Reversible magnetization and torques in anisotropic high-κ type-II superconductors

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The reversible magnetization of anisotropic high-κ type-II superconductors in an applied field **H** of arbitrary orientation with respect to the principal axes of the sample is considered in the framework of the Ginzburg-Landau theory with an anisotropic effective mass. We examine the procedure of obtaining the free-energy density F from its corresponding expression in the isotropic case by simply replacing the Ginzburg-Landau parameter κ by a $\tilde{\kappa}$ that depends on the orientation of **H** relative to the principal axes. This procedure is valid when **H** is along one of the principal axes for arbitrary values of H between H_{c1} and H_{c2} and is also valid to a good approximation when **H** is not along one of the principal axes, but only when $H \gg H_{c1}$. Because of the dependence of F on the orientation of **H**, when **H** is not parallel to one of the principal axes, the average magnetic-flux density **B** is not parallel to **H**, and a torque associated with the transverse magnetization exists, tending to orient the sample so that the value of $\tilde{\kappa}$ is the largest. Expressions for the magnetization and the torque are obtained from a variational model that permits the analytic calculation of F in the Ginzburg-Landau regime including, in addition to the supercurrent kinetic energy and the magnetic-field energy, the kinetic-energy and the condensation-energy terms arising from suppression of the order parameter in the vortex core. It is also pointed out that a comparison of the present theory with torque measurements can provide a way to estimate the upper critical field $H_{c2}(\theta,\phi)$, the thermodynamic field H_c, and the ratio $m_1{:}m_2{:}m_3$ (m_i, $i=1,2,3$, are the principal values of the effective-mass tensor m_{ij}) in the temperature region where the Ginzburg-Landau theory is appropriate.

I. INTRODUCTION

The goal of this paper is to derive expressions for the reversible magnetization and torques in anisotropic high-κ type-II superconductors. Although our theoretical starting point is the anisotropic Ginzburg-Landau theory, analytic results can be obtained only near the upper critical field. We therefore use a physically motivated model that enables us to obtain results for the reversible magnetization and torques. These results, despite their relative simplicity, should be valid to good approximation over a wide range of temperature and magnetic field.

The effects of anisotropy on the magnetic properties of type-II superconductors can be most simply accounted for in the framework of the Ginzburg-Landau theory[1] by introducing a phenomenological effective-mass tensor M_{ij},[2-7] which has the principal values M_i ($i=1,2,3$). It is convenient to define a normalized mass tensor $m_{ij}=M_{ij}/\overline{M}$ with principal values $m_i=M_i/\overline{M}$, where the mean mass $\overline{M}=(M_1M_2M_3)^{1/3}$; then $m_1m_2m_3=1$.

In Ref. 8 the reversible magnetization is considered for an isotropic type-II superconductor and for an anisotropic type-II superconductor when the applied field **H** is oriented along one of the principal axes of the sample, in the entire field range between the lower critical field H_{c1} and the upper critical field H_{c2}. In this paper we consider an anisotropic type-II superconductor for the general case that **H** is applied along an arbitrary direction with respect to the principal axes. For an infinite sample in the mixed state, the Ginzburg-Landau free energy per unit volume over cross-sectional area A in an plane perpendicular to the vortices, measured relative to that of the Meissner state, can be expressed in dimensionless form as[1,8-10]

$$F=F_c+F_{kg}+F_{kj}+F_f \ , \tag{1}$$

where

$$F_c=\frac{1}{A}\int d^2\rho \tfrac{1}{2}(1-f^2)^2 \ , \tag{2}$$

$$F_{kg}=\frac{1}{A}\int d^2\rho \frac{1}{\kappa^2} m_{ij}^{-1}(\partial_i f)(\partial_j f) \ , \tag{3}$$

$$F_{kj}=\frac{1}{A}\int d^2\rho\, f^2 m_{ij}^{-1} a_{si}a_{sj}, \quad \left[\mathbf{a}_s=\mathbf{a}+\frac{1}{\kappa}\nabla\gamma\right] \ , \tag{4}$$

and

$$F_f=\frac{1}{A}\int d^2\rho\, b^2 \tag{5}$$

are the *condensation* energy, *kinetic* energy associated with *gradients* in the magnitude of order parameter, *kinetic* energy associated with *supercurrent*, and *magnetic-field* energy; f and γ are the normalized magnitude and phase of the order parameter $\Psi=\Psi_0 f e^{i\gamma}$ (Ψ_0 is the magnitude of the order parameter in the absence of a field); κ is the Ginzburg-Landau parameter; m_{ij}^{-1} is the normalized inverse mass tensor ($m_{ij}m_{jk}^{-1}=\delta_{ik}$), $\partial_i\equiv\partial/\partial x_i$; **a** is the vector potential satisfying $\nabla\cdot\mathbf{a}=0$; $\mathbf{b}=\nabla\times\mathbf{a}$ is the local magnetic-flux density; and the two-dimensional integral is taken over A. The convention of summing over repeated indices is employed.

Here the dimensionless units correspond to measuring the magnitude of order parameter in units of Ψ_0, length in units of the mean penetration depth $\lambda=(\overline{M}c^2/16\pi e^2\Psi_0^2)^{1/2}$, magnetic field in units of $\sqrt{2}H_c$, vector potential in units of $\sqrt{2}H_c\lambda$ and energy in units of $H_c^2/4\pi$, where H_c is the thermodynamic critical field. The mean coherence length ξ and κ are expressed in terms of $\overline{M}$ by the usual relations $\xi=\phi_0/2\sqrt{2}\pi\lambda H_c$ and $\kappa=\lambda/\xi$, where $\phi_0=hc/2e=2.07\times10^{-7}$ G cm^2 is the flux quantum (ϕ_0 corresponds to $2\pi/\kappa$ in the dimensionless expressions). The mean values of ξ and λ are related to the values $\xi_i=\xi/\sqrt{m_i}$ and $\lambda_i=\lambda\sqrt{m_i}$ for spatial variation of order parameter and supercurrent, respectively, along the principal directions i $(i=1,2,3)$ via $\xi=(\xi_1\xi_2\xi_3)^{1/3}$ and $\lambda=(\lambda_1\lambda_2\lambda_3)^{1/3}$.

The second Ginzburg-Landau equation is

$$j_i=-f^2 m_{ij}^{-1} a_{sj} \ , \tag{6}$$

where $\mathbf{j}$ is the supercurrent density.

For a vortex centered on the x_3 axis, in terms of cylindrical coordinates (ρ,ϕ,x_3) we have $\nabla\gamma=-\nabla\phi$, $\mathbf{a}_s=\mathbf{a}-\hat{\boldsymbol{\phi}}(1/\kappa\rho)$, and therefore

$$\nabla\times\mathbf{a}_s=\mathbf{b}-\hat{\mathbf{x}}_3\frac{2\pi}{\kappa}\delta(\boldsymbol{\rho}) \ . \tag{7}$$

For an array of vortices at positions $\boldsymbol{\rho}_n$, we have

$$\nabla\times\mathbf{a}_s=\mathbf{b}-\hat{\mathbf{x}}_3\frac{2\pi}{\kappa}\sum_n\delta(\boldsymbol{\rho}-\boldsymbol{\rho}_n) \ , \tag{8}$$

where $\delta(\boldsymbol{\rho})$ is a two-dimensional δ function and each term in the summation represents one vortex carrying one flux quantum of mgnetic flux centered at $\boldsymbol{\rho}_n$. We choose hereafter a coordinate system such that the x_3 axis is parallel to the averaged magnetic flux density $\mathbf{B}$, i.e., $\hat{\mathbf{x}}_3=\hat{\mathbf{B}}$, so that $\mathbf{b}=\mathbf{b}(x_1,x_2)$.

In general the direction of $\mathbf{b}(\boldsymbol{\rho})$ is not a constant,[4,6] and $\mathbf{b}$ has a component transverse to $\mathbf{B}$, of which the average is zero (since the average of $\mathbf{b}$ is $\mathbf{B}$ by definition).

Using Eqs. (6) and (8), and with the help of the Ampère's law

$$\mathbf{j}=\nabla\times\mathbf{b} \tag{9}$$

and the divergence theorem, we find that the electromagnetic free energy per unit volume $F_{\rm em}=F_{kj}+F_j$ can be simply expressed as

$$F_{\rm em}=\mathbf{B}\cdot\mathbf{b}(0)=Bb_3(0) \ , \tag{10}$$

where $B=2\pi/\kappa A_{\rm cell}$, $A_{\rm cell}$ is the unit cell area of the two-dimensional flux-line lattice ($B=\phi_0/A_{\rm cell}$ in conventional units), and $\mathbf{b}(0)$ is the local magnetic-flux density at the center of a vortex.

Note that so far the discussion and the equations are the generalizations of the special cases that were considered in Ref. 8.

From Eqs. (6), (8), and (9) we obtain the coupled equations for the components of $\mathbf{b}$:

$$b_i=\epsilon_{iks}\epsilon_{jlt}m_{kl}\partial_s\left[\frac{1}{f^2}\partial_t b_j\right]+\delta_{i3}\frac{2\pi}{\kappa}\sum_n\delta(\boldsymbol{\rho}-\boldsymbol{\rho}_n) \ , \tag{11}$$

where ϵ_{ijk} is the Levi-Civitá symbol.

Since f is also an unknown in Eqs. (11), in principle one has to solve these equations and the first Ginzburg-Landau equation[1-3,7] simultaneously. Because of the nonlinearity, however, these equations cannot be solved analytically. The simplest approximation is the London model,[9,11] in which the order parameter is assumed constant, i.e., $f=1$. Although the London model can give a good qualitative account of the mixed state in the low and intermediate field regions, it suffers from its inability to account for the effects of the depression of the order parameter to zero at the vortex center, a deficiency that can produce a significant quantitative error in calculations of the magnetization. This point will be discussed in detail in a forthcoming paper.[12]

As an improvement of the London model, one of the authors proposed a variational model[13] for an isolated vortex, which reduces to the London model well outside the vortex core but has the added advantage of yielding realistic results in the vortex-core vicinity. This model was later extended to the case of a flux-line lattice.[8,14] It permits one to calculate the free energy analytically including, in addition to the term $F_{\rm em}$, the terms F_c and F_{kg} arising from the suppression of the order parameter in the vortex core, and is able to produce results that are not only qualitatively but also quantitatively good approximations to the solutions to the Ginzburg-Landau equations.[8,13,14]

In the next section we first use the London model to show that, when $H\gg H_{c1}$, the free-energy density of an anisotropic type-II superconductor in a field $\mathbf{H}$ that is arbitrarily oriented with respect to the principal axes of the sample can be obtained from its corresponding expression in the isotropic case by simply replacing κ by an angle-dependent $\tilde{\kappa}$ that depends on the orientation of $\mathbf{H}$. In Sec. III we then apply the variational model of Ref. 8 and calculate the magnetization, which is not parallel to $\mathbf{H}$ in general as a consequence of the dependence of F on the orientation of the flux-line lattice. In Sec. IV we evaluate the torque that arises from the transverse component of the magnetization and discuss its dependence upon the magnitude and the orientation of the applied field and upon the temperature. In Sec. V we summarize our results.

II. THE LONDON EQUATIONS

It is shown in Ref. 5 that the free-energy density and the Ginzburg-Landau equations can be transformed to isotropic forms by a simple transformation of variables if κ is replaced by $\tilde{\kappa}$ that depends on the orientation of the vortices with respect to the principal axes. This transformation was later shown to be valid only when the applied field $\mathbf{H}$ is along one of the principal axes[6,7] and to be approximately valid for $\kappa\gg1$ and $\mathbf{H}$ near $\mathbf{H}_{c2}$ when $\mathbf{H}$ is not along one of the principal axes.[7] In this section we show that this transformation is valid to good approximation for arbitrary orientation of $\mathbf{H}$ for the case that $\kappa\gg1$ and $H\gg H_{c1}$ (including intermediate and high fields). The London model is used for the derivation of our conclusion.

When $f=1$, Eqs. (11) reduce to the London equations,[6] which read

$$b_1=m_{33}\Delta b_1+(m_{23}\partial_1\partial_2-m_{13}\partial_2^2)b_3 , \quad (12)$$

$$b_2=m_{33}\Delta b_2+(m_{13}\partial_1\partial_2-m_{23}\partial_1^2)b_3 , \quad (13)$$

$$b_3=(m_{22}\partial_1^2-2m_{12}\partial_1\partial_2+m_1\partial_2^2)b_3-m_{13}\Delta b_1 -m_{23}\Delta b_2+\frac{2\pi}{\kappa}\sum_n\delta(\boldsymbol{\rho}-\boldsymbol{\rho}_n) , \quad (14)$$

where $\Delta=\partial_1^2+\partial_2^2$, $\nabla\cdot\mathbf{b}=0$, $\partial_z b_i=0$ and $m_{ij}=m_{ji}$. The quantity we want to solve for is $b_3(0)$ as a function of B, since $F_{\rm em}=Bb_3(0)$.

The system of Eqs. (12)–(14) can be solved by introducing the Fourier transforms

$$b_i(\boldsymbol{\rho})=\int\frac{d^2k}{(2\pi)^2}\tilde{b}_i(\mathbf{k})e^{i\mathbf{k}\cdot\boldsymbol{\rho}} \quad (i=1,2,3) , \quad (15)$$

and

$$\frac{2\pi}{\kappa}\sum_n\delta(\boldsymbol{\rho}-\boldsymbol{\rho}_n)=\frac{2\pi}{\kappa}\sum_n\int\frac{d^2k}{(2\pi)^2}e^{i\mathbf{k}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_n)} =\int\frac{d^2k}{(2\pi)^2}(2\pi)^2B\sum_{\mathbf{G}}\delta(\mathbf{k}-\mathbf{G})e^{i\mathbf{k}\cdot\boldsymbol{\rho}} . \quad (16)$$

Here $\mathbf{G}$ is the reciprocal lattice vector of the corresponding two-dimensional flux-line lattice, such that $e^{i\mathbf{G}\cdot\boldsymbol{\rho}_n}=1$, and we have used the relations

$$A_{\rm cell}\sum_n e^{-i\mathbf{k}\cdot\boldsymbol{\rho}_n}=(2\pi)^2\sum_{\mathbf{G}}\delta(\mathbf{k}-\mathbf{G}) \quad (17)$$

and $B=2\pi/\kappa A_{\rm cell}$. We find

$$\tilde{b}_3(\mathbf{k})=\frac{1+m_{33}k^2}{d(\mathbf{k})}(2\pi)^2B\sum_{\mathbf{G}}\delta(\mathbf{k}-\mathbf{G}) , \quad (18)$$

where

$$d(\mathbf{k})=(1+m_{22}k_1^2-2m_{12}k_1k_2+m_{11}k_2^2)(1+m_{33}k^2) -k^2(m_{23}k_1-m_{13}k_2)^2 , \quad (19)$$

and therefore

$$b_3(0)=\int\frac{d^2k}{(2\pi)^2}\tilde{b}_3(\mathbf{k}) =B\left[1+\sum_{\mathbf{G}\neq0}\frac{1+m_{33}G^2}{d(\mathbf{G})}\right] . \quad (20)$$

In all terms of the summation, because $|\mathbf{G}|_{\rm min}$ is the order of the inverse of the intervortex spacing L ($|\mathbf{G}|_{\rm min}\approx\lambda/L$ in conventional units), $G_{\rm min}^2$ is the order of κ^2B/H_{c2}. When $H\gg H_{c1}$, $G_{1,2}^2\gg1$, and we may expand Eq. (20) in powers of the small quantity G^{-2}, obtaining to first order[15]

$$b_3(0)\approx B\left[1+\sum_{\mathbf{G}\neq0}\frac{1}{\overline{m}_2G_1^2-2\overline{m}_{12}G_1G_2+\overline{m}_1G_2^2}\right] , \quad (H\gg H_{c1}) , \quad (21)$$

where

$$\overline{m}_1=m_{11}-m_{13}^2/m_{33} , \quad (22)$$

$$\overline{m}_2=m_{22}-m_{23}^2/m_{33} , \quad (23)$$

$$\overline{m}_{12}=m_{12}-m_{13}m_{23}/m_{33} . \quad (24)$$

We can diagonalize the denominators of the terms in the summation of Eq. (21) by choosing the axes x_1 and x_2 properly so that $\overline{m}_{12}=0$, or

$$m_{12}m_{33}-m_{13}m_{23}=0 . \quad (25)$$

We determine such axes in the Appendix. Then Eq. (21) becomes

$$b_3(0)\approx B\left[1+\sum_{\mathbf{G}\neq0}\frac{1}{\overline{m}_2G_1^2+\overline{m}_1G_2^2}\right], \quad (H\gg H_{c1}) . \quad (26)$$

We notice that Eq. (26) could have been obtained by solving the equation

$$b_3=(\overline{m}_2\partial_1^2+\overline{m}_1\partial_2^2)b_3+\frac{2\pi}{\kappa}\sum_n\delta(\boldsymbol{\rho}-\boldsymbol{\rho}_n) , \quad (27)$$

by the method of Fourier transformation under the same condition that $H\gg H_{c1}$. This proves the useful property that, for $H\gg H_{c1}$, the system of Eqs. (12)–(14) and Eq. (27) are equivalent in calculating $b_3(0)$, and therefore $F_{\rm em}$. Equation (27) is of the same form as that for the case when $\mathbf{B}$ is parallel to a principal axis, as can be seen from Eq. (14) when the coordinate axes are the principal axes, so that $m_{ij}=m_i\delta_{ij}$. This means that we may use the approximation

$$m_{ij}\approx\overline{m}_i\delta_{ij} \quad \text{for } H\gg H_{c1} , \quad (28)$$

where $\overline{m}_1$ and $\overline{m}_2$ are given by Eqs. (22) and (23),

$$\overline{m}_3=m_{33} , \quad (29)$$

and

$$\overline{m}_1\overline{m}_2\overline{m}_3=1 , \quad (30)$$

which can be shown using Eq. (25) and $\det(m_{ij})=1$, in analogy to the relation $m_1m_2m_3=1$. $\overline{m}_i$ $(i=1,2,3)$ depending on the orientation of $\mathbf{B}$, and reduce to m_i when $\mathbf{B}$ is aligned along a principal axis.

Using a simple transformation of variables,[5,8,16]

$$\tilde{x}_i=\sqrt{\overline{m}_3\overline{m}_i}\,x_i, \quad (i=1,2), \quad (31)$$

we can transform Eq. (27) into the isotropic form

$$b_3=(\tilde{\partial}_1^2+\tilde{\partial}_2^2)b_3+\frac{2\pi}{\tilde{\kappa}}\sum_n\delta(\tilde{\boldsymbol{\rho}}-\tilde{\boldsymbol{\rho}}_n) , \quad (32)$$

where

$$\tilde{\kappa}=\kappa/\sqrt{\overline{m}_3} \quad (33)$$

and $\delta(ax)=a^{-1}\delta(x)$.

It is expected from Eq. (32) that the final expressions for $b_3(0)$ and $F_{\rm em}$ are the same as those of the isotropic case, except that κ is replaced by $\tilde{\kappa}$. This means that the procedure of obtaining the free-energy density of an anisotropic superconductor from its isotropic counterpart by replacing κ by $\tilde{\kappa}$, a procedure which is valid for the case that **B** is parallel to one of the principal axes for arbitrary values of H between H_{c1} and H_{c2},[5,8] is also valid for the case that **B** is not parallel to one of the principal axes, but only if $H \gg H_{c1}$. Although this result was obtained using the London model, we expect it also to be valid in the context of the Ginzburg-Landau theory.

For H close to H_{c1}, when the intervortex spacing is comparable to or less than the penetration depth, the vortex interactions for the case that **B** is not parallel to one of the principal axes differ strongly from the case that **B** lies along a principal axis. For example, it has been argued[17–19] that parallel vortices in uniaxial layered superconductors ($m_1 = m_2 < m_3$) can attract each other when the vortices are not parallel to one of the principal axes and the intervortex spacing is of the order of the penetration depth.

III. THE VARIATIONAL MODEL AND THE REVERSIBLE MAGNETIZATION

In the following we consider only the case that $H \gg H_{c1}$, and simply apply the conclusion obtained above using the London model to the variational model in Ref. 8. Using the approximation of Eq. (28) in Eq. (11), we get

$$b_3 = \overline{m}_2 \partial_1 \left[\frac{1}{f^2} \partial_1 b_3 \right] + \overline{m}_1 \partial_2 \left[\frac{1}{f^2} \partial_2 b_3 \right] + \frac{2\pi}{\kappa} \sum_n \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_n), \tag{34}$$

and Eq. (3) becomes

$$F_{kg} = \frac{1}{A} \int d^2\rho \frac{1}{\kappa^2 \overline{m}_i} (\partial_i f)^2 . \tag{35}$$

Following Ref. 8, if we assume for the order parameter the trial function

$$f = \frac{\tilde{\rho}}{\sqrt{\tilde{\rho}^2 + \xi_v^2}} f_\infty \tag{36}$$

with

$$\tilde{\rho}^2 = \frac{x_1^2}{\overline{m}_2} + \frac{x_2^2}{\overline{m}_1} , \tag{37}$$

where ξ_v and f_∞ are the two variational parameters representing the effective core radius and the suppression of the order parameter due to overlapping of vortices, respectively, then by the simple transformation of Eq. (31) we can transform Eq. (2) for F_c, Eq. (35) for F_{kg}, and Eq. (34) for $b_3(0)$ and thus for F_{em} into the corresponding isotropic forms with κ replaced by $\tilde{\kappa}$. The final result for F is therefore obtained simply by replacing κ by $\tilde{\kappa}$ in the expression for the isotropic case obtained in Ref. 8, i.e.,

$$F_c = \tfrac{1}{2}(1 - f_\infty^2)^2 + \frac{B\tilde{\kappa}\xi_v^2 f_\infty^2}{2} \left[(1 - f_\infty^2) \ln \left[\frac{2}{B\tilde{\kappa}\xi_v^2} + 1 \right] + \frac{f_\infty^2}{2 + B\tilde{\kappa}\xi_v^2} \right], \tag{38}$$

$$F_{kg} = \frac{B f_\infty^2 (1 + B\tilde{\kappa}\xi_v^2)}{\tilde{\kappa}(2 + B\tilde{\kappa}\xi_v^2)^2} , \tag{39}$$

$$F_{\text{em}} = B^2 + \frac{B f_\infty K_0(\xi_v \sqrt{f_\infty^2 + 2B\tilde{\kappa}})}{\tilde{\kappa}\xi_v K_1(f_\infty \xi_v)} . \tag{40}$$

and F is the sum of F_c, F_{kg}, and F_{em}. For the details of the calculations, the reader is referred to Ref. 8.

Note that F depends on the orientation of **B** only through $\tilde{\kappa}$ or $\overline{m}_3$, which is found in the Appendix to be

$$\overline{m}_3 = m_1 \sin^2\theta \cos^2\phi + m_2 \sin^2\theta \sin^2\phi + m_3 \cos^2\theta , \tag{41}$$

where θ and ϕ are the polar and azimuthal angles of **B** with respect to the principal axes.

In principle the variational parameters f_∞ and ξ_v are determined as functions of B and $\tilde{\kappa}$ by solving

$$\frac{\partial F}{\partial f_\infty} = 0 \tag{42}$$

and

$$\frac{\partial F}{\partial \xi_v} = 0 \tag{43}$$

simultaneously, but this involves a significant amount of numerical analysis. Instead the following approximations can be used[8] for $\tilde{\kappa} \gg 1$:

$$f_\infty^2 = 1 - \left[\frac{B}{\tilde{\kappa}} \right]^4 , \tag{44}$$

$$\left[\frac{\xi_v}{\xi_{v0}} \right]^2 = \left[1 - 2 \left[1 - \frac{B}{\tilde{\kappa}} \right]^2 \frac{B}{\tilde{\kappa}} \right] \left[1 + \left[\frac{B}{\tilde{\kappa}} \right]^4 \right] , \tag{45}$$

where ξ_{v0} satisfes

$$\tilde{\kappa}\xi_{v0} = \sqrt{2} \left[1 - \frac{K_0^2(\xi_{v0})}{K_1^2(\xi_{v0})} \right]^{1/2} , \tag{46}$$

from which we see that $\tilde{\kappa}\xi_{v0} \approx \sqrt{2}$ for $\tilde{\kappa} \gg 1$. Note that $\tilde{\kappa} = H_{c2} = B_{c2}$ in the dimensionless units used here.

The thermodynamic magnetic field **H** is given by

$$\mathbf{H} = \tfrac{1}{2} \nabla_{\mathbf{B}} F , \tag{47}$$

and the magnetization **M** is

$$-4\pi \mathbf{M} = \mathbf{H} - \mathbf{B} . \tag{48}$$

Equations (47) and (48) give us the implicit function **M**(**H**). Note that **H** is the internal field, which is equal to the applied field only when the demagnetization effect can be neglected. For $\kappa \gg 1$ and $H \gg H_{c1}$, the demagnetization effect is unimportant since the magnetization is small compared with the applied field.

In terms of spherical coordinates, with unit vectors $\hat{\mathbf{B}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ we have

$$\mathbf{H}=(H_B,H_\theta,H_\phi)\ , \tag{49}$$

$$-4\pi\mathbf{M}=(H_B-B,H_\theta,H_\phi)\ , \tag{50}$$

where

$$H_B=\frac{1}{2}\frac{\partial F}{\partial B}\ , \tag{51}$$

$$H_\theta=\frac{1}{2B}\frac{\partial F}{\partial \theta}\ , \tag{52}$$

$$H_\phi=\frac{1}{2B\sin\theta}\frac{\partial F}{\partial \phi}\ . \tag{53}$$

We see that the longitudinal magnetization $-4\pi M_B=H_B-B$ is the same as that for isotropic case (except that κ is replaced by $\tilde{\kappa}$), which has been considered in Ref. 8, so we consider here only the transverse components $-4\pi M_\theta=H_\theta$ and $-4\pi M_\phi=H_\phi$. A straightforward calculation gives

$$H_\theta=-\frac{1}{4B}Q(B,\tilde{\kappa})\frac{(m_3-m_1\cos^2\phi-m_2\sin^2\phi)\sin2\theta}{\overline{m}_3}\ , \tag{54}$$

$$H_\phi=-\frac{1}{4B}Q(B,\tilde{\kappa})\frac{(m_1-m_2)\sin\theta\sin2\phi}{\overline{m}_3}\ . \tag{55}$$

The quantity $Q(B,\tilde{\kappa})$, which we call the torque function, is given by

$$Q(B,\tilde{\kappa})=-\tilde{\kappa}\frac{\partial F}{\partial\tilde{\kappa}}\ , \tag{56}$$

$$Q(B,\tilde{\kappa})=\frac{Bf_\infty^2}{Q(B,\tilde{\kappa}}\left\{-\tilde{\kappa}^2\xi_v^2\left[\frac{1-f_\infty^2}{2}\ln\left[\frac{2}{B\tilde{\kappa}\xi_v^2}+1\right]-\frac{1-f_\infty^2}{2+B\tilde{\kappa}\xi_v^2}+\frac{f_\infty^2}{(2+B\tilde{\kappa}\xi_v^2)^2}\right]+\frac{2+3B\tilde{\kappa}\xi_v^2+2B^2\tilde{\kappa}^2\xi_v^4}{(2+B\tilde{\kappa}\xi_v^2)^3}\right.$$
$$\left.+\frac{1}{f_\infty\xi_vK_1(f_\infty\xi_v)}\left[K_0(\xi_v\sqrt{f_\infty^2+2B\tilde{\kappa}})+\frac{B\tilde{\kappa}\xi_vK_1(\xi_v\sqrt{f_\infty^2+2B\tilde{\kappa}})}{\sqrt{f_\infty^2+2B\tilde{\kappa}}}\right]\right\}\ , \tag{57}$$

where the three terms within the large curly braces correspond to F_c, F_{kg}, and $F_{\rm em}$, respectively.

The torque function Q is found to be positive. Therefore, if $m_3>m_1>m_2$, for example, from Eqs. (54) and (55) we have $H_\theta<0$ and $H_\phi<0$, which means $\theta'<\theta$ and $\phi'<\phi$, where θ' and ϕ' are the polar and azimuthal angles of **H**. This shows that, as compared with **B**, **H** orients closer to the axes along which the corresponding principal values of the mass tensor are larger.

For $\tilde{\kappa}\gg1$ and $H\gg H_{c1}$, we have $\tilde{\kappa}\xi_{v0}\approx\sqrt{2}$, $2B\tilde{\kappa}\gg f_\infty^2$, and $f_\infty\xi_vK_1(f_\infty\xi_v)\approx1$, such that $Q(B,\tilde{\kappa})$ reduces to

$$Q(b)=bf_\infty^2\left\{-g\left[(1-f_\infty^2)\ln\left[\frac{1}{bg}+1\right]-\frac{1-f_\infty^2}{1+bg}+\frac{f_\infty^2}{2(1+bg)^2}\right]+\frac{1+3bg+4b^2g^2}{4(1+bg)^3}+K_0(2\sqrt{bg})+\sqrt{bg}K_1(2\sqrt{bg})\right\}\ , \tag{58}$$

where $b=B/\tilde{\kappa}$, and f_∞^2 and $g=(\xi_v/\xi_{v0})^2$ are given by Eqs. (44) and (45).

In order to calculate $-4\pi\mathbf{M}(\mathbf{H})$ from $\mathbf{H}(\mathbf{B})$ [Eq. (47)] and $-4\pi\mathbf{M}(\mathbf{B})$ [Eq. (48)], one must first calculate numerically **B** for a given **H** from the relation $\mathbf{H}(\mathbf{B})$, and then compute $-4\pi\mathbf{M}$. This is obviously a tedious task. Fortunately, because the magitude of the magnetization is small compared with both H and B for $H\gg H_{c1}$, we may simply replace **B** in $-4\pi\mathbf{M}(\mathbf{B})$ by **H** to calculate the magnetization. In the following θ and ϕ are considered as the

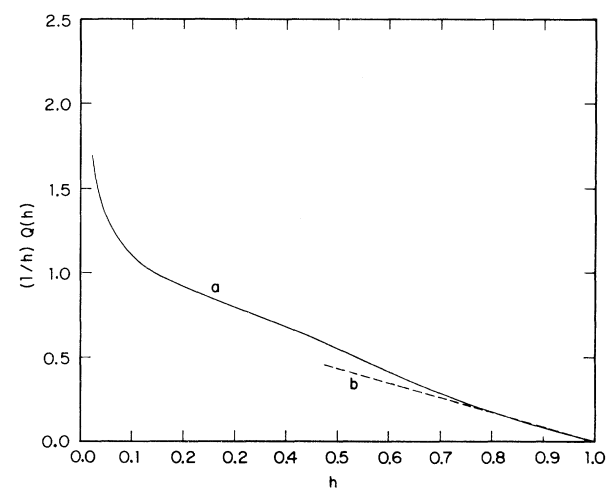

FIG. 1. Transverse magnetization function $(1/h)Q(h)$ vs the reduced applied field $h=H/H_{c2}$ in the limit $\tilde{\kappa}\gg1$: (a) this work [Eq. (58)] (solid) and (b) high-field result of Kogan and Clem (Ref. 7) [Eq. (59)] (dashed). The transverse magnetization $-4\pi M_\perp$ is proportional to $(1/h)Q(h)$ for constant T and fixed direction of **H**.

angles of **H**.

The magnitude of the transverse magnetization $M_\perp=(M_\theta^2+M_\phi^2)^{1/2}$ at constant temperature T and fixed orientation of **H** is proportional to $(1/h)Q(h)$, where $h=H/\bar{\kappa}$ is the reduced field. The calculated result of $(1/h)Q$ versus h is shown in Fig. 1, curve (a). The high-field result of Kogan and Clem[7] for a uniaxial type-II superconductor $(m_1=m_2\neq m_3)$, is also shown for comparison as curve (b). This result is the solution of the anisotropic Ginzburg-Landau equations for H near H_{c2}; for $\bar{\kappa}\gg 1$ it corresponds to replacing the torque function $Q(h)$ by

$$Q_{\rm KC}(h)=h(1-h)/\beta_A \tag{59}$$

where the Abrikosov constant $\beta_A=1.16$,[20] in Eqs. (54) and (55). Note that for the longitudinal magnetization the result of the variational model is almost identical to that of the Abrikosov high-field result in the field region $0.4\le h\le 1$,[8] while for the transverse magnetization the corresponding field region is limited to $0.8\le h\le 1$. The magnitude of the limiting slope of $-4\pi M_\perp$ versus H as $h\to 1$ is slightly less than that of Kogan and Clem.[7]

IV. TORQUES

Consider an anisotropic type-II superconducting sample in the mixed state. When the transverse magnetization is not zero, there exists a torque τ associated with it. For $\bar{\kappa}\gg 1$ and $H\gg H_{c1}$, since the magnetization is small compared with the applied field, demagnetization effects can be neglected, and it is an excellent approximation to consider the thermodynamic field H as uniform and equal to the applied fields. In this case τ is simply given by[21]

$$\tau=V\mathbf{M}\times\mathbf{H} \tag{60}$$

$$=-V\frac{H_c^2}{8\pi}Q\mathbf{P}\ , \tag{61}$$

where V is the volume of the sample, the torque function Q is given by Eq. (58), and the vector

$$\mathbf{P}=\frac{1}{\bar{m}_3}[-\hat{\theta}(m_1-m_2)\sin\theta\sin2\phi + \hat{\phi}(m_3-m_1\cos^2\phi-m_2\sin^2\phi)\sin2\theta]\ . \tag{62}$$

We return in this section to conventional (Gaussian) units. We also can express τ in terms of the unit vectors $\hat{\mathbf{X}}_i$ $(i=1,2,3)$ of the coordinate system whose axes coincide with the principal axes:

$$\mathbf{P}=\frac{1}{\bar{m}_3}[\hat{\mathbf{X}}_1(m_2-m_3)\sin2\theta\sin\phi+\hat{\mathbf{X}}_2(m_3-m_1)\sin2\theta\cos\phi + \hat{\mathbf{X}}_3(m_1-m_2)\sin^2\theta\sin2\phi]\ . \tag{63}$$

If $m_3>m_1>m_2$, for example, from Eqs. (61) and (63) and remembering that $Q>0$, we see that $\tau_1>0$, $\tau_2<0$, and $\tau_3<0$. Therefore τ, which is acting on the sample, is tending to rotate the sample to the position such that the applied field is parallel to the x_2 axis, along which the corresponding principal value of the mass tensor is the smallest. Generally, since $\bar{\kappa}$ (or H_{c2}) is inversely proportional to the effective mass, τ tends to orient the sample such that the value of $\bar{\kappa}$ (or H_{c2}) is the largest.

In the same way as for calculating $-4\pi M_\perp$, we replace **B** by **H** in Eqs. (61) and (63) for calculating τ. Note that in Eq. (61) H_c is a function of temperature T only, **P** is a function of the angles θ and ϕ and the anisotropy ratio $m_1{:}m_2{:}m_3$, and Q is a function of the reduced field $h=H/H_{c2}$ which depends on T, H, θ, ϕ, and $m_1{:}m_2{:}m_3$. Therefore, the dependences of τ upon H, upon the orientation of **H**, and upon T are determined by Q, QP $(P=|\mathbf{P}|)$, and H_c^2Q, respectively. Also note that Q and **P** are dimensionless quantities and H_c^2 has the dimension of torque per unit volume.

Torque associated with the transverse magnetization in an uniaxial anisotropic type-II superconductor was first studied by Kogan,[22] and an expression for the torque in the intermediate-field region $(H_{c1}\ll H\ll H_{c2})$ was obtained by using the London model, which corresponds to replacing the torque function $Q(h)$ by

$$Q_{\rm K}(h)=\frac{h}{2}\ln\left[\frac{\eta}{h}\right] \tag{64}$$

in Eq. (61), where η is an unknown constant of order unity.

The torque τ versus H at constant T and fixed orientation of **H**, is determined by Q the torque function. In Fig. 2 we show the calculated result of Q versus h [curve (a)]. $Q_{\rm KC}(h)$ [Eq. (59)] [curve (b)] and $Q_{\rm K}(h)$ [Eq. (64)] for various values of η [curves (c)–(e)] are also shown for comparison. Note that $Q(h)$ [therefore $\tau(h)$] has a maximum at $h\approx 0.46$, $Q_{\rm KC}(h)$ has a maximum at $h=0.50$, and $Q_{\rm K}(h)$ has a maximum at $h=\eta/e$.

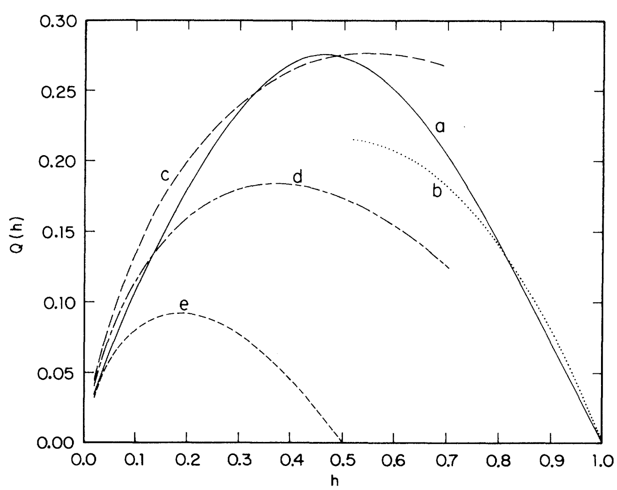

FIG. 2. Torque function $Q(h)$ vs the reduced field h in the limit $\bar{\kappa}\gg 1$: (a) this work [Eq. (58)], (b) the high-field result of Kogan and Clem (Ref. 7) [Eq. (59)], and (c)–(e) the intermediate-field results of Kogan (Ref. 22) [Eq. (64)] for $\eta=1.5$ (c), 1.0 (d), and 0.5 (e). The maximum occurs at $h\approx 0.46$ (a), $h=0.50$ (b), and $h=\eta/e$ (c)–(e).

The Kogan result[22] has been used to fit the torque measurements on the high-temperature oxide superconductors, and agreement between the theory and experiments has been reported.[23] However, we have shown in detail in another paper[12] that the London model is not adequate for describing the magnetization. The reason for the apparent agreement between the Kogan result and the experiments is as follows. The only difference between the result of the present work and that of Kogan is the difference between $Q(h)$ and $Q_K(h)$. The Kogan result was compared mainly with the measurements of τ versus θ at constant T and fixed H.[23] Since $\tau(\theta)$, as will be shown later, depends strongly upon P and much less strongly upon Q or Q_K, this comparison is not sufficient to show the validity of Q_K. Furthermore, η was treated as a fitting parameter that depends on the value of H; i.e., one was free to optimize the value of η for every fitting.[23] Also note that, since the values of H_{c2} are large in the high-temperature superconductors, the field region where the torque measurements were done[23] corresponds to small values of $h=H/H_{c2}$. As we can see from Fig. 2, for the low-field region (where h is small), ther torque function Q of this work may be approximated by a function similar to Eq. (64), i.e.,

$$Q \simeq \eta_1 \frac{h}{2} \ln \left[\frac{\eta_2}{h} \right] . \tag{65}$$

but with $\eta_1 < 1$ and $\eta_2 > 1$. Since only the normalized τ/τ_{max} was compared with theory,[23] η_1 was canceled out by taking the ratio, and the constant in the argument of the logarithmic function obtained by fitting is not η of Eq. (64) but η_2. An effective way to check the validity of Q or Q_K is to compare the theories with the measurements of τ versus H at constant T and fixed orientation of **H**.

The dependence of the torque τ upon the orientation of the applied field at constant T and fixed H is determined by QP. In Figs. 3(a)–3(c) we show the normalized τ/τ_{max} versus θ for the simple case that $m_1 = m_2 < m_3$ for values of the anisotropy ratio $\gamma = (m_3/m_1)^{1/2} = 5$ (a), 30

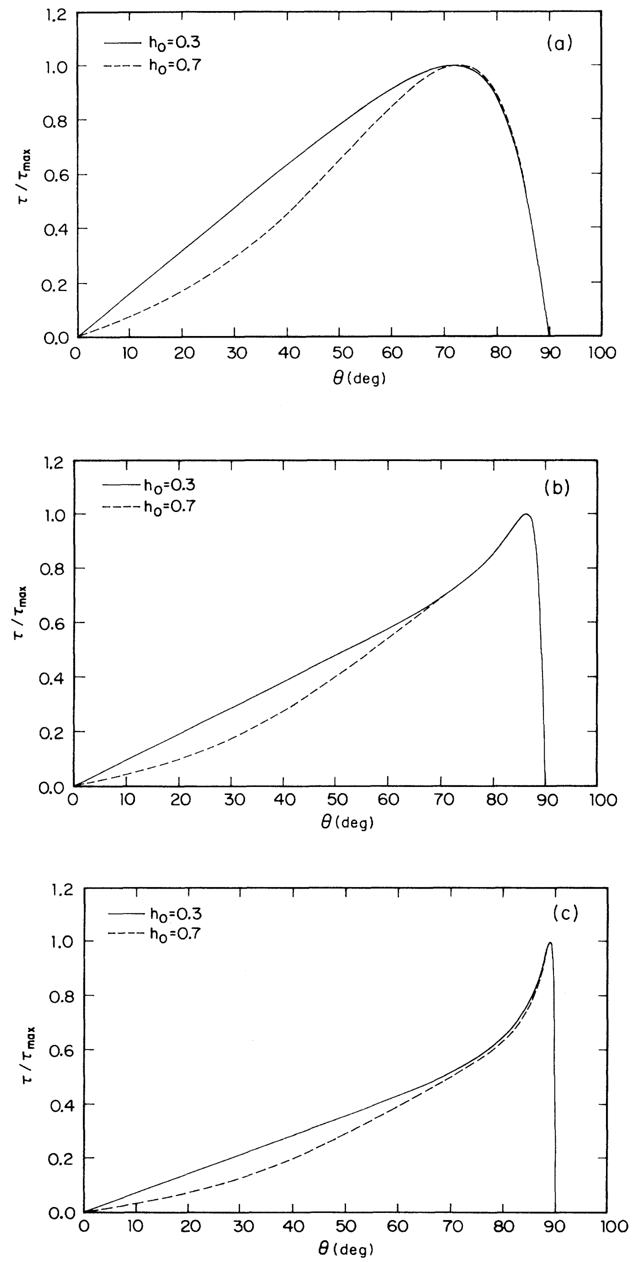

FIG. 3. Normalized torque, τ/τ_{max}, vs the orientation of **H** for constant T and fixed H for $m_1 = m_2 < m_3$ and $\gamma = 5$ (a), 30 (b) and 100 (c), where $\gamma = (m_3/m_1)^{1/2}$. θ is the angle between **H** and the X_3 axis, and $h_0 = H/H_{c2\parallel 3}$ is value of h at $\theta = 0°$ (where $H_{c2\parallel 3}$ is the upper critical field parallel to the X_3 axis).

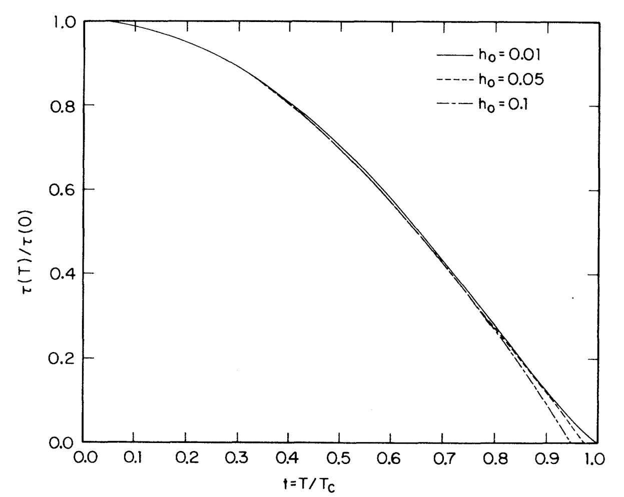

FIG. 4. Reduced torque $\tau(T)/\tau(0)$ vs reduced temperature at $t = T/T_c$ for fixed magnitude and angle θ of **H** for $h_0 = H/H_{c2}(\theta, T=0) = 0.01$, 0.05, and 0.1, assuming $1-t^2$ temperature dependences for $H_c(T)$ and $H_{c2}(\theta, T)$.

(b), and 100 (c), for values of the reduced applied field $h_0 = H/H_{c2\parallel 3} = 0.3$ (solid) and 0.7 (dashed) (where h_0 is the value of h at $\theta = 0°$, and $H_{c2\parallel 3}$ is the upper critical field parallel to the X_3 axis). $\tau(\theta)$ vanishes at $\theta = 0°$ and 90° and shows a peak at a position close to the X_1X_2 plane, and the peak becomes sharper and closer to the plane for larger anisotropy. This behavior is mainly determined by P. It is not difficult to see from Eq. (63) that the H-independent quantity $P = 0$ at $\theta = 0°$ and 90° has a maximum at $\theta = \tan^{-1}\gamma$ and the peak of the maximum becomes sharper and closer to $\theta = 90°$ as γ increases. For $\gamma = 30$, for example, $\theta = 86.3°$, at which $\tau = \tau_{\max}$ for $h_0 = H/H_{c2\parallel 3} = 0.3$, is close to $\tan^{-1}\gamma = 88.1°$, at which $P = P_{\max}$. The difference between the behaviors of $\tau(\theta)$ for different values of H arises from the field dependence of the torque function $Q(h)$.

The temperature dependence of τ at fixed **H** is determined by $H_c^2 Q$. In Fig. 4 we show $\tau(T)/\tau(0)$ versus the reduced temperature $t = T/T_c$, assuming $1 - t^2$ temperature dependences for $H_c(T)$ and $H_{c2}(T)$.

Note that $M_\perp$ has the same angular and temperature dependences as those of τ, as can be seen by comparing Eqs. (54) and (55) (when written in conventional units) with Eq. (61). This is because $\tau \propto M_\perp$ at fixed H.

V. SUMMARY AND DISCUSSION

Using the London model, we have investigated the procedure of obtaining the free-energy density of an anisotropic high-κ type-II superconductor from its corresponding expression in the isotropic case by simply replacing κ by $\tilde{\kappa}$ that depends on the orientation of **H** relative to the principal axes of the sample. This procedure is valid when **H** is along one of the principal axis for arbitrary value of H between H_{c1} and H_{c2}. We have shown in this paper that this procedure is also valid when **H** is *not* along one of the principal axes, but only for $\kappa \gg 1$ and $H \gg H_{c1}$. We expect this conclusion also to be valid in the context of the Ginzburg-Landau theory, and have applied it to the variational model of Ref. 8 and obtained expressions for the reversible magnetization and the torques associated with the transverse component of the magnetization. The theoretical expressions involve parameters $H_c(T)$, κ, and the principal values of the mass tensor m_1, m_2, and m_3, which determine the upper critical field $H_{c2}(\theta,\phi,T)$. These parameters can be obtained by comparing the theory with experimental measurements on the dependence of the torque upon the magnitude and the orientation of the applied field and on the temperature. The theory also tells that, as compared with the direction of **B**, **H**, orients closer to the axes along which the corresponding principal values of the mass tensor are larger; and that the torque tends to rotate the sample so that the applied field is parallel to the axis along which the corresponding principal value of the mass tensor is the smallest (or, along which the value of H_{c2} is the largest).

The condition that $H \gg H_{c1}$ or $(L/\lambda)^2 \ll 1$ can be easily satisfied for the case of high-κ materials, because the intervortex spacing L is comparable to or larger than λ only for small B, corresponding to H just above H_{c1}. Therefore the present theory should be valid over a large field region including intermediate and high fields.

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APPENDIX

In this appendix we determine the coordinate system (x_1,x_2,x_3) in which Eq. (25) holds. The unit vector along **B** obeys

$$\hat{\mathbf{B}} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \tag{A1}$$

in the coordinate system whose axes coincide with the principal axes X_i $(i = 1,2,3)$, where θ and ϕ are the polar and azimuthal angles of **B**. The transformation from (X_1,X_2,X_3) into (x_1,x_2,x_3) consists of two steps as follows.

First we transform (X_1,X_2,X_3) into an intermediate coordinate system (x'_1,x'_2,x'_3) by a rotation of θ about the X_2 axis and a rotation of ϕ about the X_3 axis, i.e.,

$$x'_i = P_{ij}X_j \ , \tag{A2}$$

where

$$P_{ij} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix} . \tag{A3}$$

In the (X_1,X_2,X_3) coordinates the mass tensor is $m_i\delta_{ij}$; in the (x'_1,x'_2,x'_3) coordinates it becomes $m'_{ij} = P_{ik}(m_k\delta_{kl})P^{-1}_{lj}$, or

$$m'_{11} = (m_1\cos^2\phi + m_2\sin^2\phi)\cos^2\theta + m_3\sin^2\theta \ , \tag{A4}$$

$$m'_{12} = (m_2 - m_1)\cos\theta\cos\phi\sin\phi \ , \tag{A5}$$

$$m'_{13} = (m_1\cos^2\phi + m_2\sin^2\phi - m_3)\cos\theta\sin\theta \ , \tag{A6}$$

$$m'_{22} = m_1\sin^2\phi + m_2\cos^2\phi \ , \tag{A7}$$

$$m'_{23} = (m_2 - m_1)\sin\theta\cos\phi\sin\phi, \tag{A8}$$

$$m'_{33} = (m_1\cos^2\phi + m_2\sin^2\phi)\sin^2\theta + m_3\cos^2\theta \ , \tag{A9}$$

where $m'_{ji} = m'_{ij}$.

Next we rotate the primed frame an angle γ about the x'_3 axis and obtain

$$x_i = R_{ij}x'_j \ , \tag{A10}$$

where

$$R_{ij} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{A11}$$

Note that $\hat{\mathbf{x}}_3=\hat{\mathbf{x}}'_3=\hat{\mathbf{B}}$. The mass tensor in the new frame is $m_{ij}=R_{ik}m'_{kl}R^{-1}_{lj}$, or

$$m_{11}=m'_{11}\cos^2\gamma+m'_{12}\sin2\gamma+m'_{22}\sin^2\gamma\ , \tag{A12}$$

$$m_{12}=\tfrac{1}{2}(m'_{22}-m'_{11})\sin2\gamma+m'_{12}\cos2\gamma\ , \tag{A13}$$

$$m_{13}=m'_{13}\cos\gamma+m'_{23}\sin\gamma\ , \tag{A14}$$

$$m_{22}=m'_{11}\sin^2\gamma-m'_{12}\sin2\gamma+m'_{22}\cos^2\gamma\ , \tag{A15}$$

$$m_{23}=m'_{23}\cos\gamma-m'_{13}\sin\gamma\ , \tag{A16}$$

$$m_{33}=m'_{33}\ , \tag{A17}$$

where $m_{ji}=m_{ij}$. The angle γ required to satisfy the condition of Eq. (25) is given by

$$\tan2\gamma=\frac{2(m'_{12}m'_{33}-m'_{13}m'_{23})}{(m'_{11}-m'_{22})m'_{33}-(m'^2_{13}-m'^2_{23})}, \tag{A18}$$

$$=\frac{m_3(m_2-m_1)\cos\theta\sin2\phi}{m_3m_2(\sin^2\phi-\cos^2\theta\cos^2\phi)+m_3m_1(\cos^2\phi-\cos^2\theta\sin^2\phi)-m_1m_2\sin^2\theta}\ . \tag{A19}$$

In terms of m_i $(i=1,2,3)$ and the angles θ, ϕ, and γ, the masses $\overline{m}_1$ and $\overline{m}_2$ become

$$\overline{m}_1=[m_1m_2\sin^2\theta\sin^2\gamma+m_1m_3(\cos\phi\cos\gamma-\cos\theta\sin\phi\sin\gamma)^2+m_2m_3(\sin\phi\cos\gamma+\cos\theta\cos\phi\sin\gamma)^2]/\overline{m}_3\ , \tag{A20}$$

$$\overline{m}_2=[m_1m_2\sin^2\theta\cos^2\gamma+m_1m_3(\cos\phi\sin\gamma+\cos\theta\sin\phi\cos\gamma)^2+m_2m_3(\sin\phi\sin\gamma-\cos\theta\cos\phi\cos\gamma)^2]/\overline{m}_3\ , \tag{A21}$$

where $\overline{m}_3=m_{33}=m'_{33}$ is given by (A9).

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