Dielectric formalism for a quasi-one-dimensional electron gas. II. Dielectric functions and potential correlators

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The dielectric function of an electron gas confined in a quantum wire is evaluated. The lateral confinement is modeled with a *square* or *parabolic* well and the vertical one with a *triangular* or *square* well. Screening is treated dynamically, and the electrons are assumed to interact with each other and with an external system taken to be impurities or phonons. Two cases are considered: (i) a homogeneous system and (ii) systems presenting inhomogeneity in one direction such as simple or double heterostructures. The potential correlators and dielectric functions of the external system are also evaluated. The results are useful for describing transport properties using the relevant quantum transport equations and relaxation frequencies.

I. INTRODUCTION

Electronic systems of reduced dimensionality, such as two- or one-dimensional electron gases (2DEG, 1DEG) occurring, e.g., at semiconductor interfaces, have attracted considerable theoretical and experimental attention over the past ten years. Concerning screening of the 1DEG we are aware of Refs. 1 and 2, which treat the electronic part of the dielectric function, and of Ref. 3 which treats screening by impurities as well. None of these works considers screening by phonons, nor do they give the complete expression for the total dielectric function. References 1 and 3 consider only a cylindrical wire with one or two lateral subbands occupied and Ref. 2 considers only a *square* lateral confinement. For these reasons and the fact that self-consistent calculations indicate that the confining potential can be *parabolic* in very narrow 2DEG channels,[4] we feel that additional studies of screening are necessary and valid for lateral multisubband occupation. This will be the subject of this paper both for square and parabolic lateral confinement, taking into account contributions to screening from electron-electron, electron-phonon, and electron-impurity interactions.

An additional reason for the present study is the quantum transport equations for the quasi-1DEG (Q1DEG) derived in the preceding paper and referred to hereafter as I. The collision integrals and the energy and momentum relaxation frequencies are expressed in terms of potential correlators of the external system or in terms of the dielectric functions when the latter is in equilibrium. It is therefore appropriate to complete the formalism initiated in I.

In this paper we calculate the dielectric function of the Q1DEG in the presence of electron-electron or electron-phonon interaction in Sec. II and the potential correlators, for phonons and impurities, in Sec. III. In Sec. II we consider both a homogeneous system and an inhomogeneous one along one direction, pertinent to single or double heterostructures. Concluding remarks follow in Sec. IV. Appendix A contains the dielectric function and potential correlators for a three-dimensional (3D) phonon system; these results are used in Secs. II and III. For completeness we give, in Appendix B, the electronic part of the dielectric function of the Q1DEG for lateral multisubband occupation.

II. DIELECTRIC FUNCTION OF THE Q1DEG INTERACTING WITH AN EXTERNAL SYSTEM

We consider a Q1DEG confined in a quantum wire, of length $L_x = L$, and interacting with an external system. As in I we use small Greek letters for the "longitudinal" (i.e., along x) quantum numbers and Roman letters for the "transverse" (i.e., in the yz plane) ones. Considering only one sort of particles (i.e., electrons) we can drop the index p and write the one-electron wave function as $\Psi_A(\mathbf{r}) = \psi_\alpha(x)\chi_a(\boldsymbol{\rho}), \boldsymbol{\rho} = (y,z)$. With $K = (a,b)$ the dielectric function $\epsilon^s_{KK'}(\omega, q_x)$, where ω is the frequency and q_x the wave vector, is given by Eq. (21) of I ($q \equiv q_x$) rewritten here for convenience:

$$\frac{1}{\epsilon^s_{KK'}(\omega,q_x)} = \int\int \frac{\chi^*_a(\boldsymbol{\rho})\chi_b(\boldsymbol{\rho})\chi_{a'}(\boldsymbol{\rho}')\chi^*_{b'}(\boldsymbol{\rho}')}{\epsilon_s(\omega,q_x,\boldsymbol{\rho},\boldsymbol{\rho}')} d^2\rho\, d^2\rho' \,. \tag{1}$$

The function $\epsilon_s(\omega,q_x,\boldsymbol{\rho},\boldsymbol{\rho}')$ describes the linear response of the system, homogeneous in the x direction, to a test charge with density $e\delta N$ via Eq. (15) of I.

A. Homogeneous medium

The 3D Poisson equation, in Fourier space, reads

$$\varphi(\omega,\mathbf{q}) = \frac{4\pi e^2}{q^2}\frac{N(\omega,\mathbf{q})}{\epsilon_s(\omega,\mathbf{q})}, \quad \mathbf{q} = (q_x,q_y,q_z) \,, \tag{2}$$

where $\varphi(\omega,\mathbf{q})$ is the potential created by the charge $eN(\omega,\mathbf{q})$ and $\epsilon_s(\omega,\mathbf{q})$ the 3D dielectric function. Taking the inverse Fourier transform of Eq. (2) with respect to the 2D wave vector $\mathbf{Q}=(q_y,q_z)$ we obtain

$$\frac{1}{\epsilon_s(\omega,q_x,\rho,\rho')}=\frac{1}{\epsilon_s(\omega,q_x,\rho-\rho')}=\frac{1}{\pi}\int\frac{e^{i\mathbf{Q}\cdot(\rho-\rho')}}{(Q^2+q_x^2)\epsilon_s(\omega,\mathbf{q})}d^2Q\ . \tag{3}$$

If the system is *isotropic*, $\epsilon_s(\omega,\mathbf{q})=\epsilon_s(\omega,q)$, and the integration over the angle between $\mathbf{Q}$ and $\rho-\rho'$ gives

$$\frac{1}{\epsilon_s(\omega,q_x,\rho)}=2\int_0^\infty\frac{J_0(Q\rho)Q}{q^2\epsilon_s(\omega,q)}dQ\ , \tag{4}$$

where $J_0(x)$ is the Bessel function. In the particular case that $\epsilon_s(\omega,q)$ does not depend on q we have $\epsilon_s(\omega,q)=\epsilon_s(\omega)$ (the medium has *no dispersion*) and Eq. (4) reduces to

$$\frac{1}{\epsilon_s(\omega,q_x,\rho)}=\frac{2}{\epsilon_s(\omega)}K_0(|q_x|\rho)\ , \tag{5}$$

with $K_0(x)$ being the modified Bessel function.

We now evaluate $\epsilon_s(\omega,q_x,\rho)$ when phonons constitute the external system. In reality, however, screening is determined not only by phonons but also by electron shells, etc. We shall assume that these other sources of screening contribute to the real part of the total lattice dielectric function as described by the phenomenological equation $\mathrm{Re}\epsilon_S=\epsilon_L$, where S represents the "sum" of all these sources. The imaginary part of $\epsilon_s(\omega,\mathbf{q})$ contributes to scattering and we shall be interested only in additive values of the jth phonon modes.

From Eq. (A6), in the isotropic case, we obtain

$$\mathrm{Im}\frac{1}{\epsilon_j(\omega,\mathbf{q})}=\frac{q^2}{4\hbar}|\phi_{j\mathbf{q}}|^2[\delta(\omega+\omega_{j\mathbf{q}})-\delta(\omega-\omega_{j\mathbf{q}})]\ . \tag{6}$$

Using Eqs. (4) and (6) we find, for DA, PA, and PO phonons, respectively, as discussed in Appendix A, the following results:

$$\mathrm{Im}\frac{1}{\epsilon_{\mathrm{DA}}(\omega,q_x,\rho)}=-\mathrm{sgn}\omega(C_{\mathrm{DA}}\omega^2/2\hbar^3s^3)J_0(\rho\sqrt{X}/s)\Theta(X)\ , \tag{7}$$

$$\mathrm{Im}\frac{1}{\epsilon_{\mathrm{PA}}(\omega,q_x,\rho)}=-\mathrm{sgn}\omega(C_{\mathrm{PA}}/2\hbar s)J_0(\rho\sqrt{X}/s)\Theta(X)\ , \tag{8}$$

$$\mathrm{Im}\frac{1}{\epsilon_{\mathrm{PO}}(\omega,q_x,\rho)}=(C_{\mathrm{PO}}/2\hbar)K_0(\rho|q_x|)[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]\ , \tag{9}$$

where $X=\omega^2-s^2q^2$ and $\Theta(X)$ is the step function.

The dielectric function $\epsilon^s_{KK'}(\omega,q_x)$ can now be obtained from Eqs. (1) and (3). We have

$$\frac{1}{\epsilon^s_{KK'}(\omega,q_x)}=\frac{1}{\pi}\int\frac{\phi_K(\mathbf{Q})\phi^*_{K'}(\mathbf{Q})}{(Q^2+q_x^2)\epsilon_s(\omega,\mathbf{q})}d^2Q\ , \tag{10}$$

with

$$\phi_K(\mathbf{Q})=\int\chi^*_a(\rho)e^{i\mathbf{Q}\cdot\rho}\chi_b(\rho)d^2\rho\ . \tag{11}$$

We consider only the case $\chi_a(\rho)=Z_0(z)Y_n(y)$; the subscript 0 indicates that only the lowest subband is assumed to be occupied. For confinement along the z direction we consider a *triangular* (T) well, with the usual variational wave function, or a *square* (S) well of height a_0. That is, we take

$$Z_0^T(z)=(b_0^3/2)^{1/2}ze^{-b_0z/2}\ , \tag{12}$$

or

$$Z_0^S(z)=(2/a_0)^{1/2}\sin(\pi z/a_0),\quad 0\le z\le a_0\ . \tag{13}$$

For the y direction we consider a *parabolic* (P) or a *square* (S) well of width W:

$$Y_n^P(y)=(\sqrt{\pi}2^n n\lambda)^{-1/2}e^{-y^2/2\lambda^2}H_n(y/\lambda)\ ,\quad n=0,1,\ldots\ , \tag{14}$$

$$Y_n^S(y)=(2/W)^{1/2}\sin[\pi n(y/W+\tfrac{1}{2})]\ ,\quad -W/2\le y\le W/2\ ,\quad n=1,2,\ldots\ , \tag{15}$$

where $H_n(x)$ is a Hermite polynomial and n is the energy quantum number. In Eq. (14) $\lambda=\sqrt{\hbar/m^*\omega'}$ and ω' specifies the parabolic confining potential $V(y)=m^*\omega'^2y^2/2$. From Eq. (11) we obtain

$$\phi_K(\mathbf{Q})=\Psi_0(q_z)\phi_{nn'}(q_y)\ , \tag{16}$$

with

$$\Psi_0^T(q_z)=(1-iq_z/b_0)^{-3}\ , \tag{17}$$

$$\Psi_0^S(q_z)=4\pi^2i[a_0q_z(4\pi^2-a_0^2q_z^2)]^{-1}(1-e^{ia_0q_z})\ , \tag{18}$$

and

$$\phi^P_{nn'}(q_y)=\left[\frac{n'!}{n!}\right]^{1/2}(i\sqrt{2}Y)^{n-n'}e^{-Y^2}L_n^{n-n'}(2Y^2)\ ,\quad n'\le n\ , \tag{19}$$

$$\phi^S_{nn'}(q_y)=-8\pi^2nn'Z\frac{(-1)^{n+n'}e^{iZ}-e^{-iZ}}{(2nn'\pi^2)^2-[4Z^2-\pi^2(n^2+n'^2)]^2}\ , \tag{20}$$

where $Y=\lambda q_y/2$ and $Z=Wq_z/2$. Notice that $\Psi^{S,T}(q_z)\to1$ for $q_z\to0$,

$$\phi^P_{nn'}(q_y)|_{n<n'}=\phi^P_{n'n}(q_y)|_{n'>n}\ , \tag{20'}$$

and

$$\phi^S_{nn'}(q_y)\to\delta_{nn'}\ \text{for}\ q_y\to0\ .$$

From Eqs. (7), (10), (11), and (16)–(20) we obtain for *acoustical* phonons (DA)

$$\mathrm{Im}\frac{1}{\epsilon^{\mathrm{DA}}_{n_1n_2n_3n_4}(\omega,q_x)}=-\mathrm{sgn}\omega\frac{\theta(X)}{\pi\hbar}C_{\mathrm{DA}}\omega^2 s^{-2}\int_0^{\sqrt{X}/s}\frac{|\Psi_0[(X-s^2q_y^2)^{1/2}/s]|^2}{(X-s^2q_y^2)^{1/2}}\phi_{n_1n_2}(q_y)\phi^*_{n_3n_4}(q_y)dq_y \ . \tag{21}$$

For *piezoelectrical* phonons (PA) we obtain Eq. (21) with $C_{\mathrm{DA}}\omega^2 s^{-2}$ replaced by C_{PA}. For media without spatial dispersion, $\epsilon_s(\omega,q)=\epsilon_s(\omega)$, and Eq. (10) gives

$$\frac{1}{\epsilon^s_{n_1n_2n_3n_4}(\omega,q_x)}=\frac{1}{\epsilon_s(\omega)}\int\frac{\phi_{n_1n_2}(q_y)\phi^*_{n_3n_4}(q_y)F^{T,S}[(q_x^2+q_y^2)^{1/2}]}{(q_x^2+q_y^2)^{1/2}}dq_y \ , \tag{22}$$

with

$$F^T(x)=(1+9x/8b_0+3x^2/8b_0^2)(1+x/b_0)^{-3} \tag{23}$$

and

$$F^S(x)=\frac{3r^2+8\pi^2}{r(r^2+4\pi^2)}-\frac{32\pi^4}{r^2(r^2+4\pi^2)^2}(1-e^{-r}) \ , \qquad r=xa_0 \ . \tag{24}$$

For *polar optical* phonons (PO) we obtain

$$\frac{1}{\epsilon_{\mathrm{PO}}(\omega)}=\frac{1}{\epsilon_\infty}\frac{\omega^2-\omega_0(\omega_0-\epsilon_\infty C_{\mathrm{PO}}/2\pi\hbar)}{(\omega+i\delta)^2-\omega_0^2} \ , \tag{25}$$

where ϵ_∞ is the high-frequency lattice dielectric constant.

B. System inhomogeneous in one direction

We consider a 3D system homogeneous in the xy plane and inhomogeneous in the z direction. It is convenient to specify the dielectric properties of this system with the function $\epsilon_s(\omega,\mathbf{k},z,z')$, $\mathbf{k}=(q_x,q_y)$. It is related to the potential $\varphi(\omega,\mathbf{k},z)$, created by the charge $eN(\omega,\mathbf{k},z)$, by

$$\varphi(\omega,\mathbf{k},z)=\frac{2\pi e}{k}\int\frac{N(\omega,\mathbf{k},z')}{\epsilon_s(\omega,\mathbf{k},z,z')}dz' \ . \tag{26}$$

We need the following functions defined in terms of $\epsilon_s(\omega,\mathbf{k},z,z')$:

$$\frac{1}{\epsilon^s_{n_1n_2n_3n_4}(\omega,q_x)}=\int\frac{\phi_{n_1n_2}(q_y)\phi^*_{n_3n_4}(q_y)}{k\epsilon_s(\omega,\mathbf{k})}dq_y \ , \tag{27}$$

and

$$\frac{1}{\epsilon_s(\omega,\mathbf{k})}=\int\int\frac{|Z_0(z)|^2|Z_0(z')|^2}{\epsilon_s(\omega,\mathbf{k},z,z')}dz\,dz' \ . \tag{28}$$

The function $\epsilon_s(\omega,\mathbf{k},z,z')$ has been evaluated in Refs. 5 and 6 for the models shown in Figs. 1 and 2. In Fig. 1, ϵ_-, ϵ_0, and ϵ_+ denote the lattice dielectric constants of the corresponding regions. At $z=-l$ we impose the condition

$$-\beta(\omega,\mathbf{k})D_z(\omega,\mathbf{k},-l+0)=\epsilon_- k\varphi(\omega,\mathbf{k},-l+0) \ , \tag{29}$$

where $D_z(\ )$ is the displacement vector at an infinitesimally small distance to the right of the plane $z=-l$.

The function $\beta(\omega,\mathbf{k})$ is specified by the properties of the region $z<-l$. For example, if we place a metallic electrode (i.e., a gate) at $z=-l$ we have $\beta(\ )=0$. For a medium with dielectric constant ϵ_L in the region $z<-l$, we have $\beta(\ )=\epsilon_-/\epsilon_L$. When a 3D electron gas occupies the region $z<-l$, $\beta(\ )$ is given in Refs. 5 and 6.

When the well in the z direction is *square*, Eq. (13) with Eq. (27) of Ref. 5, and Eq. (28) give

$$\frac{1}{\epsilon_s(\omega,\mathbf{k})}=\frac{1}{\epsilon_0}\left[\frac{3r^2+8\pi^2}{r(r^2+4\pi^2)}-\frac{32\pi^4}{r^2(r^2+4\pi^2)^2}\frac{2\epsilon_+\tilde{\epsilon}_-(\cosh r-1)+\epsilon_0(\epsilon_++\tilde{\epsilon}_-)\sinh r}{\epsilon_0(\epsilon_++\tilde{\epsilon}_-)\cosh r+(\epsilon_0^2+\epsilon_+\tilde{\epsilon}_-)\sinh r}\right] \ , \tag{30}$$

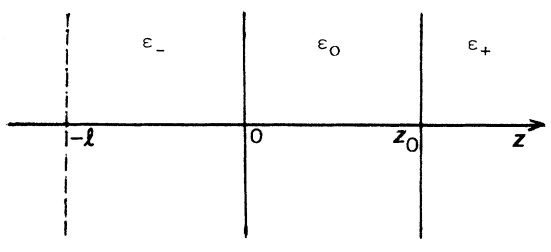

FIG. 1. Inhomogeneous system modeling double heterostructures schematically. The two interfaces are at $z=0$ and at $z=z_0$. ϵ_-, ϵ_0, and ϵ_+ are the lattice dielectric constants of the corresponding regions.

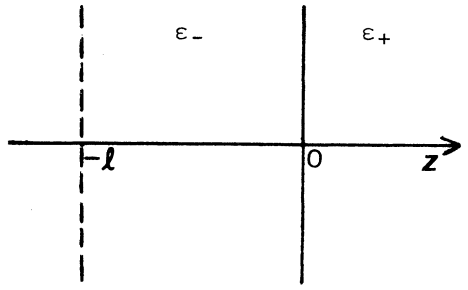

FIG. 2. Inhomogeneous system modeling a single heterostructure schematically. The interface is at $z=0$; ϵ_- and ϵ_+ are the lattice dielectric constants to the left and right of the interface, respectively.

where $r=ka_0$ and

$$\bar{\epsilon}_0 \equiv \bar{\epsilon}_-(\omega,\mathbf{k}) = \epsilon_- \frac{\beta(\omega,\mathbf{k})\tanh(kl)+1}{\beta(\omega,\mathbf{k})+\tanh(kl)} \ . \tag{31}$$

If the well is *triangular*, Eqs. (12) and (29), with Eq. (27) of Ref. 5, give

$$\epsilon_s(\omega,\mathbf{k}) = \frac{\epsilon_+(\epsilon_+ + \bar{\epsilon}_-)(1+t)^6}{\epsilon_+ - \bar{\epsilon} + (\epsilon_+ + \epsilon_-)(1+t)^3(1+9t/8+3t^2/8)} \ , \quad t = k/b_0 \ . \tag{32}$$

The last three expressions determine the dielectric function $\epsilon^s_{KK'}(\omega,q_x)$, cf. Eq. (27), which enters the expressions for the collision integrals of I.

III. POTENTIAL CORRELATORS OF THE EXTERNAL SYSTEM

In this section we give the expressions for the correlators $\langle \delta\varphi_s(\boldsymbol{\rho})\delta\varphi_s(\boldsymbol{\rho})\rangle_{\omega,q_x}$ and $\langle \delta\varphi_s\delta\varphi_s\rangle^{KK'}_{\omega,q_x}$ of external potentials acting on the Q1DEG.

In the case of a uniform external system characterized by the correlator $\langle \delta\varphi_s\delta\varphi_s\rangle_{\omega,\mathbf{q}}$ we have

$$\langle \delta\varphi_s(\boldsymbol{\rho})\delta\varphi_s(\boldsymbol{\rho}')\rangle_{\omega,q_x} \equiv \langle \delta\varphi_s\delta\varphi_s\rangle^{\boldsymbol{\rho}-\boldsymbol{\rho}'}_{\omega,q_x} = \int \frac{e^{i\mathbf{Q}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}\langle \delta\varphi_s\delta\varphi_s\rangle_{\omega,\mathbf{q}}}{4\pi^2} d^2Q \ . \tag{33}$$

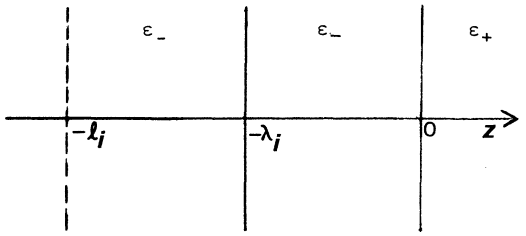

FIG. 3. Inhomogeneous system modeling a single heterostructure schematically. The interface is at $z=0$; ϵ_- and ϵ_+ are the lattice dielectric constants to the left and right of the interface, respectively. A sheet of impurities with surface density n_i^s is situated in the plane $z=-\lambda_i$.

Using Eq. (29) of I and Eqs. (13)–(21) we obtain

$$\langle \delta\varphi_s\delta\varphi_s\rangle^{n_1n_2n_3n_4}_{\omega,q_x} = \int \frac{\langle \delta\varphi_s\delta\varphi_s\rangle_{\omega,\mathbf{q}} |\Psi_0^{T,S}(q_z)|^2}{4\pi^2} \times \phi^{S,P}_{n_1n_2}(q_y)\phi^{*S,P}_{n_3,n_4}(q_y)d^2Q \ . \tag{34}$$

A. Phonons

For phonons the relevant correlators are obtained by substituting Eq. (A7) into Eqs. (33) and (34).

B. Impurities

We consider a configuration of impurities macroscopically homogeneous in the xy plane and inhomogeneous in the z direction and with density $n_i=\sum_i \delta(\mathbf{r}-\mathbf{R}_i)$. It is convenient to describe such a system with the correlator $\langle \varphi^2\rangle^{z,z'}_{\mathbf{k}}$, $\mathbf{k}=(q_x,q_y)$, the Fourier transform having been taken along the x and y directions. It is related to $\langle \varphi^2\rangle^{KK'}_{\omega,q_x}$ in the manner

$$\langle \varphi_i^2\rangle^{K,\bar{K}'}_{\omega,q_x} = \delta(\omega)\int\int\int e^{iq_y(y-y')}\langle \varphi_i^2\rangle^{z,z'}_{\mathbf{k}}\chi^*_a(\boldsymbol{\rho})\chi_b(\boldsymbol{\rho})\chi_{a'}(\boldsymbol{\rho}')\chi^*_{b'}(\boldsymbol{\rho}')dq_y d^2\rho\, d^2\rho' \ . \tag{35}$$

Using Eq. (26) we obtain after averaging the product of potentials

$$\langle \varphi_i(\mathbf{k},z)\varphi_i(\mathbf{k}',z')\rangle = \frac{4\pi^2e^2}{kk'}\int\int \frac{\langle n_i(\mathbf{k},\zeta)n_i(\mathbf{k}',\zeta')\rangle}{\epsilon_s(0,\mathbf{k},z,\zeta)\epsilon_s(0,\mathbf{k}',z',\zeta')}d\zeta\, d\zeta' \ . \tag{36}$$

Allowing for homogeneity in the xy plane this gives

$$\langle \varphi_i^2\rangle^{z,z'}_{\mathbf{k}} = \frac{4\pi^2e^2}{k^2}\int\int \frac{\langle n_i^2\rangle^{\zeta,\zeta'}_{\mathbf{k}}}{\epsilon_s(0,\mathbf{k},z,\zeta)\epsilon_s(0,-\mathbf{k},z',\zeta')}d\zeta\, d\zeta' \ . \tag{37}$$

We apply now this general result to the situation depicted in Fig. 3; ϵ_+ and ϵ_- are the dielectric constants to the right and left of the plane $z=0$, respectively. Assuming z and z' to be on opposite sides of this plane, we have

$$\epsilon_s(\omega,k,z,\zeta) = \bar{\epsilon}e^{k|z-\zeta|}, \quad \bar{\epsilon} = \frac{\epsilon_+ + \epsilon_-}{2} \ . \tag{38}$$

Let the noncorrelated impurities occupy the sheet at $z=-\lambda_i$ with surface density n_i^s. Other configurations can be obtained by integrating over λ_i. We then have $\langle n^2\rangle^{z,z'}_{\mathbf{k}} = n_i^s\delta(z-\lambda_i)\delta(z'-\lambda_i)$ and, from Eqs. (37) and (38),

$$\langle \delta\varphi_i^2\rangle^{z,z'}_{\mathbf{k}} = \left[\frac{2\pi e}{\bar{\epsilon}k}\right]^2 n_i^s e^{-k(2\lambda_i+z+z')} \ . \tag{39}$$

Integrating Eq. (39) over λ_i gives the potential correlator of impurities occupying the half space $z<-l_i$ with volume

density n_i. When this is added to Eq. (39) we obtain the correlator for volume and sheet impurities:

$$\langle \delta\varphi_i^2 \rangle_{\mathbf{k}}^{z,z'} = \left[\frac{2\pi e}{\bar{\epsilon} k} \right]^2 \left[n_i^s e^{-2k\lambda_i} + \frac{n_i}{2k} e^{-2kl_i} \right] e^{-k(z+z')} . \tag{40}$$

Then Eqs. (12)–(20) and Eq. (35) give

$$\langle \varphi_i^2 \rangle_{\omega,q_x}^{n_1 n_2 n_3 n_4} = \left[\frac{2\pi e}{\bar{\epsilon}} \right]^2 \delta(\omega) \int \frac{|\Psi_0^{T,S}(ik)|^2 \phi_{n_1 n_2}^{P,S}(q_y) \phi_{n_3 n_4}^{*P,S}(q_y)}{k^2} \left[n_i^s e^{-2k\lambda_i} + \frac{n_i}{2k} e^{-2kl_i} \right] dq_y . \tag{41}$$

We notice that when $n_1+n_2+n_3+n_4$ is an odd integer, the correlator $\langle \delta\varphi^2 \rangle_{\omega,q_x}^{n_1 n_2 n_3 n_4}$ vanishes.

IV. SUMMARY AND CONCLUDING REMARKS

In this paper we have derived expressions for the dielectric functions and potential correlators pertinent to a Q1DEG. These expressions are necessary for a rigorous description of the transport properties of the Q1DEG as they appear in the collision integrals of the quantum transport equations of I as well as in the formulas for the relaxation frequencies appearing therein.

Apart from completing I, our results, compared with those of the literature of which we are aware, are more complete in several respects: (i) they are valid for lateral multisubband occupation and for *square* or *parabolic* confinement; (ii) they are valid for *triangular* or *square* confinement along the z direction and pertinent to single (cf. Fig. 2) or double (cf. Fig. 1) heterostructures; and (iii) they account for dynamical screening of the electron-phonon interaction previously not considered.

Concerning point (iii) a word of caution is in order: the Q1D results, say along the x direction, were obtained from the 3D ones of Appendix A after integrating out the coordinates along y and z, cf. Eqs. (1)–(10). This is equivalent to neglecting any modifications of the usual electron-phonon interaction brought about by the confined geometries such as slab modes or interface phonons.[7] This neglect may not always be justified, but it was made for the sake of definiteness and clarity of the results. The same remark applies to the phenomenological constant ϵ_L introduced in Sec. II A. In any case, the electron-phonon Hamiltonian in systems of reduced dimensionality is not well established; we expect that its modifications, if any, do not substantially influence the transport results, e.g., the relaxation frequencies of I which are our main concern.

We further notice that other scattering mechanisms expected to be relevant in restricted geometries, such as surface roughness and boundary scattering, are left out from the present study. They will be dealt with in a separate publication.

Finally, we stress the pertinence of the results of Sec. II B for inhomogeneous systems. We have used some of them to describe mutual Coulomb scattering between 3D and 2D electron-gas layers.[8]

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APPENDIX A

Below we evaluate the dielectric function and potential correlators for a 3D electron-phonon system taking into account electron-electron interaction for the potential created by all charges. In the terminology of I we have only one sort of particle, i.e., electrons, and we can drop the index p from Eq. (1) of I; hence for the present case the Hamiltonian takes the form

$$H = \sum_A \hbar\omega_A a_A^\dagger a_A + \tfrac{1}{2} \sum_{A,A',B,B'} V_{AA'BB'} a_A^\dagger a_{A'}^\dagger a_B a_{B'} + \sum_{j,\mathbf{q}} \hbar\omega_{j\mathbf{q}} b_{j\mathbf{q}}^\dagger b_{j\mathbf{q}} + e \sum_{j,A,B} \tilde{\varphi}_{jAB} a_A^\dagger a_B , \tag{A1}$$

where

$$\tilde{\varphi}_j \equiv \tilde{\varphi}_j(\mathbf{r},t) = \frac{1}{\sqrt{V}} \sum_{\mathbf{q}} [\phi_{j\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} b_{j\mathbf{q}}(t) + \phi_{j\mathbf{q}}^* e^{-i\mathbf{q}\cdot\mathbf{r}} b_{j\mathbf{q}}^\dagger(t)] \tag{A2}$$

is the operator form of the potential created by phonons of the jth mode. The corresponding frequency is $\omega_{j\mathbf{q}}$, V is the volume of the system, and $b^\dagger$, and b are the creation and annihilation operators for phonons, respectively. The first term in the second line of (A1) is the Hamiltonian of the phonon system and the second term is that of the electron-phonon interaction.

In Fourier space the potential $\varphi(\omega,\mathbf{q})$, created by all charges in the system, is given by

$$\varphi(\omega,\mathbf{q}) = \varphi_e(\omega,\mathbf{q}) + \sum_j \tilde{\varphi}_j(\omega,\mathbf{q}) . \tag{A3}$$

Solving the Heisenberg equations of motion for the operators $b^\dagger$ and b and substituting the result in (A2) we can write (A3) as

$$\varphi(\omega,\mathbf{q}) = \sum_j \varphi_j(\omega,\mathbf{q}) + \frac{4\pi e}{q^2} \frac{N(\omega,\mathbf{q})}{\epsilon_{\rm ph}(\omega,\mathbf{q})} ; \tag{A4}$$

the potential $\varphi_j(\omega,\mathbf{q})$ of the jth mode, not correlated with electrons, is given by

$$\varphi_j(\omega,\mathbf{q}) = i\sqrt{V} \left[\frac{\phi_{j\mathbf{q}} b_{j\mathbf{q}}(t=0)}{\omega-\omega_{j\mathbf{q}}+i\delta} + \frac{\phi_{j,-\mathbf{q}}^* b_{j,-\mathbf{q}}^\dagger(t=0)}{\omega+\omega_{j,-\mathbf{q}}+i\delta} \right] . \tag{A5}$$

The phonon dielectric function $\epsilon_{\rm ph}(\omega,\mathbf{q})$ is then given by

$$\frac{1}{\epsilon_{\rm ph}(\omega,\mathbf{q})} = 1+\frac{q^2}{4\pi\hbar}\sum_j\left[\frac{|\phi_{j\mathbf{q}}|^2}{\omega-\omega_{j\mathbf{q}}+i\delta}-\frac{|\phi_{j,-\mathbf{q}}|^2}{\omega+\omega_{j,-\mathbf{q}}+i\delta}\right] . \tag{A6}$$

We must underline the fact that $\epsilon_{\rm ph}(\omega,\mathbf{q})$ does not depend on the phonon occupation numbers $n_{j\mathbf{q}}=b^\dagger_{j\mathbf{q}}b_{j\mathbf{q}}$. The dependence on $n_{j\mathbf{q}}$ appears only when phonon-phonon interaction is included in (A1).

From (A5) we obtain the nonsymmetrized correlator of phonon potentials:

$$\langle\varphi_j(\omega,\mathbf{q})\varphi_{j'}(\omega,\mathbf{q}')\rangle = 16\pi^4\delta(\omega+\omega')\delta(\mathbf{q}+\mathbf{q}')\delta_{j,j'}\langle\varphi_j\varphi_j\rangle_{\omega,\mathbf{q}} , \tag{A7}$$

with

$$\langle\varphi_j\varphi_j\rangle_{\omega,\mathbf{q}}=2\pi[|\phi_{j\mathbf{q}}|^2\delta(\omega-\omega_{j\mathbf{q}})(n_{j\mathbf{q}}+1) + |\phi_{j,-\mathbf{q}}|^2\delta(\omega+\omega_{j,-\mathbf{q}})n_{j,-\mathbf{q}}] . \tag{A8}$$

The quantity $\phi_{j\mathbf{q}}$, the Fourier transform of the electron-phonon interaction potential, is specified in the usual manner. For *acoustical* phonons in the deformation potential model (DA) and for *piezoelectrical* phonons (PA) we have $\omega_{\mathbf{q}}=sq$, where s is the sound velocity. For optical (DO) and *polar optical* (PO) phonons, $\omega_q=\omega_0$. With C_i denoting the interaction constant ($i\equiv$DA,PA,DO,PO) we have

$$|\phi^{\rm DA}_{\mathbf{q}}|^2=C_{\rm DA}q,\quad |\phi^{\rm PA}_{j\mathbf{q}}|^2=C^j_{\rm PA}/q,\quad |\phi^{\rm DO}_{\mathbf{q}}|^2=C_{\rm DO} , \quad |\phi^{\rm PO}_{\mathbf{q}}|^2=C_{\rm PO}/q . \tag{A9}$$

The potential correlators pertaining to the total potential $\varphi(\omega,\mathbf{q})$, as given by Eq. (A4), are obtained from Eqs. (A7)–(A9).

APPENDIX B

Below we calculate the dielectric function $\Delta\epsilon_K(\omega,q_x)$ of a Q1DEG as given by Eq. (23) of I. Considering only one sort of particle (electrons) we can drop the index p and write [$K=(a,b)$, $q_x=q$]

$$\Delta\epsilon_{ab}(\omega,q)=W^+_a(\omega,q)-W^-_b(\omega,q) , \tag{B1}$$

where

$$W^\pm_a(\omega,q)=\frac{m^*e^2}{\pi\hbar^2q}\int\frac{f_{ak}}{k^\pm-k+i\delta}dk , \tag{B2}$$

with $k^\pm=(m^*/q\hbar)(\omega-\omega_{ab})\pm q/2$. We remind that f_{ak} is the electron distribution function pertinent to the ath subband and $\omega_{ab}=\omega_a-\omega_b$ is the energy difference between subbands a and b. As for the imaginary part of $\Delta\epsilon(\ )$ we obtain

$$\mathrm{Im}\Delta\epsilon_{ab}(\omega,q)=\frac{m^*e^2}{\hbar^2q}(f_{bk^-}-f_{ak^+}) . \tag{B3}$$

For explicit expressions of the functions defined by Eqs. (B1)–(B3) we must specify the function f_{ak}. We assume it to be the equilibrium Fermi-Dirac function.

For *nondegenerate* electrons we have $f_{ak}=(n_a\pi/k_T)\exp(-k^2/k_T^2)$, where $k_T=2m^*k_BT/\hbar$ is the thermal wave vector and $n_a=N_a/L$ is the concentration of the Q1DEG in the ath subband. This leads to

$$W^\pm_a(\omega,q)=\frac{e^2n_a}{\hbar v_Tq}[F(x^\pm)-i\sqrt{\pi}e^{-(x^\pm)^2}] , \tag{B4}$$

where $v_T=\hbar k_T/m^*$ is the thermal velocity, $x^\pm=(\omega-\omega_{ab}\pm\hbar q^2/m^*)/qv_T$, and $F(x)=\int\exp(z^2)dz/(x-z)\sqrt{\pi}$. For $|x|\gg1$, $F(x)\approx1/x$ and for $|x|\ll1$, $F(x)\approx2x$. Equation (B4) specifies $\mathrm{Re}\Delta\epsilon(\ )$ and $\mathrm{Im}\Delta\epsilon(\ )$. In particular, for intrasubband transitions we have $a=b$ and

$$\mathrm{Im}\Delta\epsilon_{aa}(\omega,q)=\frac{2\sqrt{\pi}e^2n_a}{\hbar v_Tq}\sinh\left[\frac{\hbar\omega}{2k_BT}\right]e^{-(\omega/qv_T)^2-(q/2k_T)^2} . \tag{B5}$$

The real part of $\Delta\epsilon(\ )$ is not very simple for $a=b$; but if $|\omega\pm\hbar q^2/2m^*|\ll qv_T$ we have

$$\mathrm{Re}\Delta\epsilon_{aa}(\omega,q)\cong\frac{e^2n_a}{k_BT}\cong\epsilon_L\eta ; \tag{B6}$$

ϵ_L is the lattice dielectric constant and η the plasma parameter. For a weakly nonideal electron gas, $\eta\ll1$. We notice that for $\omega\le qv_T$, a condition appropriate for transport problems, we have $|\Delta\epsilon_{aa}(\omega,q)|\le\epsilon_L\eta\ll\epsilon_L$, i.e., the screening of intrasubband transitions by electrons is weak.

For *degenerate* electrons, at zero temperature, Eq. (B2) becomes

$$W^\pm_a(\omega,q)=\frac{m^*e^2}{\pi\hbar^2q}\left[\ln\left|\frac{\omega_q+q^\pm}{\omega_q-q^\mp}\right|-i\pi\theta(-q^\pm<\omega_q<q^\mp)\right] , \tag{B7}$$

where $\omega_q=2m^*(\omega-\omega_{ab})/\hbar q$ and $q^\pm=2k_{Fa}\pm q$, k_{Fa} being the Fermi wave vector of the ath subband. Moreover, $\theta(x<y<z)=1$ for $x<y<z$ and equals zero otherwise. For intrasubband transitions $\omega_q=2m^*\omega/\hbar q$, and the static dielectric function ($\omega=0$) takes the simple form[1]

$$\Delta\epsilon_{aa}(q)=\frac{2m^*e^2}{\pi\hbar^2q}\ln\left|\frac{2k_{Fa}+q}{2k_{Fa}-q}\right| . \tag{B8}$$

Concerning $\mathrm{Im}\Delta\epsilon_{aa}$, which enters the expressions (69) and (70) of I, for the relaxation frequencies, we remark that, due to the factor $\omega_T/\sinh(\omega_T)$ in these expressions, only the frequency region $0<\omega<k_BT/\hbar\ll\mu-\hbar\omega_a$ gives the dominant contribution to the integrals. In this case we have

$$\mathrm{Im}\Delta\epsilon_{aa}(\omega,q)\cong\frac{2e^2q}{\hbar}\delta(\omega-qv_{Fa})+\frac{m^*e^2\omega}{\hbar(\mu-\hbar\omega_a)}\delta(q-2k_{Fa}) . \tag{B9}$$

Detailed calculations will be reported elsewhere.

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