

## Vortex pinning by twin boundaries in copper oxide superconductors

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We investigate the pinning properties of twin boundaries in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  single crystals within a collective-pinning theory. For small angles  $\vartheta$  between the magnetic field and the twin boundaries, the vortex lattice adjusts to the array of twinning planes. Above the bulk depinning temperature  $T_p$ , the pinning properties of the twin boundaries are enhanced as compared with the bulk pinning. The combined effects lead to an increased pinning of the vortex lattice at small angles  $\vartheta$ , explaining the narrow angular response observed recently in the resistive transition and in torque measurements.

Recently, the question of the interplay between vortex pinning and twin boundaries in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  has attracted a lot of interest.<sup>1-3</sup> Kwok *et al.*<sup>2</sup> observed an enhanced pinning due to twin boundaries in resistivity measurements on an  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  single crystal. Their results have been confirmed with torque measurements by Gyorgy *et al.*<sup>3</sup> Besides their importance for the pinning properties of the material, the twinning planes (TP) provide a testing ground for the theory of vortex pinning in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ . In this paper we attempt to explain the experimental findings of Kwok *et al.*<sup>2</sup> and of Gyorgy *et al.*<sup>3</sup> within the framework of a collective-pinning theory. We first briefly collect the experimental results and then present a model describing the pinning properties of the TP.

Recent experiments on single crystals of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  reveal a kink in the resistive transition (i.e., a sharp change in  $\partial\rho/\partial T$  at a temperature  $T_K < T_c$ ) for a magnetic field applied perpendicularly to the current-flow direction. A particularly sharp kink has been observed by Kwok *et al.*<sup>2</sup> for the case where both the magnetic field and current are in the  $a$ - $b$  plane of the crystal. For a magnetic field of 1.5 T, the kink is  $\approx 1$  K below the zero-field critical temperature [ $T_c(H=0 \text{ T})=92.5 \text{ K}$ ] and separates the resistive transition into an Ohmic regime above  $T_K=91.6 \text{ K}$  and a non-Ohmic part below  $T_K$ . Upon lowering the temperature below  $T_K$ , a sharp drop appears in the resistivity whenever the applied magnetic field is aligned with the twin boundaries. In the following we attempt to model the effects of the TP on the pinning properties of the crystal. First, we determine the equilibrium configuration of the vortex lattice in the presence of an array of attracting TP. Next, we discuss the effects of point-pinning centers and analyze the dynamic properties

of a vortex in an applied force field within a collective-pinning theory. We will demonstrate that the enhanced pinning by twin boundaries is a *dimensional effect*.

To begin with, we consider a strongly simplified model. We assume the twin boundaries to form a periodic lattice (with spacing  $d$ ) and the external field  $\mathbf{H}$  to lie in the  $a$ - $b$  plane, enclosing an angle  $\vartheta$  with the TP. Furthermore, we neglect all effects of point-pinning centers for the time being. The TP attract the vortices because of the suppression of the order parameter<sup>4</sup> and each vortex deforms in order to adjust itself to the twinning lattice (see Fig. 1). For small angles  $\vartheta$ , the vortices will follow the TP over a distance  $r$  and point along the external-field direction only in the average over one period. First we consider small magnetic fields such that we can neglect the interaction between the vortices. The line tension for a vortex lying in the  $a$ - $b$  plane is<sup>5</sup>

$$\varepsilon_l = (\Phi_0/4\pi\lambda_{ab})^2 \ln(\kappa\sqrt{\Gamma})/\sqrt{\Gamma},$$

where  $\Gamma = m_c/m_{ab}$  denotes the mass-anisotropy factor,  $\lambda_{ab}$  is the magnetic-penetration depth in the  $a$ - $b$  plane, and  $\kappa = \lambda_{ab}/\xi_{ab}$  is the Ginzburg-landau parameter ( $\xi_{ab}$  = coherence length). Due to the suppression of the order parameter, the line tension is reduced to  $\varepsilon_l - \Delta\varepsilon_l$ , with  $\Delta\varepsilon_l > 0$ , within the TP. The energy difference (per period) between an adjusted and a straight vortex ( $r=0$ ) is

$$E(r, \vartheta) = (r+s-t)\varepsilon_l - r\Delta\varepsilon_l \\ \approx \frac{1}{2}\varepsilon_l \left[ \frac{1}{4} - \frac{\vartheta}{d} \right]^{-1} \vartheta^2 - r\Delta\varepsilon_l \quad (\vartheta \rightarrow 0).$$

The lowest-energy configuration for the vortex is found

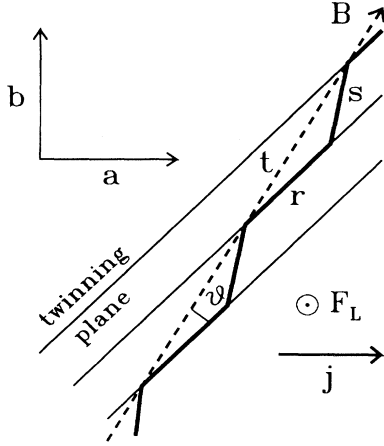


FIG. 1. Deformation of a single vortex by an attractive twin boundary potential. A current  $\mathbf{j}$  pointing along the  $a$  axis exerts a Lorentz force  $\mathbf{F}_L$  parallel to the  $c$  axis, leading to vortex motion within the plane above the depinning temperature  $T_p$ .

by minimizing  $E(r, \vartheta)$  with respect to  $r$  (with  $\vartheta$  fixed) and we obtain the result

$$r(\vartheta) = d \left[ \frac{1}{\vartheta} - \frac{1}{\vartheta^*} \right] \Theta(\vartheta^* - \vartheta),$$

$$E(r(\vartheta), \vartheta) = -\frac{1}{2} \varepsilon_l d \frac{(\vartheta^* - \vartheta)^2}{\vartheta} \Theta(\vartheta^* - \vartheta),$$

where the critical angle is given by  $\vartheta^* = \sqrt{2\Delta\varepsilon_l/\varepsilon_l} [\Theta(x)$  is the Heaviside unit-step function]. We find that, for small angles  $\vartheta < \vartheta^*$ , the vortex deforms in order to lower its energy (see Fig. 2). The vortex binding energy rapidly approaches its asymptotic value  $\Delta\varepsilon_l$  as  $\vartheta \rightarrow 0$ . For angles above  $\vartheta^*$ , the vortex remains straight,  $r(\vartheta > \vartheta^*) = 0$ . The crucial parameter determining the critical angle  $\vartheta^*$  is the relative lowering of the line tension in the TP,  $\Delta\varepsilon_l/\varepsilon_l$ . Using the decoration experiments of Dolan *et al.*,<sup>6</sup> we can determine  $\Delta\varepsilon_l/\varepsilon_l$  for  $\mathbf{H} (= 40 \text{ G}) \parallel c$  at  $T_1 = 4.2 \text{ K}$ :

$$\Delta\varepsilon_l \approx V(a_0^{\text{TP}}) - V(a_0^b).$$

Here

$$V(r) = 2(\Phi_0/4\pi\lambda_{ab})^2 K_0(r/\lambda_{ab})$$

is the interaction energy between vortices and  $a_0^{\text{TP}} \approx 0.75 \mu\text{m}$  and  $a_0^b \approx 1.3 \mu\text{m}$  are the nearest-neighbor distances between the vortices within the TP and in the bulk, respectively, as extracted from Fig. 4 of Ref. 6. From the value

$$(\Delta\varepsilon_l/\varepsilon_l)_{\text{H} \parallel c}(T_1 = 4.2 \text{ K}) \approx 2 \cdot 10^{-3},$$

we can find an estimate for

$$(\Delta\varepsilon_l/\varepsilon_l)_{\text{H} \parallel c}(T_2 = 90 \text{ K}),$$

assuming a suppression of the order parameter<sup>4,7</sup> to be at the origin of the attractive interaction  $\Delta\varepsilon_l$ . First, turning the field  $\mathbf{H}$  into the  $a$ - $b$  plane, a factor  $\ln\kappa/\ln(\kappa\sqrt{\Gamma}) \approx 0.7$  is picked up. Second, the suppression of the order parameter

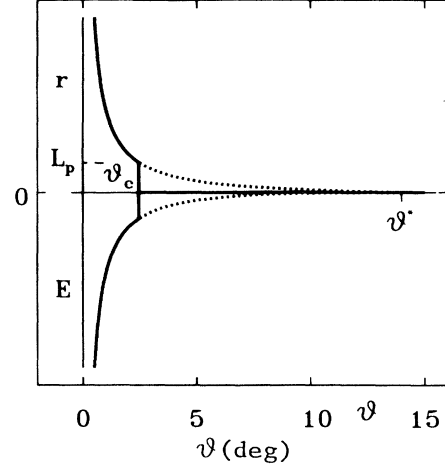


FIG. 2. Deformation  $r$  and pinning potential  $E$  of a vortex for a field enclosing an angle  $\vartheta$  with the TP. For angles  $\vartheta > \vartheta^*$ , the vortex remains aligned with the field everywhere. The pinning properties of the planes are only effective for a sufficiently large deformation,  $r(\vartheta) > L_p$ , where  $L_p$  is the pinning length in the planes. This reduces the critical angle for the observation of enhanced pinning by twin boundaries to the value  $\vartheta_c$ .

parameter increases with temperature  $\propto \xi^{-1}(T)$  and, hence,

$$\Delta\varepsilon_l = \varepsilon_{\text{cond}}^b - \varepsilon_{\text{cond}}^{\text{TP}} \propto [(1-t) - (1-t)^2] = t(1-t),$$

$t = T/T_c$ . With  $\varepsilon_l \propto (1-t)$ , we obtain the result

$$(\Delta\varepsilon_l/\varepsilon_l)_{\text{H} \parallel c}(T_2 = 90 \text{ K}) \approx 0.03.$$

The limit where the magnetic pinning is less than the maximal core pinning is given by

$$\Delta\varepsilon_l/\varepsilon_l < [4 \ln(\kappa\sqrt{\Gamma})]^{-1} \approx 0.04,$$

thus, our result is consistent with the above assumption of core pinning. The resulting critical angle is  $\vartheta^* \approx 14^\circ$ .

Next we consider an arbitrary strength for the magnetic field, supposing that  $l, d < \lambda_c$  with  $l = (\sqrt{3}\Gamma\Phi_0/2H)^{1/2}$  the mean distance between the vortices in the plane.<sup>8</sup> The critical angle  $\vartheta^*$  can be found from the balance between the energy gain due to trapping by TP and the cost of elastic energy,

$$\frac{1}{2} c_{44}(\mathbf{k}) \vartheta^{*2} \approx \frac{\Delta\varepsilon_l}{lh} f,$$

where

$$c_{44}(\mathbf{k}) \approx \frac{H^2}{4\pi} \frac{1}{(\lambda_c k_\perp)^2}, \quad H_{c1} \ll H \ll H_{c2}$$

is the tilt modulus for the field  $\mathbf{H}$  parallel to the  $a$ - $b$  plane,  $f$  is the fraction of vortices trapped by the TP, and  $h = \Phi_0/Bl$  is the vortex separation along the  $c$  axis. The wave vector  $k_\perp$  denotes the cutoff for transverse fluctuations of the vortices. For large fields,  $l \ll d$ , the trapped fraction is  $f \approx l/d$ , the cutoff  $k_\perp$  is given by the spacing  $d$  of the twin boundaries,  $k_\perp \approx \pi/d$ , and we obtain a reduction of the critical angle

$$\vartheta^* \simeq \left[ \frac{\pi \ln(\kappa\sqrt{\Gamma})}{2\sqrt{3}} \frac{2 \Delta \varepsilon_l}{\varepsilon_l} \right]^{1/2} \left[ \frac{l}{d} \right]^{3/2}.$$

With  $H \simeq 1.5$  T and assuming a distance<sup>2</sup>  $d \simeq 10^3$  Å between adjacent TP, we are at the crossover between the low- and high-field regime and the reduction of the critical angle is not yet effective.

Having discussed the equilibrium configuration of the vortex, we now study its dynamic properties in an applied force field. In the experiment by Kwok *et al.*,<sup>2</sup> the current applied along the  $a$  axis produces a Lorentz force pointing along the TP (see Fig. 1). We have to include the point-pinning centers in our model which prevent the vortex from moving in this force field. The mobility of the vortices in the TP is reduced as compared with the bulk: First, strain fields associated with the twin boundaries make them probable locations for atomic defects, leading to an increased density of pinning centers. Second, thermal fluctuations lead to a smoothening of the pinning potential above the depinning temperature  $T_p$  defined by the condition  $\langle u^2 \rangle (T_p) = \xi^2$  ( $u$  is the displacement of the vortex line from its equilibrium position). This smoothening is more effective for three-dimensional (3D) bulk pinning than for 2D pinning within the TP: For single-vortex pinning and  $T > T_p$ , the bulk critical current density drops exponentially with increasing temperature,<sup>9</sup>

$$j_c^{3D}(T) \sim j_c^{3D}(0) \exp[-(T/T_p)^3],$$

whereas the decrease is only algebraic for the critical current in the twin boundaries,<sup>10</sup>

$$j_c^{2D}(T) \sim j_c^{2D}(T_p) (T/T_p)^{-7},$$

where  $j_c^{3D}(0)$  and  $j_c^{2D}(T_p)$  denote the critical current densities in the bulk (at  $T=0$ ) and in the TP (at  $T=T_p$ ), respectively. Hence, a dimensional reduction of the vortex motion by trapping into the TP potential well enhances the barriers for motion at temperatures  $T > T_p$ . In  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ ,  $T_p \simeq 30$  K for a field of  $H \simeq 1$  T.<sup>9</sup>

The enhanced pinning properties of the TP will be experimentally manifest if the vortices move *within* the twinning planes and do not prefer to jump out of the twin boundaries into the bulk, where their motion is less hindered. First note that pinning within the TP becomes effective only if the trapped length of the vortex,  $r(\vartheta)$ , is larger than the collective-pinning length  $L_p$ .<sup>11</sup> Hence, the critical angle where the pinning properties of the TP become relevant is reduced to

$$\vartheta_c = \vartheta^* / (1 + \vartheta^* L_p / d),$$

where  $\vartheta^*$  has to be taken in rad. The pinning length  $L_p$  is not easy to determine accurately, but a reasonable estimate<sup>12</sup> for the situation in the experiment of Kwok *et al.* is  $L_p \simeq 50a_0$ , where  $a_0 = \sqrt{\Phi_0/B}$ . With  $d \sim 10^3$  Å we obtain a critical angle  $\vartheta_c \simeq 2.5^\circ$ .

Let us now study the motion of the vortex under the influence of the Lorentz force produced by the applied current density  $j$ . The thermally activated motion of the vortex along the TP will proceed in a sequence of optimal

jumps<sup>13</sup> with a length  $L_{\text{opt}}^{2D}$  which depends on the applied current density,  $L_{\text{opt}}^{2D} = L_p (j_c^{2D}/j)^\mu$ ,  $\mu = \frac{3}{4}$ . Here,  $j_c^{2D}$  denotes the critical current density of the TP. The energy barrier  $E_B^{\text{in}}$  for this jump within the TP is given by the energy difference of two neighboring metastable states in the random-pinning environment,

$$E_B^{\text{in}} = U (L_{\text{opt}}^{2D} / L_p)^{1/3}$$

[ $U$  denotes the pinning energy and  $r(\vartheta) > L_{\text{opt}}^{2D} > L_p$ ]. On the other hand, the vortex could also jump out of the plane and move in the region between the TP, where the pinning is reduced. The vortex then moves in the bulk in a sequence of optimal jumps<sup>13</sup> with

$$L_{\text{opt}}^{3D} = L_p (j_c^{3D}/j)^{\mu'},$$

$\mu' \simeq \mu$ . The barrier for a minimal segment of length  $L_{\text{opt}}^{3D}$  against jumping out of the plane is  $E_B^{\text{out}} = \Delta \varepsilon_l L_{\text{opt}}^{3D}$ . For large current densities  $j$ , such that  $E_B^{\text{in}} \geq E_B^{\text{out}}$ , the vortex prefers to move between the TP where it is less pinned. The criterion for this dynamic instability is

$$j \geq j^{\text{esc}} \simeq j_c^{2D} \left[ \frac{j_c^{3D}}{j_c^{2D}} \right]^{3/2} \left[ \frac{L_p \Delta \varepsilon_l}{U} \right]^2.$$

To estimate the energy ratio

$$L_p \Delta \varepsilon_l / U = L_p (\Delta \varepsilon_l / \varepsilon_l) \varepsilon_l / U,$$

we express the line energy  $\varepsilon_l$  by the pinning energy  $U$  and the pinning length  $L_p$ ,  $\varepsilon_l = UL_p / (\sqrt{\Gamma} \xi_c^2)$ , with  $\xi_c$  the coherence length along the  $c$  axis. We obtain

$$L_p \Delta \varepsilon_l / U = (\Delta \varepsilon_l / \varepsilon_l) (L_p / \xi_{ab})^2.$$

Our analysis applies to the situation of single-vortex pinning which is restricted to low fields and low temperatures (for  $H = 1.5$  T the temperature range is estimated<sup>13</sup> to be  $T < 40$  K). At the crossover between single-vortex pinning and collective lattice pinning, a consistent set of parameters is  $L_p \simeq a_0 \simeq 360$  Å,  $\Delta \varepsilon_l / \varepsilon_l = 0.02$ , and  $\xi_{ab} \simeq 30$  Å and we obtain the result  $L_p \Delta \varepsilon_l / U \simeq 3$ . This ratio measures the relative importance of the TP pinning and the pinning by point defects. Since  $L_p \Delta \varepsilon_l / U \rightarrow 0$  for  $T \rightarrow 0$ , we find that there exists a crossover temperature above which the pinning potential becomes dominated by the TP, leading to the trapping of the vortex into the TP potential well below the critical angle  $\vartheta_c$  calculated above. The factor  $(j_c^{3D}/j_c^{2D})^{3/2}$  accounts for the reduced pinning in the bulk. For a reduction<sup>3</sup>  $j_c^{3D}/j_c^{2D} \simeq 0.2$ , we find that  $j^{\text{esc}} \simeq j_c^{2D}$ . The experiments by Kwok *et al.*<sup>2</sup> have been performed at high temperatures in the regime of collective lattice pinning. The pinning length  $L_p$  grows rapidly with temperature above the depinning temperature  $T_p$  and we expect the ratio  $L_p \Delta \varepsilon_l / U$  to be even larger in this case. We conclude that, at high temperatures, the vortex remains pinned by the TP and the dimensional reduction remains effective for all experimentally relevant current densities. At small temperatures our model predicts a *dynamic instability* such that enhanced pinning by twin boundaries disappears for large current densities.

Let us now return to the experimental findings by Kwok *et al.*<sup>2</sup> (resistive transition) and by Gyorgy *et al.*<sup>3</sup> (torque measurements). The sharp drop in the resistivity for magnetic fields aligned with the direction of the twin boundaries can be understood in the light of the above model: As the angle between the field and the TP decreases below the critical angle  $\vartheta_c$ , the vortices become partially trapped by the TP and their net velocity is reduced by the enhanced pinning in the planes. The vortices are pinned collectively; thus, if a fraction of the vortices is strongly pinned, the motion of the whole lattice is reduced<sup>14</sup> and the resistivity drops. The smallest onset width of the resistivity drop in the experiment of Kwok *et al.*<sup>2</sup> is  $\sim 6^\circ$ . In our model we find an estimate for the onset width  $2\vartheta_c \simeq 5^\circ$  ( $H \simeq 1.5$  T,  $d \simeq 10^3$  Å,  $\Delta\epsilon_l/\epsilon_l \simeq 0.03$ ), which compares well with the experimental results.

As discussed above, the drop in the resistivity for  $\mathbf{H}$  aligned with the twin boundaries is a consequence of their enhanced pinning strength. Hence, this drop is expected to disappear above the temperature where pinning becomes ineffective. The transition to the regime of flux flow has been described as proceeding in two stages:<sup>15</sup> The glass temperature  $T_g < T_K$  separates the region of collective creep from the region of thermally assisted flux flow (TAFF). Using the experiments by Koch *et al.*,<sup>16</sup> we estimate  $T_g(H = 1.5 \text{ T}) \simeq 85$  K.<sup>17</sup> Below  $T_g$  the vortex lattice is assumed to be pinned in a glassy state which is characterized by infinitely growing energy barriers  $E_B(j)$  ( $j \rightarrow 0$ ) for creep,  $E_B \sim j^{-\alpha}$ ,  $\alpha > 0$ , and the linear resistivity is zero.<sup>9</sup> Above  $T_g$ , the vortices can move in a force field but their motion is still restricted by the existence of large but finite barriers (plastic flow). The vortex motion still proceeds via thermally activated hops and the resistivity is non-Ohmic at large current densities and becomes Ohmic as  $j \rightarrow 0$  with an exponentially small resistivity. Finally, at  $T_K$ , all influence of pinning disappears, the vortices flow freely, and the transition becomes Ohmic. The above scenario for the depinning transition agrees with the experimental observation that the resistive drop for  $\vartheta < \vartheta_c$  disappears for temperatures  $T > T_K$ .

Additional support of the above model is provided by the torque measurements of Gyorgy *et al.*<sup>3</sup> For magnetic fields parallel to the TP, a sixfold increase is observed in the torque as compared with the background. The peaks (width at onset  $10^\circ$ – $15^\circ$ ) disappear at low temperatures ( $T = 27$  K). Assuming a critical-state model for the current flow producing the magnetic moment, we have to consider the two critical current densities  $j_c^{abc}$  and  $j_c^c$  flowing in the planes and along the direction of the  $c$  axis,

respectively (we use the notation of Ref. 3). As the magnetic field points along the TP,  $j_c^{abc}$  produces a Lorentz force pushing the vortices in the direction of the planes and thus  $j_c^{abc}$  is increased by the enhanced pinning properties of the twin boundaries as described above. The current density flowing parallel to the  $c$  axis produces a force which tends to push the vortices out of the TP potential well. The effective trapping force decreases slowly with temperature above  $T_p$  [ $\sim T^{-1}$ , compared with  $j_c^{2D,3D}(T)$  above] and thus the critical current  $j_c^c$  will be enhanced due to the trapping ( $\sim \Delta\epsilon_l$ ) of the vortices by the twin boundaries if  $\vartheta < \vartheta^*$ . This increase of the critical currents  $j_c^{abc}$  and  $j_c^c$  by the TP explains the observed peaks in the measurements. Upon cooling, the relative importance of the TP pinning decreases as compared with the pinning by point defects as shown above. Below the depinning temperature, the vortices are pinned in the bulk and the contribution of the twin boundaries becomes unobservable.

In summary, we have described a model which explains the enhanced pinning produced by twin boundaries within the framework of a collective-pinning theory. We have found that, at small angles  $\vartheta < \vartheta_c$ , the vortex lattice accommodates to the lattice of twinning planes, resulting in a decreased resistivity<sup>2</sup> or in an enhanced critical current density,<sup>3</sup> depending on the experimental situation. These improved superconducting properties are a dimensional effect which is limited to the temperature regime between the depinning temperature  $T_p$  and the temperature of free flux flow,  $T_K$ , where the pinning strength in the twinning planes is enhanced over the bulk pinning. For low temperatures we have found a dynamic instability where the pinning by twinning planes disappears for high current densities. In our model we have studied the lowest-energy configuration of the vortices. At finite temperatures one has to consider a statistical distribution of configurations involving also higher energies, leading to a smearing of the transition at  $\vartheta_c$ . We have based our analysis on the theory of collective pinning and have found good agreement with experiment. It is an open problem of whether or not single-impurity pinning can also provide a similarly consistent picture.

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