\Lambda)^{-1}\ . \tag{11}$$

When this is substituted back into Eqs. (4) and (7), the resulting integral yields

$$K_\phi(\rho)=(c\phi_0/8\pi\Lambda^2)[H_1(\rho/\Lambda)-Y_1(\rho/\Lambda)-2/\pi]\ , \tag{12}$$

where H_1 is the Struve function and Y_1 the Bessel function of the second kind.[28-29] Limiting expressions (for $\rho \ll \Lambda$ and $\rho \gg \Lambda$) for the sheet current density $K_\phi(\rho)$ and the magnetic flux $\Phi_z(\rho,0)$ up through a circle of radius ρ are given in the Appendix. The field

$$\mathbf{b}=\hat{\rho}b_\rho(\rho,z)+\hat{\mathbf{z}}b_z(\rho,z)$$

is sketched in Fig. 1.

The repulsive Lorentz force exerted by this vortex on a second vortex of the same sign is $F_\rho(\rho)=K_\phi(\rho)\phi_0/c$, and the corresponding interaction potential, obtained by integrating from ρ to ∞, is[21,22]

$$U_0(\rho)=(\phi_0^2/8\pi\Lambda)[H_0(\rho/\Lambda)-Y_0(\rho/\Lambda)]\ , \tag{13}$$

which reduces to the following results in the limits shown:

$$U_0(\rho)\approx(\phi_0^2/4\pi^2\Lambda)\ln(\Lambda/\rho),\quad \rho\ll\Lambda\ , \tag{14a}$$

$$\approx\phi_0^2/4\pi^2\rho,\quad \rho\gg\Lambda\ . \tag{14b}$$

At temperatures so close to T_c that Λ becomes larger than the linear dimensions of the specimen, the dominant logarithmic form enables a Kosterlitz-Thouless transition to occur.[30-36]

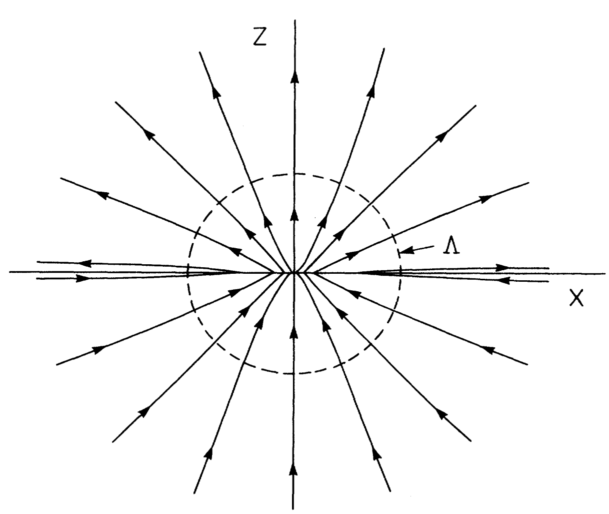

FIG. 1. Sketch of the magnetic field **b** generated by a 2D pancake vortex in an isolated and superconducting layer with thin-film screening length Λ (Refs. 21 and 22). Note different behavior for $r<\Lambda$ and $r>\Lambda$. The field for $z>0$ and $r>\Lambda$ resembles that of a magnetic monopole (flux ϕ_0 into solid angle 2π) such that $b=\phi_0/2\pi r^2$.

III. 2D PANCAKE VORTEX IN A STACK OF THIN FILMS

We next apply an extension[23-26] of the above approach to study a vortex in an infinite stack of parallel, thin (Josephson-decoupled) superconducting layers in the planes $z=z_n$, where $z_n=ns$ $(n=0,\pm1,\pm2,\ldots)$. Although we consider here, for simplicity, only the case of very thin, equally spaced, superconducting layers, it would be straightforward (using the method of Refs. 23 and 25) to generalize to the case of layers of arbitrary thickness $d<s$, or for unit cells containing unequally spaced layers of different thicknesses.

Let us consider the magnetic field and currents generated throughout all space by a 2D pancake vortex in the layer $n=0$ only; the other layers $(n\neq0)$ contain no vortices but do carry screening currents. We choose a gauge such that $\gamma=-\phi$ in the central $(n=0)$ layer and $\gamma=0$ in all the other $(n\neq0)$ layers; $\mathbf{a}=\hat{\boldsymbol{\phi}}a_\phi(\rho,z)$. The general solution of desired symmetry has the form

$$a_\phi(\rho,z)=\int_0^\infty dq\ A(q)J_1(q\rho)Z(q,z)\ , \tag{15}$$

where $Z(q,z)$ between each pair of layers obeys

$$Z(q,z)=\alpha e^{-qz}+\beta e^{qz} \tag{16}$$

and, in addition, has the symmetries $Z(q,-z)=Z(q,z)$ and

$$Z(q,z_n)=\exp(-Q|z_n|)$$

$(n=0,\pm1,\pm2,\ldots)$. The value of Q is determined from Eqs. (4) and (8) and the condition that $\gamma=0$ in the layers with $n\neq0$. We obtain, for example,

$$Z(q,z)=[\sinh q(s-z)+e^{-Qs}\sinh qz]/\sinh qs\ ,\qquad 0\le z\le s\ , \tag{17a}$$

$$Z(q,z)=e^{-Qs}[\sinh q(2s-z)+e^{-Qs}\sinh q(z-s)]/\sinh qs,\qquad s\le z\le 2s\ , \tag{17b}$$

where

$$\cosh Qs=\cosh qs+(q\Lambda)^{-1}\sinh qs\ . \tag{18}$$

The scallops in $Z(q,z)$ as a function of z are necessary to describe the discontinuities in $b_\rho(\rho,z)$ arising from the layer sheet currents $K_\phi(\rho,z_n)$. However, for values of s and Λ expected for the high-temperature superconductors, the discontinuities in $b_\rho(\rho,z_n)$ for $n\neq 0$ are very tiny and it is, for most purposes, an excellent approximation to simply replace $Z(q,z)$ by e^{-Qz}.

From Eqs. (4) and (8) we then obtain

$$\int_0^\infty dq\ A(q)(q\Lambda \sinh Qs/\sinh qs)J_1(q\rho)=\phi_0/2\pi\rho\ , \tag{19}$$

from which the Hankel transform yields, with the help of Eq. (18),

$$A(q)=(\phi_0/2\pi)[1+2q\Lambda\coth qs+(q\Lambda)^2]^{-1/2}\ . \tag{20}$$

As expected, in the limit $s\rightarrow\infty$, $A(q)$ reduces to the result appropriate for an isolated film [Eq. (11)], but in the opposite limit, $s\rightarrow 0$, we obtain

$$A(q)\approx(\phi_0/2\pi\Lambda)(q^2+\lambda_\parallel^{-2})^{-1/2}\ , \tag{21}$$

where the effective penetration depth for decay of fields associated with currents flowing parallel to the layers is given by

$$\lambda_\parallel=(s\Lambda/2)^{1/2}\ . \tag{22}$$

In terms of the quantities in Eq. (6), we note that

$$\lambda_\parallel^{-2}=4\pi\langle n_{3D}^*\rangle e^{*2}/m^*c^2\ , \tag{23}$$

where $\langle n_{3D}^*\rangle=(n_{3D}^*d/s)$ is the average pair density in the stack of superconducting layers.

To help us visualize which are the important lengths for the high-temperature superconductors, we assume the following orders of magnitude for the above lengths: $d\approx 6$ Å, $s\approx 12$ Å, $\lambda_s\approx 1000$ Å, $\lambda_\parallel\approx 1400$ Å, and $\Lambda\approx 10^2$ μm. Thus,

$$s/2\lambda_\parallel=\lambda_\parallel/\Lambda\approx 10^{-3}\ .$$

Other values of d and s should be chosen to model specific compounds with various numbers of CuO_2 layers per unit cell, but in each instance we expect the following inequalities to hold: $d<s\ll\lambda_s<\lambda_\parallel\ll\Lambda$.

Substitution of the exact result for $A(q)$ [Eq. (20)] into Eq. (15) yields a complicated integral which cannot be evaluated analytically. The complexity arises chiefly because of the fine features on the scale of s. If we give up information on this length scale, however, and examine only the spatial variation on the scale of $\lambda_\parallel$ and larger, we can obtain analytic results. For values of ρ or $|z|$ much larger than s, the important q's in the integrand of Eq. (15) are of order ρ^{-1} or $|z|^{-1}\ll s^{-1}$, and it is an excellent approximation to replace Eq. (20) by Eq. (21) and to replace $Z(q,z)$ by $\exp(-Q|z|)$, where, as can be shown from Eq. (18), $Q=(q^2+\lambda_\parallel^{-2})^{1/2}$. The result is

$$a_\phi(\rho,z)=(\phi_0\lambda_\parallel/2\pi\Lambda\rho)(e^{-|z|/\lambda_\parallel}-e^{-r/\lambda_\parallel})\ , \tag{24}$$

where $r=(\rho^2+z^2)^{1/2}$. From this result we obtain the magnetic field components

$$b_z(\rho,z)=(\phi_0/2\pi\Lambda r)e^{-r/\lambda_\parallel}\ , \tag{25a}$$

$$b_\rho(\rho,z)=(\phi_0/2\pi\Lambda\rho)[(z/|z|)e^{-|z|/\lambda_\parallel}-(z/r)e^{-r/\lambda_\parallel}]\ , \tag{25b}$$

sketched in Fig. 2. [As mentioned above, the approximation of replacing $Z(q,z)$ by $\exp(-Q|z|)$ results in the loss of the small (down by a factor $\sim\lambda_\parallel/\Lambda$) discontinuities in $b_\rho(\rho,z)$ across the layers with $n\neq 0$. If desired, these can be recovered with the help of Ampere's law and Eq. (28).] The magnetic flux

$$\Phi_z(\rho,z)=2\pi\rho a_\phi(\rho,z)$$

up through a layer at height z within a circle of radius ρ is

$$\Phi_z(\rho,z)=(\phi_0\lambda_\parallel/\Lambda)(e^{-|z|/\lambda_\parallel}-e^{-r/\lambda_\parallel})\ . \tag{26}$$

The total flux up through a layer at height z is

$$\Phi_z(\infty,z)=(\phi_0\lambda_\parallel/\Lambda)\exp(-|z|/\lambda_\parallel)\ .$$

The flux up through the central layer

$$\Phi_z(\infty,0)=(\phi_0\lambda_\parallel/\Lambda)$$

is much less than ϕ_0. From Eq. (4), the sheet current density in the central ($n=0$) layer

$$K_\phi(\rho,0)=(c/2\pi\Lambda)[\phi_0/2\pi\rho-a_\phi(\rho,0)]$$

is found to be

$$K_\phi(\rho,0)=(c\phi_0/4\pi^2\Lambda\rho)[1-(\lambda_\parallel/\Lambda)(1-e^{-\rho/\lambda_\parallel})]\ . \tag{27}$$

Note that the (negative) flux term is relatively small, since

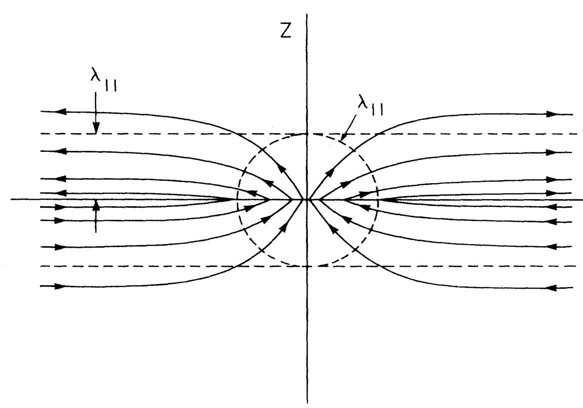

FIG. 2. Sketch of the magnetic field **b** generated by a 2D pancake vortex in only the central ($z=0$) layer of an infinite stack of parallel superconducting layers (not shown) of spacing $s\ll\lambda_\parallel$. Because of the screening currents in the other ($z_n=ns\neq 0$) layers, the vortex's magnetic flux $\phi_0(\lambda_\parallel/\Lambda)$ is guided radially out to infinity essentially within a disk of thickness $\lambda_\parallel$. For $\rho\gg\lambda_\parallel$, the field just above the central layer is $b_\rho=\phi_0/2\pi\Lambda\rho$ [flux $\phi_0(\lambda_\parallel/\Lambda)$ through area $2\pi\rho\lambda_\parallel$].

$\lambda_\parallel/\Lambda \approx 10^{-3}$. In all the other ($n \neq 0$) layers,

$$K_\phi(\rho,z_n) = -(c\phi_0\lambda_\parallel/4\pi^2\Lambda^2\rho)(e^{-|z_n|/\lambda_\parallel} - e^{-r_n/\lambda_\parallel}) , \quad (28)$$

where $z_n = ns$ and $r_n = (\rho^2 + z_n^2)^{1/2}$.

Given the above solution for the field and current distribution generated by a single 2D pancake vortex at the origin of the central layer, it is now a relatively simple matter to add other 2D pancake vortices at various positions in other layers and to calculate their interactions. In the present model, these vortices do not interact via Josephson coupling, since here J_0 is set equal to zero, but they do interact electromagnetically. For example, it is easy to see from $F_\rho = K_\phi\phi_0/c$ and Eq. (27) that the repulsive interaction energy between two 2D pancake vortices of the same sense in the central ($n=0$) layer is logarithmic to all distances, not just within Λ. Similarly, a consequence of Eq. (28) is that the interaction between 2D pancake vortices in different layers is weak, but attractive, favoring coaxial alignment.

IV. STACK OF ALIGNED 2D PANCAKE VORTICES

We consider next the magnetic field and current distribution produced by a stack of 2D pancake vortices whose centers lie along a common axis tilted at an arbitrary angle θ relative to the z axis.

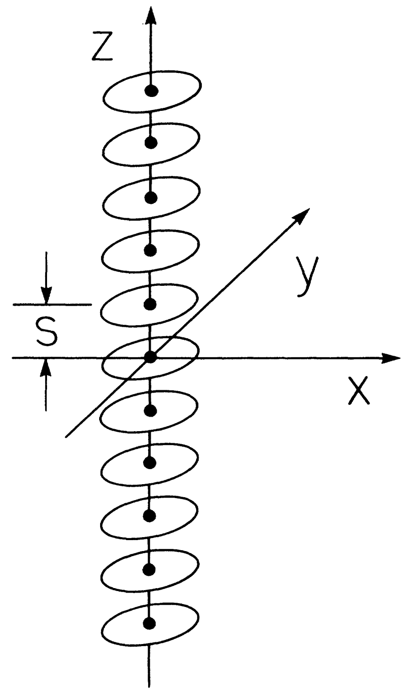

FIG. 3. Stack of 2D pancake vortices aligned along the z axis.

A. Axis perpendicular to layers: $\theta=0$

For simplicity, we first examine the case for which the 2D vortices are aligned along the z axis, as shown in Fig. 3. The vector potential can be written as an infinite sum of contributions like that of Eq. (24), but centered on different layers,

$$a_\phi(\rho,z) = \sum_n (\phi_0\lambda_\parallel/2\pi\Lambda\rho)(e^{-|\delta z_n|/\lambda_\parallel} - e^{-\delta r_n/\lambda_\parallel}) , \quad (29)$$

where $\delta z_n = z - z_n$ and $\delta r_n = (\rho^2 + \delta z_n^2)^{1/2}$. Converting the sum over n to an integral over z' yields[37]

$$a_\phi(\rho) = (\phi_0/2\pi\rho)[1 - (\rho/\lambda_\parallel)K_1(\rho/\lambda_\parallel)] , \quad (30)$$

where K_n is the modified Bessel function of order n. The corresponding magnetic field has only a z component,

$$b_z(\rho) = (\phi_0/2\pi\lambda_\parallel^2)K_0(\rho/\lambda_\parallel) , \quad (31)$$

which is the well-known solution[3] for the field of a London-model vortex in a medium with penetration depth $\lambda_\parallel$. The sheet-current density in each superconducting layer, found from Eqs. (4) and (30), is

$$K_\phi(\rho) = (c\phi_0/4\pi^2\Lambda\lambda_\parallel)K_1(\rho/\lambda_\parallel) . \quad (32)$$

To compute the lower critical field, we note that, in general, the Helmholtz free energy associated with layer n, (obtained by integrating the magnetic-field energy density and the supercurrent kinetic-energy density and performing a partial integration) can be expressed as

$$E_n = \phi_0 I_n/2c , \quad (33)$$

where I_n is the total supercurrent flowing in the counterclockwise direction around the vortex axis in the nth layer. For a stack of pancake vortices aligned along the z axis, each of the E_n's obtained by combining Eqs. (32) and (33) is equal to

$$E(0) = (\phi_0^2/8\pi^2\Lambda)K_0(\xi_\parallel/\lambda_\parallel) , \quad (34)$$

where the radial integral's lower limit is taken to be $\xi_\parallel$, the Ginzburg-Landau coherence distance for spatial variation parallel to the CuO_2 planes. Since the energy per unit length of an aligned stack perpendicular to the layers is

$$\varepsilon_1(\theta=0) = E(0)/s = (\phi_0/4\pi)H_{c1}(0) ,$$

we may make use of Eqs. (22) and (34) to recover the familiar London-model result for the lower critical field[3]

$$H_{c1}(\theta=0) = (\phi_0/4\pi\lambda_\parallel^2)K_0(\xi_\parallel/\lambda_\parallel) . \quad (35)$$

B. Tilted stack: $0<\theta<\pi/2$

We consider next the magnetic field and current distribution of a leaning tower of 2D pancake vortices whose line of centers is tilted at an angle θ relative to the z axis, as shown in Fig. 4. In principle, this can be obtained from the vector potential **a** expressed as a linear superposition of pancake-vortex contributions [Eq. (24)] centered on different layers. The resulting magnetic flux density **b** has a longitudinal component parallel to the line of centers. In addition, the vortex currents **K**, which flow in roughly circular patterns but are confined to the lay-

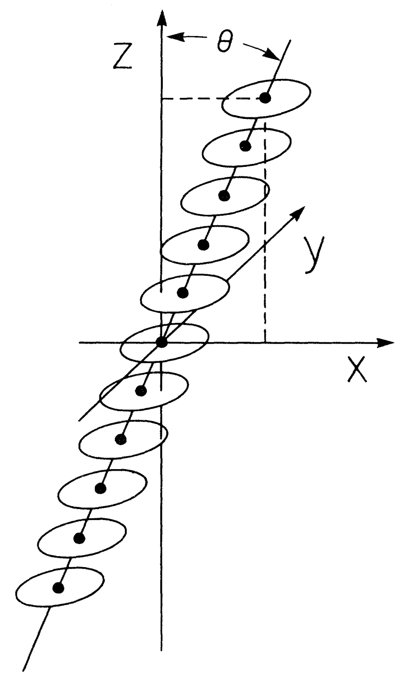

FIG. 4. Aligned stack of 2D pancake vortices tilted at angle θ relative to the z axis.

ers, generate components of $\mathbf{b}$ perpendicular to the line of centers, as in the anisotropic effective-mass description.[13,14]

An artifact of the present model, which totally neglects Josephson coupling between layers ($\lambda_\perp = \infty$), is that, although the integral of $\mathbf{b}\cdot d\mathbf{S}$ over the plane $z=$const (parallel to the layers) gives total magnetic flux ϕ_0, as expected, the integrals of the magnetic flux through the planes $x=$const or $y=$const (perpendicular to the layers) are *zero*. For any finite value of $\lambda_\perp$, however, the corresponding interlayer Josephson currents must produce a net flux component parallel to the layers, generally following the line of centers, such that the magnetic flux through any plane intersecting the tilted stack (even the planes $x=$const or $y=$const perpendicular to the layers) is ϕ_0.

We can determine the lower critical field $\mathbf{H}_{c1}(\theta)$ for the case $0<\theta<\pi/2$ without calculating $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{K}$ using an energy approach. To do this, we need to calculate the additional energy per layer [above and beyond that of Eq. (34)] required to tilt the line of centers to an angle θ relative to the z axis. We first compute the work required to misalign just a pair of 2D pancake vortices in two different layers. With the help of Eqs. (4) and (15), we can find the screening supercurrent $K_\phi(\rho,z_n)$ generated in layer n ($z=z_n=ns$) in response to the vortex in the central ($n=0$) layer. This current, which is clockwise when viewed from above, gives rise to an inward restoring force $F_\rho(\rho,z_n)=K_\phi(\rho,z_n)\phi_0/c$ on the vortex in the nth layer. The energy required to misalign the axis of the 2D vortices in layers 0 and n [obtained from the integral of $-F_\rho(\rho,z_n)$] is thus

$$\Delta E(\rho_n,z_n)=(\phi_0/2\pi\Lambda)\int_0^\infty dq\, q^{-1}A(q)\times[1-J_0(q\rho_n)]e^{-Q|z_n|}\ , \qquad (36)$$

where $\rho_n=|z_n|\tan\theta$. The interaction energy per layer required to tilt the entire stack of 2D pancake vortices through an angle θ is therefore

$$\Delta E(\theta)=\tfrac{1}{2}\sum_n{}'\,\Delta E(\rho_n,z_n)\ , \qquad (37)$$

where the factor of $\frac{1}{2}$ is to correct for double counting; the prime indicates that the term $n=0$ is excluded. Using Eq. (21) and $Q=(q^2+\lambda_\parallel^{-2})^{1/2}$, and converting the sum over n to an integral over z', we obtain

$$\Delta E(\theta)=(\phi_0^2\, 8\pi^2\Lambda)\ln[(1+\cos\theta)/2\cos\theta]\ . \qquad (38)$$

The total electromagnetic energy per layer stored in the tilted stack of vortices is, from Eqs. (34) and (38),

$$E(\theta)=E(0)+\Delta E(0)\ .$$

Since the energy per unit length of vortex is

$$\varepsilon_1(\theta)=E(\theta)\cos\theta/s\ ,$$

we obtain to logarithmic accuracy when $\lambda_\parallel \gg \xi_\parallel$:

$$\varepsilon_1(\theta)=\{(\phi_0/4\pi\lambda_\parallel)^2\ln[(\lambda_\parallel/\xi_\parallel)(1+\cos\theta)/2\cos\theta]\}\cos\theta\ . \qquad (39a)$$

For an array of line vortices at the common angle (θ,ϕ) in spherical coordinates, producing a macroscopic flux density $\mathbf{B}=B\hat{\mathbf{B}}$, where

$$\hat{\mathbf{B}}=\hat{\mathbf{x}}\sin\theta\cos\phi+\hat{\mathbf{y}}\sin\theta\sin\phi+\hat{\mathbf{z}}\cos\theta\ ,$$

the thermodynamic magnetic field must be calculated from the Helmholtz free-energy density as $\mathbf{H}=4\pi\nabla_B F(\mathbf{B})$. At very low flux density, close to the lower critical field, we have $F(\mathbf{B})=B\varepsilon_1(\theta)/\phi_0$. Since

$$\mathbf{H}_{c1}(\theta)=H_{c1B}(\theta)\hat{\mathbf{B}}+H_{c1\theta}(\theta)\hat{\boldsymbol{\theta}}\ ,$$

where

$$H_{c1B}(\theta)=4\pi\partial[B\varepsilon_1(\theta)/\phi_0]/\partial B\ ,$$

$$H_{c1\theta}(\theta)=4\pi\partial[\varepsilon_1(\theta)/\phi_0]/\partial\theta\ ,$$

and

$$\hat{\boldsymbol{\theta}}=\hat{\mathbf{x}}\cos\theta\cos\phi+\hat{\mathbf{y}}\cos\theta\sin\phi-\hat{\mathbf{z}}\sin\theta\ ,$$

we have

$$H_{c1B}(\theta)=(\phi_0/4\pi\lambda_\parallel^2)\times\{\ln[(\lambda_\parallel/\xi_\parallel)(1+\cos\theta)/2\cos\theta]\}\cos\theta\ , \qquad (39b)$$

$$H_{c1\theta}(\theta)=-(\phi_0/4\pi\lambda_\parallel^2)\{\ln[(\lambda_\parallel/\xi_\parallel)(1+\cos\theta)/2\cos\theta]-(1+\cos\theta)^{-1}\}\sin\theta\ . \qquad (39c)$$

For large $\lambda_\parallel/\xi_\parallel$, Eq. (39) tells us that $H_{c1}(\theta)$ points very nearly along the z axis for all θ in the range $0<\theta<\pi/2$.

With our approximation that $\lambda_\perp=\infty$, the energy cost for magnetic flux to penetrate parallel to the layers vanishes. Thus, $H_{c1}(\pi/2)=0$ when **B** is exactly parallel to the layers.

V. MISALIGNMENT OF 2D PANCAKE VORTICES IN DIFFERENT LAYERS

A. Thermal disruption of a stack initially perpendicular to the layers

In this section we show that the energies required to misalign the 2D pancake vortices are so small that, as T approaches T_c, thermal energies $k_B T$ can strongly disrupt the alignment and break up a straight stack. To calculate the energies involved, we suppose that an infinite stack of 2D vortex pancakes, initially aligned along the z axis, is perturbed by displacing the central-layer ($n=0$) vortex to a distance ρ from the z axis, as shown in Fig. 5. The resulting vector potential can be obtained by superposing contributions of the form of Eq. (24). We are more interested, however, in the restoring force on the displaced vortex. This is easily calculated from

$$F_\rho(\rho)=K_{0\phi}(\rho)\phi_0/c \ ,$$

where $K_{0\phi}(\rho)$ is the sheet-current density in the central layer induced by the vortices in all the other ($n\neq0$) layers. Although the current generated in the central layer by the central-layer vortex alone flows counterclockwise when viewed from above, $K_{0\phi}(\rho)$ flows clockwise. To compute $K_{0\phi}(\rho)$, we note that this is just the current density of Eq. (32) (which includes the contribution of the vortex in the central layer) less the central-layer vortex contribution of Eq. (27). We therefore find, ignoring terms of order $\lambda_\parallel/\Lambda$,

$$F_\rho(\rho)=-(\phi_0^2/4\pi^2\Lambda\rho)[1-(\rho/\lambda_\parallel)K_1(\rho/\lambda_\parallel)] \ . \tag{40}$$

Expansion of K_1 reveals that F_ρ is nearly linear in ρ for $\rho\ll\lambda_\parallel$. The magnitude of the restoring force reaches a maximum at $\rho\approx1.1\lambda_\parallel$, and for $\rho\gg\lambda_\parallel$, we have $F_\rho(\rho)\approx-\phi_0^2/4\pi^2\Lambda\rho$. The corresponding potential well for the misaligned vortex is

$$U_0(\rho)=(\phi_0^2/4\pi^2\Lambda)[\gamma+\ln(\rho/2\lambda_\parallel)+K_0(\rho/\lambda_\parallel)] \ , \tag{41}$$

where $\gamma=0.5772\ldots$ is Euler's constant. This potential is approximately quadratic for small ρ but is logarithmic beyond $\rho\sim\lambda_\parallel$.

At very low temperatures, where $k_BT\ll\phi_0^2/4\pi^2\Lambda$, the root-mean-square displacement $\rho_{\rm rms}$ of a 2D pancake vortex obeys $\rho_{\rm rms}\ll\lambda_\parallel$, and a stack of 2D pancake vortices thus withstands thermal agitation, holds itself together, and produces a field distribution very similar to that of a straight 3D vortex threading through an isotropic type-II superconductor. As the temperature increases, we can expect from the form of Eq. (41) that $\rho_{\rm rms}^2$ will initially be approximately proportional to k_BT. A change in behavior occurs, however, when k_BT approaches $\phi_0^2/4\pi^2\Lambda$ and $\rho_{\rm rms}$ exceeds $\lambda_\parallel$, such that the behavior of $\rho_{\rm rms}$ is determined primarily by the logarithmic form of $U(\rho)$. We find that a divergence of $\rho_{\rm rms}$ occurs at a temperature which can be estimated from the expression

$$\rho_{\rm rms}^2=\int d^2\rho\,\rho^2 e^{-U(\rho)/k_BT}\Big/\int d^2\rho\, e^{-U(\rho)/k_BT} \ . \tag{42}$$

Since, at high temperatures, the behavior of the integrals in this expression is dominated by large values of ρ, where $U(\rho)$ is logarithmic, we can replace $U(\rho)/k_BT$ in both the numerator and denominator by

$$(\phi_0^2/4\pi^2\Lambda k_BT)\ln\rho=\ln\rho^\alpha \ ,$$

where $\alpha=\phi_0^2/4\pi^2\Lambda k_BT$, and take the lower limit of the integral to be $\lambda_\parallel$. The resulting expression for $\rho_{\rm rms}$ is

$$\rho_{\rm rms}\approx[(\alpha-2)/(\alpha-4)]^{1/2}\lambda_\parallel \ . \tag{43}$$

According to this expression, since α is large at low temperatures and decreases with increasing temperature, we see that $\rho_{\rm rms}$ increases with temperature and diverges when α decreases to the value 4 or T increases to the value T_b, where

$$T_b=\phi_0^2/16\pi^2k_B\Lambda \ . \tag{44}$$

It is remarkable that this condition for the thermally induced breakup of an isolated stack of 2D pancake vortices is exactly the same as that for the Kosterlitz-Thouless transition of an isolated superconducting thin film of screening length Λ.[32] We stress that the results in Eqs. (43) and (44) depend critically upon the logarithmic ρ dependence of $U_0(\rho)$ for large ρ, which occurs only when there is no interlayer Josephson coupling. The smallest Josephson coupling, however, will produce a linear ρ dependence of $U_0(\rho)$ for large ρ.

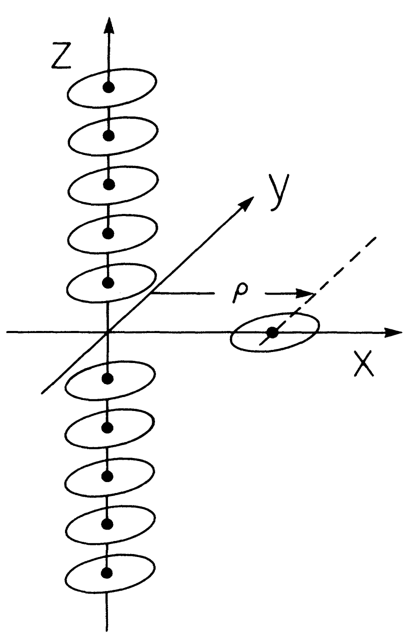

FIG. 5. Stack of 2D pancake vortices aligned along the z axis, but with central-layer vortex displaced by ρ in the x direction.

B. Thermal decoupling of vortices in different layers when $B \gg H_{c1}$

We next consider the vortex structure when the average magnetic flux density B (perpendicular to the layers) is considerably larger than the lower critical field H_{c1}. The intervortex spacing, which is roughly $(\phi_0/B)^{1/2}$, is then less than $\lambda_\parallel$, and the total local magnetic-field distribution $\mathbf{b}$ can be regarded as a superposition of overlapping individual vortex contributions. Let us consider, for reference, a periodic vortex array in which 2D pancake vortices in each layer form a triangular lattice at the lattice points $\boldsymbol{l}$ and the vortices in adjacent layers are perfectly aligned. We use the notation of Ref. 23, in which the direct- and reciprocal-lattice vectors are $\boldsymbol{l}=m\mathbf{a}_1+n\mathbf{a}_2$ and $\mathbf{g}=2\pi(m\mathbf{b}_1+n\mathbf{b}_2)$, where m and n are integers and the fundamental lattice vectors are $\mathbf{a}_1=a_0\hat{\mathbf{x}}$, $\mathbf{a}_2=(a_0/2)(\hat{\mathbf{x}}+\hat{\mathbf{y}}\sqrt{3})$, $\mathbf{b}_1=a_0^{-1}(\hat{\mathbf{x}}-\hat{\mathbf{y}}/\sqrt{3})$, and $\mathbf{b}_2=(2/a_0\sqrt{3})\hat{\mathbf{y}}$. The area of the unit cell is thus $A=\phi_0/B=a_0^2\sqrt{3}/2$, and the length of a reciprocal-lattice vector is

$$g_{mn}=(4\pi/a_0\sqrt{3})(m^2-mn+n^2)^{1/2} .$$

We next consider the displacements of the central-layer vortex lattice similar to that shown in Fig. 6. The work per vortex required to rigidly displace the lattice of 2D pancake vortices in the central ($n=0$) layer to the position $\boldsymbol{\rho}=\hat{\mathbf{x}}x+\hat{\mathbf{y}}y$ is

$$E_c(\boldsymbol{\rho})=(B/\phi_0)\sum_{\mathbf{g}} U_0(\mathbf{g})[\exp(i\mathbf{g}\cdot\boldsymbol{\rho})-1] , \tag{45}$$

where

$$U_0(q)=-\phi_0^2/2\pi\Lambda\lambda_\parallel^2 q^2(q^2+\lambda_\parallel^{-2}) \tag{46}$$

is the 2D Fourier transform of the potential of Eq. (41).

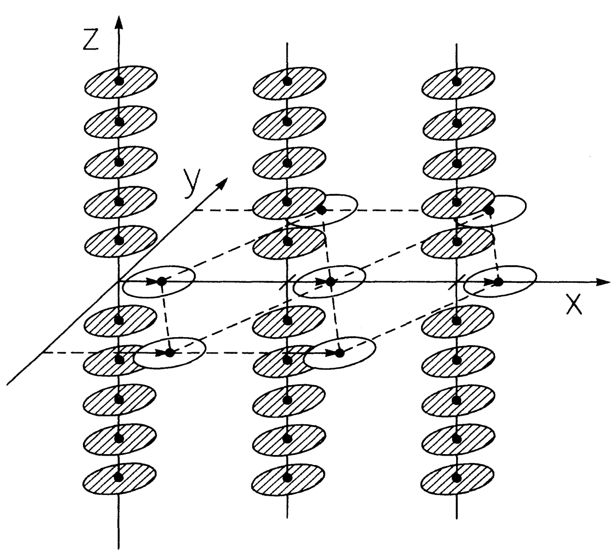

FIG. 6. Triangular lattice of aligned stacks of 2D pancake vortices, but with 2D central-layer vortex lattice displaced in the x direction. (For clarity, only the aligned stacks in the xz plane are shown.)

That is,

$$U_0(\boldsymbol{\rho})=(2\pi)^{-2}\int d^2q\, U_0(q)[\exp(i\mathbf{q}\cdot\boldsymbol{\rho})-1] . \tag{47}$$

Equation (45) has been obtained using the method introduced in Refs. 23 and 25 for calculating the periodic coupling energy between the primary- and secondary-film vortex lattices in a dc superconducting transformer. Similarly, $E_c(\boldsymbol{\rho})$ in Eq. (45) can be regarded as the coupling energy between the 2D vortex lattice in the central layer and the aligned 2D vortex lattices in all the other ($n\neq 0$) layers. Note that $E_c(\boldsymbol{\rho})$ has the periodicity of the lattice, i.e., $E_c(\boldsymbol{\rho}+\boldsymbol{l})=E_c(\boldsymbol{\rho})$.

When $B \ll H_{c1}$, it is a good approximation to replace $\mathbf{g}$ by $\mathbf{q}$ and to convert the sum over $\mathbf{g}$ to an integral over $\mathbf{q}$. For $\rho \ll a_0$, Eq. (45) then gives back $E_c(\boldsymbol{\rho})\approx U_0(\boldsymbol{\rho})$. When $B > H_{c1}$, however, it is necessary to sum over the reciprocal-lattice vectors. For large B, this summation involves values of $\mathbf{g}$ for which $U_0(\mathbf{g})$ decreases rapidly with increasing $\mathbf{g}$. It is then a good approximation to retain only those terms involving the six shortest reciprocal-lattice vectors of length $g_{10}=4\pi/a_0\sqrt{3}$. This one-reciprocal-lattice vector approximation[23–26] yields

$$E_c(X,Y)\approx 2u_{10}[3-\cos 2\pi X-\cos 2\pi Y-\cos 2\pi(X+Y)] , \tag{48}$$

where $u_{10}=-(B/\phi_0)U_0(g_{10})$ and $\boldsymbol{\rho}$ is expressed in terms of new coordinates X and Y, defined via

$$\boldsymbol{\rho}=\hat{\mathbf{x}}x+\hat{\mathbf{y}}y=\mathbf{a}_1X+\mathbf{a}_2Y .$$

Contours of constant E_c within the unit cell are shown in Fig. 1 of Ref. 23. Within the unit cell ($0\le X\le 1$, $0\le Y\le 1$), the minimum values of E_c, ($E_c=0$) occur when $(X,Y)=(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$. The maximum values of E_c ($E_c=E_{c\max}=9u_{10}$) occur when the central-layer vortices are at $(X,Y)=(\frac{1}{3},\frac{1}{3})$ and $(\frac{2}{3},\frac{2}{3})$, the centers of the equilateral triangles formed by the $n\neq 0$ vortices. Saddle points, where $E_c=8u_{10}$, occur when $(X,Y)=(\frac{1}{2},0)$, $(0,\frac{1}{2})$, $(\frac{1}{2},\frac{1}{2})$, $(1,\frac{1}{2})$, and $(\frac{1}{2},1)$, where the central-layer vortices are at the midpoints of lines connecting $n\neq 0$ nearest-neighbor vortices.

We now consider the effects of thermal agitation. When $k_BT \ll 8u_{10}$, we expect that thermal effects will be too weak to cause a significant amount of deregistration of the vortex lattices in different layers. On the other hand, we should expect decoupling of the vortex arrays in different layers to occur at a temperature given by $k_BT_d=\gamma(8u_{10})$, where γ is a constant of order unity. Using this criterion and Eq. (46), we obtain the following condition for the interlayer decoupling transition temperature in the one-reciprocal-lattice-vector approximation:

$$T_d=(8\gamma\sqrt{3}/\pi)(\phi_0^2/16\pi^2k_B\Lambda)/(1+B/B_1) , \tag{49}$$

where $B_1=(\sqrt{3}/2\pi)(\phi_0/4\pi\lambda_\parallel^2)$ is [see Eq. (39)] a little smaller than $H_{c1}(0)$. Let us set $\gamma=\pi/8\sqrt{3}=0.23$ to obtain the interpolation formula

$$T_d(B)\approx(\phi_0^2/16\pi^2k_B\Lambda)/(1+B/B_1) , \tag{50}$$

which reduces to $T_d(0)=T_b$ [Eq. (44)] when $B\to 0$. At high fields, assuming $\Lambda \approx 10^2$ μm and $\lambda_\parallel \approx 1400$ Å, Eq. (50) yields $T_d \approx (0.5 \text{ KT})/B$, suggesting that magnetic coupling of the vortex lattices in adjacent layers can be destroyed by thermal agitation even at quite low temperatures. Such an effect would be consistent with observations that the magnetic coupling force in granular-aluminum dc transformers is strongly suppressed by magnetic fields.[26]

VI. CONCLUSIONS

In this paper I have considered an infinite stack of insulated thin superconducting layers and calculated the magnetic field and current distribution [Eqs. (25)–(28) generated by a 2D pancake vortex in just one of the layers. In Sec. IV, I have shown how this solution can be used as a building block in computations of the magnetic field and current distribution generated by 3D vortex lines represented as stacks of 2D pancake vortices. In Sec. V, I showed how the 2D-pancake-vortex model can be used to calculate the extent to which thermal agitation can shake up the stack and even cause the stack to break up at a temperature that corresponds to the Kosterlitz-Thouless transition temperature of an isolated film.[30–36]

I also showed in Sec. V how the application of a magnetic field perpendicular to the layers strongly suppresses the electromagnetic coupling of the 2D pancake vortices in adjacent layers and reduces the temperature at which interlayer decoupling occurs. The latter result has implications for the field-temperature phase diagram of layered superconductors. In particular, it suggests that there will be a region of the phase diagram (at relatively low temperatures and high applied magnetic fields) for which the 2D pancake vortices in adjacent layers are thermally decoupled (melted in the direction perpendicular to the layers) but for which the 2D pancake vortices in each layer form 2D solids. For such a case the problem of determining the flux-lattice melting temperature reduces to the previously solved problem of 2D flux-lattice melting in an isolated film.[38–40] Note that, as shown in Ref. 38, the melting temperature T_M is nearly independent of ma~~gnetic field~~ over a broad range of fields, while the interlayer decoupling temperature T_d [Eqs. (49) and (50)] is strongly suppressed by a magnetic field.

The extent to which the above model, in which Josephson coupling is neglected, can approximate the behavior of vortices in highly anisotropic high-temperature superconductors, such as those in the Bi-Sr-Ca-Cu-O and Tl-Ba-Ca-Cu-O systems, deserves further study and awaits experimental confirmation of their anisotropy. Surely a model setting $\lambda_c = \infty$ would *not* be a good approximation for $YBa_2Cu_3O_{7-\delta}$, for which vortex-lattice decoration experiments[41,42] have shown that $\lambda_a{:}\lambda_b{:}\lambda_c \approx 1.2{:}1{:}5.5$. For the most anisotropic copper-oxide superconductors, however, certain features of the resistive transition may require an interpretation in terms of 2D pancake vortices, rather than in terms of 3D vortex lines.[43–46]

The neglect of the Josephson-coupling energy relative to the electromagnetic coupling energy is a severe approximation, one that requires very large λ_c. One way to estimate the importance of the Josephson coupling energy is to consider the energy of the displaced vortex sketched in Fig. 5. When Josephson coupling is included,[47–49] the energy given in Eq. (41) must also include the energy of a Josephson vortex-antivortex pair extending from the z axis to the displaced vortex. Since the energy per unit length of a Josephson vortex is[20] $(\phi_0/4\pi)^2/\lambda_\parallel\lambda_\perp$ (aside from a logarithmic factor), the cost in coupling energy of a Josephson vortex-antivortex pair stretched to a length $\lambda_\parallel$ must be of order $(\phi_0/4\pi)^2/\lambda_\perp$. Since the electromagnetic energy given in Eq. (41) is of order $\phi_0^2/4\pi^2\Lambda$ at $\rho \approx \lambda_\parallel$, this suggests that the Josephson-coupling energy will be negligible by comparison with the electromagnetic energy only when $\lambda_\perp \gg \Lambda = 2\lambda_\parallel^2/s$.

The surest way to guarantee that Josephson coupling will be extremely weak is to artificially introduce insulating layers between superconducting layers, such as in recent work using multilayer structures of $PrBa_2Cu_3O_{7-\delta}$ interposed between $YBa_2Cu_3O_{7-\delta}$ layers.[50–52] I call the reader's attention to closely related calculations of Artemenko and Kruglov,[53] of which I learned after the completion of this work.

ACKNOWLEDGMENTS

I thank M. R. Beasley, M. W. Coffey, L. Dobrosavljevic, D. K. Finnemore, A. F. Hebard, D. Huse, P. Kes, V. G. Kogan, A. M. Goldman, Z. Hao, and A. P. Malozemoff for suggestions and helpful discussions. Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under Contract No. W-7405-Eng-82. This work was supported by the Director for Energy Research, Office of Basic Energy Sciences.

APPENDIX

The properties of the currents and fields for a vortex in a single, isolated film are such that the sheet-current density circulating around the axis obeys[22,23]

$$K_\phi(\rho) \approx c\phi_0/4\pi^2\Lambda\rho, \quad \rho \ll \Lambda , \tag{A1a}$$

$$\approx c\phi_0/4\pi^2\rho^2, \quad \rho \gg \Lambda . \tag{A1b}$$

From the fluxoid quantization condition [Eq. (4)], the magnetic flux $\Phi_z(\rho)$ through a circle of radius ρ in the film is easily found to be

$$\Phi_z(\rho) \approx \phi_0(\rho/\Lambda), \quad \rho \ll \Lambda , \tag{A2a}$$

$$\approx \phi_0(1-\Lambda/\rho), \quad \rho \gg \Lambda . \tag{A2b}$$

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