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Orientational and positional order in flux lattices of type-II superconductors

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A detailed theory of a hexatic vortex glass, recently observed in high-T_c superconductors, is developed. The vortex lattice in this phase is characterized by short-range positional order, which decays as $\exp(-\alpha r)$ in three dimensions (3D) and as $\exp(-\beta r^2)$ in 2D, and by extended orientational correlations, which may be long range in a 3D sample and decay algebraically in a 2D film. For 2D and 3D the angular and field dependence of positional and orientational correlation functions is obtained; these may be easily tested experimentally.

I. INTRODUCTION

There is now a considerable effort to understand the equilibrium phases and the dynamics of vortex lattices in high-T_c superconductors. Depending on temperature and the magnetic field, these systems exhibit a variety of properties, including glassy[1,2] and liquid[3,4] behavior of the magnetic flux. Nelson has pointed out[5] that high-temperature behavior must be determined by a significant thermal fluctuation effect,[6] which has not been taken into account in the standard theory of the Abrikosov flux-line lattice. The low-temperature, low-field phase of the magnetic flux, on the contrary, must be dominated by static disorder in the material, rather than by thermal effects. In this paper we will concentrate on the low-temperature phase of the vortex lattice. Larkin and co-workers[7] argued that pinning in high-T_c oxides should be due to the collective effect of a large number of randomly arranged centers, presumably oxygen vacancies. This collective pinning destroys translational order in the flux-line lattice,[8] giving rise to a vortex-glass state.[9] But even for strong disorder, triangular correlations can still persist in the local arrangement of vortices. This allows one to introduce local crystallographic axes. It has been shown,[10] that while correlations in the position of vortices decay on a short scale, correlations in the orientation of locally defined axes may persist on a much greater scale. This phase of the vortex lattice has been called a hexatic vortex glass[10] (HVG). A high-temperature liquid counterpart of that phase has been studied by Marchetti and Nelson.[11] In their model, disorder in the flux-line lattice is generated by thermally activated dislocation loops. The corresponding phase of mobile flux lines is analogous to a Halperin-Nelson hexatic phase in a theory of two-dimensional 2D melting.[12] It has been called a hexatic flux liquid[11] (HFL).

Recently, Murray *et al.*[13] demonstrated that the low-temperature, low-field static phase of magnetic flux lattices in high quality Bi-Sr-Ca-Cu-O single crystals is the HVG phase. In accordance with theoretical predictions, they found that the positional order decays exponentially with a correlation length of a few lattice constants, while the orientational order persists for hundreds of lattice constants. The authors of Ref. 13 have suggested two ways in which one can understand their observations. In the first scenario, the observed HVG state is a vestige of the high-temperature HFL state[11] "which then gets quenched-in as the temperature is lowered" due to the low mobility of flux lines. In the second scenario,[10] the HVG is the true low-temperature ground state, produced by the competition between the interaction of flux lines with pinning centers, and elastic interactions between the lines.

This paper is intended to develop the second scenario in greater detail and to elucidate other features of the HVG state. To simplify the problem, a few potentially important effects have been left out of the picture. First, dislocations in the flux-line lattice has been neglected. Second, isotropic elastic moduli, for which one has well established expressions, are considered. Third, only distances large compared to the lattice spacing and large enough to ignore the effect of nonlocal elasticity on positional correlations are studied. Fourth, our study will be restricted to low temperatures, when any dynamics of flux lines is irrelevant. This work must, therefore, be considered as a first approximation for the problem. With the above restrictions, properly defined,[12] positional and orientational correlation functions are calculated in two and three dimensions, and their magnetic field and angular dependences are obtained. The model is discussed in Sec. II. Positional and orientational order are studied in Sec. III and Sec. IV, respectively. The effect of the random orientational field on hexatic correlations is considered in Sec. V. The HVG phase in a thin superconducting film is studied in Sec. VI. The relevance of theoretical results to experiments is discussed in Sec. VII.

II. THE MODEL

Suppose the undisturbed flux lines are parallel to the direction of the field, Z, and form a perfect triangular lattice in any cross-section parallel to the XY plane. In the presence of pinning centers, the lattice is deformed. A zero-temperature local equilibrium configuration of flux lines is determined by the competition between the elastic interactions between the lines and their interactions with pinning centers. There are two types of local deformations of the lattice: the local displacement $\mathbf{u}(x,y,z)$, which has two components, u_x and u_y, and the local rotation, $\theta_b(x,y,z)$, about the Z axis, which is given by

$$\theta(\mathbf{r})=\tfrac{1}{2}(\partial_x u_y-\partial_y u_x) \ . \tag{1}$$

The free energy of the system is[14]

$$F=\tfrac{1}{2}\int d^3r[(C_{11}-C_{66})(\partial_\alpha u_\alpha)^2+C_{66}(\partial_\alpha u_\beta)^2+C_{44}(\partial_z u_\alpha)^2]-\int d^3r\, u_\alpha f_\alpha \ , \tag{2}$$

where α,β can be x and y; $\mathbf{f}(\mathbf{r})$ is the random pinning force; C_{11},C_{44}, and C_{66} are the elastic moduli of the triangular vortex lattice. We shall assume a Gaussian distribution for the probability of a given configuration $\mathbf{f}(\mathbf{r})$,

$$P[\mathbf{f}(\mathbf{r})]\propto\exp\left[-\frac{1}{2W}\int d^3r[\mathbf{f}(\mathbf{r})]^2\right] . \tag{3}$$

At low temperature the statistical mechanics associated with the energy (2) is dominated by extremal configurations of the displacement field $\mathbf{u}(\mathbf{r})$. The Fourier transform of $\mathbf{u}$ satisfies

$$u_\alpha(\mathbf{q})=G_{\alpha\beta}(\mathbf{q})f_\beta(\mathbf{q}) \ , \tag{4}$$

where

$$G_{\alpha\beta}(\mathbf{q})=(C_{11}q_\perp^2+C_{44}q_z^2)^{-1}\frac{q_\alpha q_\beta}{q_\perp^2}+(C_{66}q_\perp^2+C_{44}q_z^2)^{-1}\left[\delta_{\alpha\beta}-\frac{q_\alpha q_\beta}{q_\perp^2}\right] \tag{5}$$

is the Green's function of the vortex lattice, $q_\perp^2=q_x^2+q_y^2$. Correspondingly, the bond-angle field satisfies

$$\theta_b(\mathbf{q})=\frac{i}{2}(C_{66}q_\perp^2+C_{44}q_z^2)^{-1}\epsilon_{\alpha\beta}q_\alpha f_\beta(\mathbf{q}) \ , \tag{6}$$

where $\epsilon_{\alpha\beta}$ is the unit antisymmetric tensor, $\epsilon_{xy}=1$.

The dependence of the elastic moduli on $\mathbf{q}$ and the magnetic field has been obtained by Brandt[15] and Larkin and Ovchinnikov.[16] I will consider the limit of large Ginzburg-Landau parameter κ. Magnetic field is considered to be small in comparison with B_{c2}, but strong enough to provide a significant interaction between the vortices. This gives

$$1\ll\kappa,\quad \frac{2}{\kappa^2}<b\ll 1 \ , \tag{7}$$

where $b=B/B_{c2}$. The general expressions for the elastic moduli[15] then reduce to

$$C_{11}=\frac{B^2}{4\pi}\left[1+\frac{q^2}{q_h^2}\right]^{-1}+C_{66} \ ,$$
$$C_{44}=\frac{B^2}{4\pi}\left[\left[1+\frac{q^2}{q_h^2}\right]^{-1}+\frac{q_h^2}{q_{\rm BZ}^2}\right] \ , \tag{8}$$
$$C_{66}=\frac{B^2}{4\pi}(8b\kappa^2)^{-1} \ ,$$

where

$$q_h=(2b\kappa^2)^{-1/2}q_{\rm BZ}<q_{\rm BZ} \ ,$$
$$q_{\rm BZ}=(8\pi)^{1/2}/3^{1/4}a \ , \tag{9}$$
$$a=(2^{1/2}/3^{1/4})(\Phi_0/B)^{1/2} \ .$$

Here a is the flux lattice spacing, $q_{\rm BZ}$ is the radius of the Brillouin zone, $\Phi_0=ch/2e$ is the flux quantum associated with each vortex. Note that the elastic moduli tend to finite values $c_{11}=C_{11}(0)$, $c_{44}=C_{44}(0)$, $c_{66}=C_{66}$ as $q\to 0$. In this limit, according to Eq. (7), the compression deformation of the lattice is small in comparison with the shear deformation, that is $c_{66}\ll c_{11}$.

The value of the parameter W in Eq. (3) and the explicit dependence of W on the magnetic field are defined by the nature of pinning centers, which is not well understood at the moment. There are strong statistical arguments,[16,7] however, which imply that $W\propto B$ for the collective pinning at $b\ll 1$.

III. POSITIONAL ORDER

The periodic structure of the vortex lattice may be described by a set of positional order parameters,[12]

$$\rho_{\mathbf{G}}(\mathbf{r})=\exp[i\mathbf{G}\cdot\mathbf{u}(\mathbf{r})] \ , \tag{10}$$

where $\mathbf{G}$ is any reciprocal lattice vector. The decay of the positional order due to a G component of the displacement field $\mathbf{u}$ is measured via the positional correlation function

$$g_{\mathbf{G}}(\mathbf{r})=\langle\rho_{\mathbf{G}}(\mathbf{r})\rho_{\mathbf{G}}^*(0)\rangle=\langle\exp\{i\mathbf{G}\cdot[\mathbf{u}(\mathbf{r})-\mathbf{u}(0)]\}\rangle \ . \tag{11}$$

Writing $\mathbf{u}$ as

$$u_\alpha(\mathbf{r})=\int d^3r'G_{\alpha\beta}(\mathbf{r}'-\mathbf{r})f_\beta(\mathbf{r}') \ , \tag{12}$$

we have

$$g_{\mathbf{G}}(\mathbf{r})=\left\langle\exp\left[i\int d^3r'K_\beta(\mathbf{G},\mathbf{r},\mathbf{r}')f_\beta(\mathbf{r}')\right]\right\rangle \ , \tag{13}$$

where

$$K_\beta(\mathbf{G},\mathbf{r},\mathbf{r}')=G_\alpha[G_{\alpha\beta}(\mathbf{r}-\mathbf{r}')-G_{\alpha\beta}(\mathbf{r}')] \ . \tag{14}$$

The angular brackets $\langle\cdots\rangle$ in Eq. (13) mean an average with respect to possible configurations of $\mathbf{f}(\mathbf{r})$ which occur with the probabilities given by Eq. (3). Thus, the correlation function (13) is defined by the path integral

$$g_{\mathbf{G}}(\mathbf{r})=\frac{\int D[\mathbf{f}(r')]\exp\left[i\int d^3r' K_\beta(\mathbf{G},\mathbf{r},\mathbf{r}')f_\beta(\mathbf{r}')-\frac{1}{2W}\int d^3r'[f_\beta(\mathbf{r}')]^2\right]}{\int D[\mathbf{f}(\mathbf{r})]\exp\left[-\frac{1}{2W}\int d^3r'[f_\beta(\mathbf{r}')]^2\right]} . \tag{15}$$

Integration over $\mathbf{f}(\mathbf{r})$ gives

$$g_{\mathbf{G}}(\mathbf{r})\exp\left[-\frac{W}{2}\int d^3r' K_\beta^2(\mathbf{G},\mathbf{r},\mathbf{r}')\right]=\exp\left[-\frac{W}{2}\int\frac{d^3q}{(2\pi)^3}|K_\beta(\mathbf{G},\mathbf{r},\mathbf{q})|^2\right] , \tag{16}$$

where

$$K_\beta(\mathbf{G},\mathbf{r},\mathbf{q})=G_\alpha G_{\alpha\beta}(\mathbf{q})(e^{i\mathbf{q}\cdot\mathbf{r}}-1) \tag{17}$$

is the Fourier transform of Eq. (14); $G_{\alpha\beta}(\mathbf{q})$ being given by Eq. (5). Using Eq. (5) we obtain

$$|K_\beta(\mathbf{G},\mathbf{r},\mathbf{q})|^2=2[1-\cos(\mathbf{q}\cdot\mathbf{r})]\left[\frac{1}{(C_{11}q_\perp^2+C_{44}q_z^2)^2}\frac{(\mathbf{G}\cdot\mathbf{q}_\perp)^2}{q_\perp^2}+\frac{1}{(C_{66}q_\perp^2+C_{44}q_z^2)^2}\left[\mathbf{G}^2-\frac{(\mathbf{G}\cdot\mathbf{q}_\perp)^2}{q_\perp^2}\right]\right] . \tag{18}$$

Equation (18) shows that the integral in Eq. (16) is dominated by small q. At small q, $C_{11}\gg C_{66}$, which allows one to replace Eq. (18) with

$$|K_\beta(\mathbf{G},\mathbf{r},\mathbf{q})|^2=\frac{2[1-\cos(\mathbf{q}\cdot\mathbf{r})]}{(c_{66}q_\perp^2+c_{44}q_z^2)^2}\left[\mathbf{G}^2-\frac{(\mathbf{G}\cdot\mathbf{q}_\perp)^2}{q_\perp^2}\right] , \tag{19}$$

where c_{44} and c_{66} are the values of the elastic moduli at $q=0$. Then a somewhat tedious but straightforward integration can be performed explicitly yielding

$$g_{\mathbf{G}}(\mathbf{r})=\exp\left[-\frac{G^2W}{24\pi c_{44}^{1/2}c_{66}^{3/2}}\left[(1+\sin^2\theta)(r_\perp^2+\tilde{z}^2)^{1/2}+(1-2\sin^2\theta)\frac{\tilde{z}^2}{(r_\perp^2+\tilde{z}^2)^{1/2}+\tilde{z}}\right]\right] . \tag{20}$$

where $r\gg a$. Here $\mathbf{r}_\perp=(x,y)$, $\tilde{z}=(c_{66}/c_{44})^{1/2}z$, θ is the angle between $\mathbf{G}$ and $\mathbf{r}_\perp$.

There are two characteristic lengths associated with the exponential decay of positional order, given by Eq. (20). In decoration experiments[13] one can study correlations in the position of vortices on the surface of a superconductor. If the surface is normal to the magnetic field, then z in Eq. (20) is zero, which gives

$$g_{\mathbf{G}}(\mathbf{r})=\exp\left[-\frac{r_\perp}{\xi_\perp(G,\theta)}\right] , \tag{21}$$

where

$$\xi_\perp=\frac{24\pi c_{44}^{1/2}c_{66}^{3/2}}{G^2W(1+\sin^2\theta)} . \tag{22}$$

The minimal correlation length corresponds to the first reciprocal lattice vector $G=G_0=4\pi/a\sqrt{3}$. $\xi_\perp$ depends on the angle that $\mathbf{r}_\perp$ makes with the direction of $\mathbf{G}$. According to Eq. (22), the two extreme correlation lengths, at $\theta=0$ and $\theta=\pi/2$, differ by a factor of 2.

Correlations in the direction of the field are a characteristic of the bending of flux lines. Putting $r_\perp=0$ in Eq. (20) we have

$$g_{\mathbf{G}}(\mathbf{r})=\exp\left[-\frac{z}{\xi_\parallel(G)}\right] , \tag{23}$$

where

$$\xi_\parallel=\frac{16\pi c_{44}c_{66}}{G^2W} . \tag{24}$$

The measure of the positional order is $\xi_\perp/a$ in the XY plane, and $\xi_\parallel/a$ in the Z direction. Substituting the field dependence of all parameters into Eqs. (22) and (24) one obtains

$$\frac{\xi_\perp}{a}\propto b,\quad \frac{\xi_\parallel}{a}\propto b^{3/2} . \tag{25}$$

According to Eqs. (7), (8), (22), and (24) the ratio of $\xi_\parallel/\xi_\perp$ is greater than $4\sqrt{5}/3\approx 3$ and increases with the magnetic field.

IV. ORIENTATIONAL ORDER

For hexatic symmetry, the order parameter is defined as[12]

$$\psi(\mathbf{r})=\exp[6i\theta_b(\mathbf{r})] . \tag{26}$$

The corresponding orientational correlation function is

$$g_6(\mathbf{r})=\langle\psi(\mathbf{r})\psi^*(0)\rangle=\langle\exp\{6i[\theta_b(\mathbf{r})-\theta_b(0)]\}\rangle . \tag{27}$$

With the help of Eqs. (1) and (12) we obtain

$$g_6(\mathbf{r})=\left\langle\exp\left[i\int d^3r' Q_\gamma(\mathbf{r},\mathbf{r}')f_\gamma(\mathbf{r}'\right]\right\rangle , \tag{28}$$

where

$$Q_\gamma(\mathbf{r},\mathbf{r}')=-3\epsilon_{\alpha\beta}\partial'_\alpha[G_{\beta\gamma}(\mathbf{r}-\mathbf{r}')-G_{\beta\gamma}(\mathbf{r}')] . \tag{29}$$

The averaging in Eq. (28) with respect to $\mathbf{f}(\mathbf{r})$ configurations is analogous to that made in Sec. III. The result of the averaging is

$$g_6(\mathbf{r})=\exp\left[-\frac{W}{2}\int\frac{d^3q}{(2\pi)^3}|Q_\gamma(\mathbf{r},\mathbf{q})|^2\right], \tag{30}$$

where

$$Q_\gamma(\mathbf{r},\mathbf{q})=3i\epsilon_{\alpha\beta}q_\alpha G_{\beta\gamma}(\mathbf{q})(e^{i\mathbf{q}\cdot\mathbf{r}}-1) \tag{31}$$

is the Fourier transform of Eq. (29). Substituting here $G_{\alpha\beta}(\mathbf{q})$ from Eq. (5), we have

$$|Q_\gamma(\mathbf{r},\mathbf{q})|^2=\frac{18q_\perp^2[1-\cos(\mathbf{q}\cdot\mathbf{r})]}{(C_{66}q_\perp^2+C_{44}q_z^2)^2} . \tag{32}$$

In contrast to the integral in Eq. (16), the integral in Eq. (30) is well behaved at small q. For large $q_\perp$ it must be cut off at q_{BZ}. Performing the integration with $C_{66},C_{44}(q)$ given by Eqs. (8), one obtains

$$g_6=\exp\left[-\frac{9Wq_{\mathrm{BZ}}k(\delta)}{32\pi C_{66}^2}\right], \tag{33}$$

where $\delta=q_h/q_{\mathrm{BZ}}<1$, and $k(\delta)$ is a dimensionless function whose numerical value is close to one:

$$k(\delta)=\frac{16}{\pi}\int_0^\infty dy\int_0^1\frac{x^3dx}{\{x^2+4y^2[1+(x^2+y^2+\delta^2)^{-1}]\}^2} . \tag{34}$$

Equation (33) shows that a sufficiently large pinning is required to destroy the orientational hexatic order in a 3D flux line lattice. At $2/\kappa^2\ll b\ll 1$ the exponent in Eq. (33) is proportional to $1/\sqrt{B}$, so that a stronger field favors a greater orientational order.

V. RANDOM ORIENTATIONAL FIELD

In a phenomenological description, Eq. (6) can be derived from the effective free energy of the bond-angle field,

$$F_{\mathrm{eff}}=\frac{1}{2}\int d^3r[C_{66}(\partial_\alpha\theta_b)^2+C_{44}(\partial_z\theta_b)^2] -\frac{1}{2}\int d^3r\,\epsilon_{\alpha\beta}f_\alpha\partial_\beta\theta_b . \tag{35}$$

Here the second term produces screw deformations of the flux-line lattice, while the first elastic term resists such deformations. Competition between these two effects is responsible for the orientational order of the ground state.

As has been shown in Secs. III and IV, a weak random force $\mathbf{f}(\mathbf{r})$, while destroying the long-range positional order in the lattice, cannot destroy the long-range orientational order. This may be easily understood by comparing Eqs. (2) and (35). According to these equations, $\mathbf{f}(\mathbf{r})$, while interacting directly with the displacement field $\mathbf{u}$, interacts only with the derivative of the bond-angle field θ. Correspondingly, the interaction between $\mathbf{f}$ and θ becomes negligible at small q and, therefore, does not affect long-range orientational order. The effect of a random field which acts directly on the bond-angle field would be different.[11] Such a field can be incorporated by adding a new term to F_{eff},

$$F'_{\mathrm{eff}}=F_{\mathrm{eff}}-\frac{1}{2}\int d^3r\,h\cos^2[6\theta_b(\mathbf{r})-\phi(\mathbf{r})] , \tag{36}$$

where the random field $\phi(\mathbf{r})$ singles out the preferred set of local crystallographic axes. When studying the effect of this field on the long-range order, the second term in Eq. (35) may be omitted and the elastic moduli may be replaced by their constant values at $q=0$. Then the effective free energy of the bond-angle field becomes

$$F'_{\mathrm{eff}}=\frac{1}{2}\int d^3r[(\tilde{\nabla}\theta_b)^2-h\cos^2(6\theta_b-\phi)] , \tag{37}$$

where $\tilde{\nabla}=(c_{66}^{1/2}\nabla_\perp,c_{44}^{1/2}\partial_z)$. Now the problem is the same as for a 3D XY ferromagnet with random anisotropy. The latter problem has been intensively studied.[17] It has been shown that correlations decay exponentially with a correlation length $R_c\propto h^{-2}$. There is strong reason to believe that this effect is small in oxide superconductors, so that R_c is much greater than $\xi_\perp$ and $\xi_\parallel$ of Sec. III. Indeed, if the size of the pinning centers and the average distance between them are small in comparison with the flux lattice spacing, then the cumulative effect of pinning is the generation of a random force $\mathbf{f}(\mathbf{r})$ whose direction and strength changes smoothly along any path through the lattice.[8] The rotational effect of such a force then reduces to $\epsilon_{\alpha\beta}\partial_\alpha f_\beta$, which is equivalent to the second term in Eq. (35), and cannot destroy the long-range orientational order. A weak random field that acts directly on θ, and is not reduced to $\epsilon_{\alpha\beta}\partial_\alpha f_\beta$ is not forbidden by symmetry, however. In particular, it may be generated by the intrinsic chirality of some pinning centers, e.g., screw dislocations in the atomic lattice. For one or another reason, such a field may emerge in a real sample and may eventually destroy hexatic correlations at large distances.[11]

VI. THIN FILMS

Let the magnetic field be normal to a thin ferromagnetic film. If the thickness of the film is small in comparison with $\xi_\parallel$ of Sec. III, the problem becomes effectively two dimensional. As usual, one should expect that the effect of disorder in 2D is stronger than in 3D. Indeed, in the bulk sample any deformation of the vortex lattice in a cross section normal to the field must adjust itself to the deformations in neighboring cross sections, to avoid large bending of the flux lines. This constraint is absent in the film, where the order in the vortex lattice must, therefore, decay faster than in a bulk sample.

In two dimensions Eqs. (16) and (30) become

$$g_{\mathrm{G}}(\mathbf{r})=\exp\left[-\frac{W'}{2}\int\frac{d^2q}{(2\pi)^2}|K|^2\right], \tag{38}$$

$$g_6(\mathbf{r})=\exp\left[-\frac{W'}{2}\int\frac{d^2q}{(2\pi)^2}|\mathbf{Q}|^2\right], \tag{39}$$

where

$$|\mathbf{K}|^2=2[1-\cos(\mathbf{q}\cdot\mathbf{r})]\left[\frac{(\mathbf{G}\cdot\mathbf{q})^2}{C_{11}^2q^6}+\frac{1}{C_{66}^2q^4}\left[\mathbf{G}^2-\frac{(\mathbf{G}\cdot\mathbf{q})^2}{q^2}\right]\right], \tag{40}$$

$$|\mathbf{Q}|^2=18\frac{1-\cos(\mathbf{q}\cdot\mathbf{r})}{C_{66}^2q^2}, \tag{41}$$

W' is a constant in the expression for the probability for the random force distribution,

$$P[\mathbf{f}(\mathbf{r})]\propto\exp\left[-\frac{1}{2W'}\int d^2r[\mathbf{f}(\mathbf{r})]^2\right]. \tag{42}$$

The integral in Eq. (38) diverges at $q\rightarrow 0$, and, thus, must be cut off at $q\sim 2\pi/L$, where L is the linear size of the film. The first term in Eq. (40) may be neglected at small q. Then integration gives

$$g_G(\mathbf{r})=\exp\left[-\frac{r^2}{\xi^2}\ln\frac{L}{r}\right]=\left[\frac{r}{L}\right]^{(r/\xi)^2}, \tag{43}$$

where

$$\xi=\frac{(8\pi)^{1/2}C_{66}}{G(W')^{1/2}(1+2\sin^2\theta_b)^{1/2}}. \tag{44}$$

In a weak field ξ/a must be proportional to $\sqrt{B}$, while ξ does not depend on the field. The two extreme correlation lengths in a film, at $\theta_b=0$ and $\theta_b=\pi/2$, differ by a factor of $\sqrt{3}$.

For the orientational correlation function, the integration in Eq. (39) gives

$$g_6(\mathbf{r})=\exp\left[-\frac{9W'}{2\pi C_{66}^2}\ln(\tfrac{1}{2}rq_{\rm BZ})\right]=\left[\frac{2}{rq_{\rm BZ}}\right]^{9W'/2\pi C_{66}^2}. \tag{45}$$

Equation (45) is valid for $rq_{\rm BZ}\gg 1$. It displays the algebraic decay of hexatic order in two dimensions, with an exponent that decreases as $1/\sqrt{B}$ in a weak field.

In two dimensions, the exponent governing the algebraic decay of hexatic order, $\eta=9W'/2\pi C_{66}^2$ can be expressed in terms of the positional correlation length,

$$\eta=\frac{27}{4\pi^2}\left[\frac{a}{\xi_0}\right]^2, \tag{46}$$

where ξ_0 is ξ of Eq. (44) at $\theta=0$. Remarkably, this simple formula does not contain any other parameters of the theory and may be directly tested experimentally.

VII. DISCUSSION

Recent experimental data on the high-T_c superconductor Bi-Sr-Ca-Cu-O are in agreement with the theoretical suggestion that, in presence of collective pinning, the low-temperature, low-field phase of the magnetic flux lattice is the HVG. It has been experimentally demonstrated[13] that while the positional correlations decay on the scale of a few lattice constants, orientational order persists for hundreds of lattice constants. To explain the observed slow decay of orientational correlations, it has been assumed[13] that hexatic order decays either exponentially with a correlation length $R_c\sim 250a$, or algebraically with an exponent $\eta\sim 0.06$. If the latter is true, it would indicate that the vortex lattice is effectively two dimensional. Indeed, algebraic decay occurs at marginal dimensionality, which is 2D for orientational order.[12,18] Equation (46) provides a simple test of this assumption. For $\xi_\perp\sim 3.5a$ it gives $\eta_6=0.06$ which is consistent with experiment. On the other hand, the 2D limit is true when $\xi_\parallel$ is greater than the thickness of the film, ~ 100 μm in the experiment. It would imply $\xi_\parallel\sim 100\xi_\perp$ which, according to Sec. III, is hardly possible at $B<100G$. Thus, a 3D picture with a slow exponential decay of hexatic correlations due to the effects discussed in Sec. V, may be more plausible. It should be noted, however, that the experimental situation was quite close to the crossover from 3D to 2D. Experiments with samples of greater thickness and the analysis of larger images may be necessary to clarify this issue.

Comparison between theoretical and experimental results on the angular structure and field dependence of the HVG phase might also be a challenging task. For a 3D sample, the situation when the magnetic field is normal to the plane of decoration corresponds to the positional correlation function $g_G(r_\perp)=\exp(-r_\perp/\xi_\perp)$ given by Eq. (21). This dependence of g_G on $r_\perp$ is in good agreement with the experimental data, also indicating a 3D nature of correlations rather than 2D. Varying the orientation of the field with respect to the surface of the sample, one can also test in decoration experiments a complex dependence of g_G on $r_\perp$ and z, given by the general formula, Eq. (20). This has not yet been done. A systematic study of the field dependence of positional and orientational functions would allow one to test theoretical predictions for the dependence of $\xi_\perp$, $\xi_\parallel$, and g_6 on B, which may be important for understanding the nature of pinning forces. The angular dependence of the correlation length, $\xi_\perp\propto(1+\sin^2\theta_b)^{-1}$ in 3D, and $\xi\propto(1+2\sin^2\theta_b)^{-1/2}$ in 2D, may also be tested experimentally. According to the existing data,[13] the smallest and the greatest positional correlation lengths differ by a factor of 2, which is in agreement with the above formula for $\xi_\perp(\theta_b)$, indicating again a 3D nature of correlations.

Our treatment of positional and orientational order in flux-line lattices is based upon the concept of collective pinning,[16,7] which assumes a large concentration of weak pinning centers. Observation of the HVG phase in Bi-Sr-Ca-Cu-O (Ref. 13) has demonstrated the relevance of this concept to high-T_c superconductors. Comparison of our results with experimental data on oxide superconductors must be taken with caution, since our study ignores their pronounced anisotropy. It should be noted, however, that expressions for anisotropic elastic moduli are not established well enough to give confidence to any other approach. On the contrary, experiments on 3D and 2D amorphous superconductors[19–21] have demonstrated a high degree of isotropy of elastic properties and homogeneity of the pinning field at the scale of the superconducting coherence length. Consequently, the concept of collective pinning and all our results on the HVG phase may directly apply to amorphous superconductors.

Note added. After this work was submitted, I received unpublished results by Houghton, Pelcovits, and Subdø[22] who performed the same calculations, but with nonlocal anisotropic elastic moduli.

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