Dielectric formalism for a quasi-one-dimensional electron gas. I. Quantum transport equations

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The quantum kinetic equation for the one-particle distribution function, pertinent to a quasi-one-dimensional electron gas, is derived from first principles. The electrons are assumed to interact weakly with an external system, e.g., impurities, phonons, and/or with each other. Correlations between the fluctuations of the density operator and of the scattering potential are taken into account in a manner equivalent to the polarization approximation. The derived collision integral allows for screening and is expressed in terms of the dielectric functions and potential correlators. The results are valid for weak scattering potentials of arbitrary type. From the kinetic equation energy and momentum balance equations are derived. The relevant energy and momentum relaxation rates, with the help of model distribution functions, can be expressed in terms of the dielectric functions and take a simple form suitable for practical applications.

I. INTRODUCTION

The study of quasi-one-dimensional (Q1D) systems has been intensified in recent years. When the width of a quantum wire becomes much smaller than the impurity mean free path and comparable to the de Broglie wavelength for electrons, quantum-size effects are expected and have been observed such as the oscillatory behavior of the conductivity of a quantum wire as function of the impurity density,[1] the quenching of the Hall effect,[2] etc. Besides, these systems are expected to have high electron mobilities, since the relevant k space for scattering is severely restricted along two directions, and possible device applications.[3]

Concerning transport along a quantum wire, most of the theoretical treatments of which we are aware consider δ-function or model impurity-scattering potentials starting from linear-response theory[4] or from the Boltzmann equation[5] without a thorough treatment of screening. The latter has been treated in Refs. 5 and 7 but electron transport has been considered only in the ballistic case.[7] Given the importance of these Q1D systems we feel that a rigorous treatment of transport should include a proper treatment of screening in restricted geometries and should be valid for arbitrary potentials including electron-electron interaction. This is done in this paper using the so-called dielectric formalism, which turned out to be very convenient. It consists in the averaging of the fluctuating microscopic variables and was first applied by Klimontovich and Silin to a classical electron-ion plasma.[8] Subsequently it was used in a number of three-dimensional (3D) classical and quantum plasmas.[9] A modification of this formalism allowing for nonuniform geometries, size quantization, and arbitrary scattering potentials was made by Boiko and Sirenko[10–12] who treated various problems related to the two-dimensional electron gas (2DEG), both classical[11] and quantum,[10] such as mutual Coulomb scattering, [11] nonlinear conduction,[12] etc.

In this paper, we derive, in Sec. II, the quantum kinetic equation for the one-electron distribution function applicable to a Q1D electron gas, with account taken of the screening and of particle-particle correlations in a manner equivalent to the polarization approximation.[9] The collision integral is expressed in terms of the dielectric functions and correlators which are evaluated in the next paper. From this equation follow, in Sec. III, energy and momentum balance equations as well as energy and momentum relaxation frequencies; the latter, with the help of model distribution functions, are expressed in terms only of the dielectric functions. Remarks and conclusions follow in the last section.

II. THE QUANTUM KINETIC EQUATION

A. Derivation

We consider a Q1D system, a quantum wire of length L_x with the other dimensions L_y and L_z much smaller than $L_x = L$. This system is homogeneous along the x direction and inhomogeneous in the yz plane. The confinement of an electron gas in such a system creates a Q1D electron gas (Q1DEG) with distinct energy levels (subbands). From the whole system (electrons plus lattice) we choose several sorts of particles, denoted by the subscript p (it can be one sort), and derive the quantum kinetic equation for the one-particle distribution function pertinent to p-sort particles. The rest of the system, e.g., impurities, phonons, other mobile charge carriers, is called an external system. It is assumed that the p-sort carriers interact weakly with the latter and with particles of another sort p'. For the description that follows we use as basis the eigenfunctions $\Psi_{pA}(\mathbf{r})$ of the free-particle

Hamiltonian $H_p^0(\mathbf{r})$ assuming that $\Psi_{pA}(\mathbf{r})=\Psi_\alpha(x)X_a^p(\boldsymbol{\rho})$, where $\boldsymbol{\rho}=(y,z)$, and denoting the eigenvalues of $H_p^0(\mathbf{r})$ by $\hbar\omega_A^p=\hbar\omega_\alpha^p+\hbar\omega_\alpha^p$. Here the small Greek letters α, β, and γ label the "longitudinal" eigenfunctions and eigenvalues, while the small Roman latters label the corresponding "transverse" entities; the complete set is denoted by the capital Greek letters $A=(\alpha,a)$, $B=(\beta,b)$, $\Gamma=(\gamma,g)$, etc. In the language of second quantization, with $a_{pA}^\dagger$ and a_{pA} being the creation and annihilation operators, respectively, the Hamiltonian H of the whole system is given by

$$H=H_s+\sum_{p,A}\hbar\omega_A^p a_{pA}^\dagger a_{pA}+\sum_{p,A,B}e_p(\varphi_{pAB}^{\rm ex}+\tilde{\varphi}_{pAB}^{\,s})a_{pA}^\dagger a_{pB}+\tfrac{1}{2}\sum_{\substack{p,A,B,\\ p',A',B'}}V_{AA'BB'}^{pp'}a_{pA}^\dagger a_{p'A'}^\dagger a_{p'B'}a_{pB}(2-\delta_{pp'})\ . \tag{1}$$

Here H_s is the Hamiltonian of the external system, e_p is the charge of the p-sort carriers, $\varphi^{\rm ex}=\varphi^{\rm ex}(\mathbf{r},t)$ is the potential of the external electric field [when the latter is not included in $H_p^0(\mathbf{r})$], and $\tilde{\varphi}_s=\tilde{\varphi}_s(\mathbf{r},t)$ is a potential created by the external scattering system. We have

$$\varphi_{pAB}(t)=\int\Psi_{pA}^*(\mathbf{r})\varphi(\mathbf{r},t)\Psi_{pB}(\mathbf{r})d^3r \tag{2}$$

and

$$V_{AA'B'B}^{pp'}=e_pe_{p'}\int\int\Psi_{pA}^*(\mathbf{r}_1)\Psi_{p'A'}^*(\mathbf{r}_2)|\mathbf{r}_1-\mathbf{r}_2|^{-1}\times\Psi_{p'B'}(\mathbf{r}_2)\Psi_{pB}(\mathbf{r}_1)d^3r_1d^3r_2\ , \tag{3}$$

with $\mathbf{r}_1$ and $\mathbf{r}_2$ being the position vectors of carriers p and p', respectively.

The equation of motion for the one-particle density operator

$$\rho_{AB}^p=a_{pB}^\dagger a_{pA}\ ,\qquad i\hbar\frac{\partial\rho_{AB}^p}{\partial t}=[\rho_{AB}^p,H]\ , \tag{4}$$

where [ ] denotes the commutator, after substituting in it Eq. (1), takes the form

$$i\hbar\frac{\partial\rho_{AB}^p}{\partial t}=\sum_\Gamma(\{H_{A\Gamma}^p,\rho_{\Gamma B}^p\}-\{\rho_{A\Gamma}^p,H_{\Gamma B}^p\})\ , \tag{5}$$

where $\{L,M\}=(LM+ML)/2$. H_{AB}^p is given by Eq. (2) with the change $\varphi(\mathbf{r},t)\rightarrow H^p(\mathbf{r},t)$ and

$$H^p(\mathbf{r},t)=H_p^0(\mathbf{r})+e_p\varphi^{\rm ex}(\mathbf{r},t)+e_p\varphi(\mathbf{r},t)\ , \tag{6}$$

$$\varphi(\mathbf{r},t)=\tilde{\varphi}_s(\mathbf{r},t)+\sum_p\varphi_p^0(\mathbf{r},t)\ , \tag{7}$$

$$\varphi_p^0(\mathbf{r},t)=e_p\int\frac{N_p(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}d^3r'\ . \tag{8}$$

The total number operator of p particles $N_p(\mathbf{r},t)$, is given by

$$N_p(\mathbf{r},t)=\rho^p(\mathbf{r},\mathbf{r},t)=\sum_{A,B}\Psi_{pB}^*(\mathbf{r})\Psi_{pA}(\mathbf{r})\rho_{AB}^p(t)\ , \tag{9}$$

and is related to the mean value f_{AB}^p (i.e., the one-particle distribution function) of ρ_{AB}^p by the normalization condition $\langle N_p\rangle=\sum_A\langle\rho_{AA}^p\rangle=\sum_A f_{AA}^p$.

The operators ρ^p and φ are split into average and fluctuating parts ($f_{AB}^p=\langle\rho_{AB}^p\rangle$):

$$\rho_{AB}^p=f_{AB}^p+\delta\rho_{AB}^p,\quad \varphi=\langle\varphi\rangle+\delta\varphi\ . \tag{10}$$

Then the average of Eq. (5), i.e., the quantum kinetic equation for the distribution function f_{AB}^p, reads

$$i\hbar\frac{\partial f_{AB}^p}{\partial t}=\sum_\Gamma[(H_p^0+e_p\varphi^{\rm ex}+e_p\langle\varphi\rangle)_{A\Gamma}^p f_{\Gamma B}^p-f_{A\Gamma}^p(H_p^0+e_p\varphi^{\rm ex}+e_p\langle\varphi\rangle)_{\Gamma B}^p]+i\hbar{\rm St}f_{AB}^p\ , \tag{11}$$

where

$${\rm St}f_{AB}^p=(e_p/i\hbar)\sum_\Gamma\langle\{\delta\varphi_{A\Gamma}^p,\delta\rho_{\Gamma B}^p\}-\{\delta\rho_{A\Gamma}^p,\delta\varphi_{\Gamma B}^p\}\rangle\ . \tag{12}$$

So far no assumptions were made in deriving Eqs. (11) and (12). The main task now is to express ${\rm St}f_{AB}^p$ in terms of the distribution functions f_{AB}^p. This will be done under certain assumptions and it will be shown that ${\rm St}f_{AB}^p$, when expressed in terms of f_{AB}^p, is the collision integral of the quantum kinetic equation.

Subtracting Eq. (11) from Eq. (5) we obtain

$$i\hbar\frac{\partial}{\partial t}\delta\rho_{AB}^p=\hbar\omega_{AB}^p\delta\rho_{AB}^p+(f_B^p-f_A^p)e_p\delta\varphi_{AB}^p\ , \tag{13}$$

where $\omega_{AB}^p=\omega_A^p-\omega_B^p$ and $f_A^p\equiv f_{AA}^p$. In obtaining Eq. (13) we omitted, on the right-hand side, the terms

$$\sum_\Gamma(\varphi_{A\Gamma}^{\rm ex}\delta\rho_{\Gamma B}^p-\delta\rho_{A\Gamma}^p\varphi_{\Gamma B}^{\rm ex}+C-\langle C\rangle)\ ,$$

where

$$C=\{\delta\varphi_{A\Gamma}^p,\delta\rho_{\Gamma B}^p\}-\{\delta\rho_{A\Gamma}^p,\delta\varphi_{\Gamma B}^p\}\ .$$

This neglect corresponds to the following assumptions.

(1) The collisions do not influence the fluctuations. This is valid for weak interactions.

(2) The nondiagonal elements of the density matrix are small compared to the diagonal ones.

(3) The electric field does not influence the fluctuations.

(4) Exchange interaction between identical particles is not important.

The fluctuating part $\delta\varphi$ of the potential can be written as

$$\delta\varphi(r,t)=\delta\varphi_s(r,t)+\sum_p\delta\varphi_p(r,t)\ . \tag{14}$$

The first term represents the fluctuation of the external system in the absence of the plasma and the second term arises as linear response to the density fluctuation δN_p:

$$\delta\varphi_p(\omega,q_x,\boldsymbol{\rho})=e_p\int\frac{\delta N_p(\omega,q_x,\boldsymbol{\rho}')}{\epsilon_s(\omega,q_x,\boldsymbol{\rho},\boldsymbol{\rho}')}d^2\rho' \tag{15}$$

where $\epsilon_s(\ )$ is the dielectric function of the external system. In order to simplify the notation in what follows we

put $q_x \equiv q$. Taking a one-side Fourier transform of Eq. (13), with respect to the time, involves the quantities $\delta\rho^0_{pAB}(\omega)$ and $\delta\varphi^p_{AB}(\omega)$. $\delta\rho^0_{pAB}(\omega)=i\delta\rho^p_{AB}(t=0)/(\omega-\omega^p_{AB}+i\delta)$ with $\delta\to 0+$, and $\delta\rho^p_{AB}(t=0)$ describes the fluctuation of the density operator at $t=0$. We obtain

$$\delta\rho^p_{AB}(\omega)=\delta\rho^0_{pAB}(\omega)+\frac{e_p}{\hbar}\frac{f^p_B-f^p_A}{\omega-\omega^p_{AB}+i\delta}\delta\varphi^p_{AB}(\omega)\ , \tag{16}$$

$$\delta\varphi^p_{AB}(\omega)=\int dq\,(e^{iqx})^p_{\alpha\beta}\delta\varphi^p_{ab}(\omega,q)\ , \tag{17}$$

$$\delta\varphi^p_{ab}(\omega,q)=\int \chi^{*p}_a(\rho)\chi^p_b(\rho)\delta\varphi(\omega,q,\rho)d^2\rho\ , \tag{18}$$

where

$$(e^{iqx})^p_{\alpha\beta}=\int \Psi^*_\alpha(x)\Psi_\beta(x)e^{iqx}dx\ . \tag{19}$$

With the abbreviation $K=(p,a,b)$ we obtain

$$\delta\varphi_K(\omega,q)=\delta\varphi^s_K(\omega,q)+\sum_{p'A'B'}e_{p'}\frac{(e^{-iqx})^{p'}_{\beta'\alpha'}\delta\rho^{p'}_{A'B'}(\omega)}{\epsilon^s_{KK'}(\omega,q)}\ , \tag{20}$$

with

$$\frac{1}{\epsilon^s_{KK'}(\omega,q)}=\int\int\frac{\chi^{*p}_a(\rho)\kappa_b(\rho)\kappa^{p'}_{a'}(\rho')\kappa^{*p'}_{b'}(\rho')}{\epsilon_s(\omega,q,\rho,\rho')}d^2\rho d^2\rho'\ . \tag{21}$$

We now substitute Eqs. (16) and (17) in Eq. (20) and use the definitions

$$P_{KK'}(\omega,q)=\delta_{KK'}+\Delta\epsilon_{K'}(\omega,q)/\epsilon^s_{KK'}(\omega,q)\ , \tag{22}$$

$$\Delta\epsilon_K(\omega,q)=(e_p^2/\hbar L)\sum_{\alpha,\beta}\frac{f^p_A-f^p_B}{\omega-\omega^p_{AB}+i\delta}|(e^{iqx})^p_{\alpha\beta}|^2\ , \tag{23}$$

$$R_{KK'}(\omega,q)=[P^{-1}(\omega,q)]_{KK'}\ , \tag{24}$$

and

$$\frac{1}{\epsilon_{KK'}(\omega,q)}=\sum_{K''}\frac{R_{KK''}(\omega,q)}{\epsilon^s_{K''K'}(\omega,q)}\ . \tag{25}$$

We then obtain

$$\delta\varphi^p_{AB}(\omega)=\frac{1}{L}\sum_{q,K'}(e^{iqx})^p_{\alpha\beta}\left[\frac{\delta\varphi^0_{K'}(\omega,q)}{\epsilon_{KK'}(\omega,q)}+R_{KK'}(\omega,q)\delta\varphi^s_{K'}(\omega,q)\right]\ , \tag{26}$$

where

$$\varphi^0_K(\omega,q)=e_p\sum_{\alpha,\beta}(e^{-iqx})^p_{\beta\alpha}\delta\rho^0_{pAB}(\omega)\ . \tag{26'}$$

To arrive at Eq. (22) we have used

$$(e^{iqx})^p_{\alpha\beta}(e^{-iq'x})^p_{\beta\alpha}\to|(e^{iqx})^p_{\alpha\beta}|^2\delta_{q,q'}\ ,$$

justified, for example, in a plane-wave basis.

From Eq. (12) by Fourier transformation we obtain

$$\mathrm{St}f^p_{AB}=\frac{e_p}{4\pi^2i\hbar}\sum_\Gamma\int\int\langle\{\delta\varphi^p_{A\Gamma}(\omega),\delta\rho^p_{\Gamma B}(\omega')\}-\{\delta\rho^p_{A\Gamma}(\omega),\delta\varphi^p_{\Gamma B}(\omega')\}\rangle e^{-i(\omega+\omega')t}d\omega d\omega'\ . \tag{27}$$

To obtain the explicit form of $\mathrm{St}f^p_{AB}$ we substitute Eq. (26) for $\delta\varphi^p_{AB}(\omega)$ and Eqs. (16) and (17) for $\delta\rho^p_{AB}(\omega)$ in Eq. (27).

Since the system is homogeneous in the x direction, the symmetrized potential correlator $\langle\{\ \}\rangle$ for the external system can be written[13] as

$$\langle\{\delta\varphi^s_K(\omega,q),\delta\varphi^s_{K'}(\omega',q')\}\rangle=4\pi^2\delta(\omega+\omega')\delta(q+q')\langle\delta\varphi^2_s\rangle^{K\bar K'}_{\omega,q}\ , \tag{28}$$

where $\bar K=(p,b,a)$ and

$$\langle\delta\varphi^2_s\rangle^{K\bar K'}_{\omega,q}=\int\int\chi^{*p}_a(\rho)\chi^p_b(\rho)\chi^{p'}_{a'}(\rho')\chi^{*p'}_{b'},(\rho')\langle\delta\varphi^2_s\rangle^{\rho\rho'}_{\omega,q}d^2\rho d^2\rho'\ . \tag{29}$$

For some particular cases the correlator (29) is evaluated in the next paper. As for the initial density fluctuations we use the relation[14]

$$\langle\delta\rho^0_{pAB}(\omega)\delta\rho^0_{p'A'B'}(\omega')\rangle=4\pi^2\delta(\omega+\omega')\delta(\omega-\omega^p_{AB})\delta_{pp'}\delta_{ABA'}f^p_B(1-f^p_A)\ . \tag{30}$$

With the help of Eqs. (28), (29), and (30), Eq. (27) takes the form

$$\mathrm{St}f^p_{AB}=\mathrm{St}_{ps}f^p_{AB}+\sum_{p'}\mathrm{St}_{pp'}f^p_{AB}\ . \tag{31}$$

It can be shown, using certain symmetry properties [cf. (A1)–(A3)], that

$$\mathrm{St}f^p_{AB}=(\mathrm{St}f^p_{BA})^*\ , \tag{32}$$

i.e., the collision integral is a Hermitian operator.

The general expression for $\mathrm{St}f^p_{AB}$ is rather unwieldy and will not be written down. Below we give the form for the di-

agonal function only $f^p_{AA}\equiv f^p_A$:

$$\mathrm{St}_{ps}f^p_A=\frac{e_p^2}{4\pi\hbar^2}\sum_B\int|(e^{iqx})^p_{\alpha\beta}|^2dq\sum_{K',K''}R_{KK'}(\tilde{\omega},q)R^*_{KK''}(\tilde{\omega},q)\times\left[\frac{2}{\hbar}(f^p_B-f^p_A)\langle\delta\varphi^2_s\rangle^{K'K''}_{\tilde{\omega},q}-i\left[\frac{1}{\epsilon^s_{K'K''}(\tilde{\omega},q)}-\frac{1}{\epsilon^{*s}_{K''K'}(\tilde{\omega},q)}\right]\times[f^p_A(1-f^p_B)+f^p_B(1-f^p_A)]\right] \tag{33}$$

$$\mathrm{St}_{pp'}f^p_A=\sum_{B,A',B'}W^{pp'}_{AB',BA'}[f^p_B(1-f^p_A)f^{p'}_{A'}(1-f^{p'}_{B'})-f^p_A(1-f^p_B)f^{p'}_{B'}(1-f^{p'}_{A'})]\ , \tag{34}$$

where $\tilde{\omega}=\omega^p_{AB}$ and the transition probability $W^{pp'}_{AB',BA'}$ is given by

$$W^{pp'}_{AB',BA'}=\frac{2\pi}{\hbar L^2}e_p^2e_{p'}^2\sum_q\frac{|(e^{iqx})^p_{\alpha\beta}|^2|(e^{iqx})^{p'}_{\alpha'\beta'}|^2}{|\epsilon_{KK'}(\tilde{\omega},q)|^2}\delta(\hbar\tilde{\omega}-\hbar\tilde{\omega}')\ . \tag{35}$$

Equation (34) for $p=p'$ is the familiar form of the collision integral for scattering between identical particles, e.g., electron-electron scattering. Using the property (A9) of Appendix A it can be shown that Eq. (33), representing the collisional integral for scattering of the particles by, e.g., fixed impurities, phonons, etc. can be written in the familiar form

$$\mathrm{St}_{ps}f^p_A=\sum_B[W^p_{BA}f^p_B(1-f^p_A)-W^p_{AB}f^p_A(1-f^p_B)]\ ; \tag{36}$$

the transition probability W^p_{AB} is given by

$$W^p_{AB}=\frac{e_p^2}{\hbar^2L}\sum_q|(e^{iqx})^p_{\alpha\beta}|^2\langle\delta\phi_s\delta\phi_s\rangle^K_{\tilde{\omega},q}\ , \tag{37}$$

where

$$\langle\delta\phi_s\delta\phi_s\rangle^K_{\omega,q}=\sum_{K',K''}R_{KK'}(\omega,q)R^*_{KK''}(\omega,q)\langle\delta\varphi^s_{K'}\delta\varphi^s_{\bar{K}''}\rangle_{\omega,q}\ ; \tag{37'}$$

$\langle\delta\varphi^s_K\delta\varphi^s_{K'}\rangle_{\omega,q}$ is given by Eq. (28) with the change $\langle\{\ \}\rangle\rightarrow\langle\ \rangle$. When the external system is in equilibrium, at temperature T_s, Eq. (33), with the help of Appendix A, cf. (A11), takes the form

$$\mathrm{St}_{ps}f^p_A=\frac{e_p^2}{\hbar L}\sum_{q,B}|(e^{iqx})^p_{\alpha\beta}|^2\langle\delta\phi^2_s\rangle^K_{\tilde{\omega},q}\left[f_B-f_A-[f^p_A(1-f^p_B)+f^p_B(1-f^p_A)]\tanh\left[\frac{\hbar\tilde{\omega}}{2k_BT_s}\right]\right]$$
$$=\frac{e_p^2}{\hbar L}\sum_{q,B}|(e^{iqx})^p_{\alpha\beta}|^2\Pi_K(\tilde{\omega},q)\{(f^p_B-f^p_A)\coth(\hbar\tilde{\omega}/2k_BT_s)-[f^p_A(1-f^p_B)+f^p_B(1-f^p_A)]\}\ , \tag{38}$$

or the form (36) with

$$W^p_{AB}=\frac{2e_p^2}{\hbar L}[1-\exp(-\hbar\tilde{\omega}/k_BT_s)]^{-1}\sum_q|(e^{iqx})^p_{\alpha\beta}|^2\Pi_K(\tilde{\omega},q)\ , \tag{39}$$

respectively, with

$$\Pi_K(\omega,q)=\mathrm{Im}\sum_{K'}\frac{R_{KK'}(\omega,q)}{\epsilon^*_{KK'}(\omega,q)}\ , \tag{40}$$

obeying the symmetry properties

$$\Pi_K(\omega,q)=\Pi^*_K(\omega,q)=-\Pi_{\bar{K}}(-\omega,-q)\ . \tag{41}$$

For elastic scattering the second term in $\{\ \}$ of Eq. (38) vanishes and it can be omitted.

The quantum kinetic equation for the diagonal part of the density function $f^p_A\equiv f^p_{AA}$ is given by Eq. (11) together with Eqs. (31) and (33)–(37). For $p=p'=1$ we have one equation for the Q1DEG, whose particles (electrons) interact weakly with an external system (e.g., impurities) and with each other.

A considerable simplification of all the expressions above occurs if we have one sort of carrier ($p=p'$) occupying the lowest subband ($\alpha=\beta$, $a=b$). The index K can be dropped and we have

$$\epsilon(\omega,q)=\epsilon^s(\omega,q)+\Delta\epsilon(\omega,q)\ , \tag{42}$$

$$R(\omega,q)=\epsilon^s(\omega,q)/\epsilon(\omega,q)\ , \tag{43}$$

and

$$\Pi(\omega,q)=\mathrm{Im}\epsilon^s(\omega,q)/|\epsilon(\omega,q)|^2\ . \tag{44}$$

B. Remarks

(1) In deriving the kinetic equations we use the equation for fluctuations $\delta\rho$ of the density matrix within the so-called collisionless approximation, which corresponds to the neglect of the terms in Eq. (13), responsible for the influence of collisions on the fluctuations. This approximation is equivalent to the polarization approximation in the equation for the two-particle correlation function g_2 in Bogoliubov's method.[9] Though Bogoliubov's and Klimontovich's formalisms are equivalent in this approximation, the latter is more convenient in the case of systems with long-range Coulomb interaction (instead of the equation for the matrix g_2 we solve the equation for $\delta\rho$). The employed approximation requires certain inequalities to hold. For definiteness, we shall consider the plane-wave basis.

Firstly, the interaction between particles must be weak enough that the gas can be considered as weakly nonideal, i.e.,

$$\eta \ll 1 \ ; \tag{45}$$

here $\eta = e^2 n / \epsilon_L W$ is a plasma parameter, W is the thermal or Fermi energy of the 1 DEG, and ϵ_L is the lattice dielectric constant.

Secondly, the interaction between two particles at small distances is treated in a perturbative manner. That imposes a restriction on the region of considered wave vectors:

$$q \ll 1/r_\Lambda \ . \tag{46}$$

Here the Landau length r_Λ is the distance between particles, where the Coulomb interaction energy is equal to the mean kinetic energy, $e^2/\epsilon_L r_\Lambda = W$. Therefore, our consideration will be valid if the contribution of the region $q \geq 1/r_\Lambda$ in the treated problem can be neglected (e.g., some integrals are cut off at the thermal vector, smaller than r_Λ^{-1}). If this does not hold we must cut off the integration over q at r_Λ^{-1} artificially [the resulting accuracy is within a factor of order unity in the 1D and 2D cases, and logarithmic in the 3D case—$\ln(q_{\max}/q_{\min})$]. This cutoff can be accounted for exactly with the help of the T matrix of the two-particle interaction.

Thirdly, the interaction with the external system is considered to be weak; this holds when

$$\nu_{\rm ES} \ll W/\hbar \ , \tag{47}$$

where $\nu_{\rm ES}$ is the frequency of scattering by the external system. In other words, collisional broadening of the energy levels must be smaller than the average energy.

(2) In a 1D weakly nonideal plasma the dielectric function of the 1DEG, $\Delta\epsilon$, in many cases appears to be small in comparison with the lattice dielectric constant ϵ_L. This means that screening effects are not important. To understand the reasons for this fact let us look upon the static screening in the cases of nondegenerate 3D, 2D, and 1D electron gases ($n_i, i=1,2,3$, is the electron concentration).

3D. For $k \ll k_T$ the dielectric function $\Delta\epsilon(0,\mathbf{k}) \simeq \epsilon_L / k r_D)^2$, where $\mathbf{k}=(k_x,k_y,k_z)$, $k_T = \sqrt{2mT}/\hbar$, and $r_D = (\epsilon_L T/4\pi e^2 n_3)^{1/2}$ is the Debye length. Then the potential created by an external charge $\tilde{e}$ is $\varphi(\mathbf{r}) = (\tilde{e}/\epsilon_L r)\exp(-r/r_D)$.

2D. For $q \ll k_T$ we have $\Delta\epsilon(0,\mathbf{q}) \simeq \epsilon_L / q\rho_D$, where $\mathbf{q}=(q_x,q_y)$ and $\rho_D = \epsilon_L T/2\pi e^2 n_2$. When $\rho \gg \rho_D$ the potential is $\varphi(\boldsymbol{\rho}) = (\tilde{e}/\epsilon_L)(\rho_D^2/\rho^3)$.

1D. For $|q_x| \ll k_T$ we have $\Delta\epsilon(0,q_x) = \sqrt{2}\eta\epsilon_L$, where $\eta = e^2 n_1/\epsilon_L T$ is a plasma parameter, $\eta \ll 1$. Then it can be shown that $\varphi(x) \simeq \tilde{e}/\epsilon_L (1+\sqrt{2}\eta)|x| \simeq \tilde{e}/\epsilon_L |x|$.

We see that in the 3D case a test charge is surrounded by an electron gas from all sides and this leads to a strong exponential screening. In the 2D case a test particle is surrounded only by electrons in the plane and the screening is weaker, of a power law. In the 1D case only[2] a small number of electrons in the line can screen the test charge. This leads to a renormalization of the dielectric constant now given by $\epsilon_L(1+\sqrt{2}\eta)$, with $\eta \ll 1$.

(3) Some words must be said about electron-electron collisions in the Q1DEG. From Eq. (34) it can be shown that when the Q1DEG occupies only the lowest subband, the collision integral $\mathrm{St}_{ee} f_k$ vanishes in the polarization approximation when the wave functions along the x direction are plane waves. The reason is that, due to momentum and energy conservation, two electrons moving in one dimension either do not "notice" each other or simply exchange states. Other electrons only screen this two-particle interaction. A nonzero value for $\mathrm{St}_{ee} f_k$ is obtained only beyond the polarization approximation.

III. BALANCE EQUATIONS

As is well known, energy and momentum balance equations can be derived from the quantum kinetic equation. Concerning the Q1DEG, when the wave functions along the direction x are plane waves, we have $\alpha = k_x$ (only one sort of particle is considered) and

$$\Psi_\alpha(x) = e^{ik_x x}/\sqrt{L_x} \ . \tag{48}$$

This leads to

$$(e^{iq_x x})_{\alpha\alpha'} = \delta_{q_x, k_x - k'_x} \ , \tag{49}$$

assuming that spin is conserved in the collisions. We further assume that $f_{abk_x k'_x}$ is diagonal in the "transverse" indices, i.e., $f_{abk_x k'_x} = f_{ak_x k'_x}\delta_{ab}$. This implies that the nondiagonal components of the density matrix are small compared with the diagonal ones. The distribution function is normalized as

$$2\sum_k f_{ak} = N_a \ , \tag{50}$$

where N_a is the total number of particles in the ath subband. The system is homogeneous in the x direction. In the presence of an external electric field $\mathbf{E} = E\hat{\mathbf{x}}$, $\varphi^{\rm ex}(x) = -Ex$ and the kinetic equation, with the help of Eqs. (11) and (31)–(39), takes the form ($k_x \equiv k, q_x \equiv q, L_x \equiv L$)

$$\frac{\partial f_{ak}}{\partial t} + \frac{eE}{\hbar}\frac{\partial f_{ak}}{\partial k} = \mathrm{St} f_{ak} = \mathrm{St}_{\rm ES} f_{ak} + \mathrm{St}_{ee} f_{ak} \ . \tag{51}$$

$\mathrm{St}_{ee}f_{ak}$ is given by Eq. (34) with $p=p'$, $A=ak$, etc. $\mathrm{St}_{\mathrm{ES}}f_{ak}$ is given by Eqs. (33) and (36)–(39).

Operating on Eq. (51) with $(2/L)\sum_{ka}$, $(2e/L)\sum_{ka}\hbar k/m^*$, and $(2/L)\sum_{ak}\hbar^2k^2/2m^*$, where m^* is the effective mass, we obtain

$$\frac{\partial n}{\partial t}=0, \quad \frac{\partial j}{\partial t}=\frac{en}{m^*}(eE-R) \tag{52}$$

and

$$\frac{\partial \epsilon}{\partial t}=jE-Pn \ . \tag{53}$$

Here n, j, and ϵ are the carrier density, current density, and kinetic energy of the Q1DEG, respectively. The quantities $-R$ and P are the friction force against the medium (i.e., external system) and the power transferred (per particle), respectively. Upon dropping the index ES from $\mathrm{St}_{\mathrm{ES}}$, they are given by

$$R=-(2/nL)\sum_{k,a}\hbar k\ \mathrm{St}f_{ak} \tag{54}$$

and

$$P=-(2/nL)\sum_{k,a}(\hbar^2k^2/2m^*)\mathrm{St}f_{ak} \ . \tag{55}$$

With the help of Eqs. (36) and (37′) we can write, in an obvious notation,

$$\begin{bmatrix}R\\P\end{bmatrix}=-\frac{1}{nL}\sum_{aa',kq}\begin{bmatrix}\hbar q\\ \hbar\tilde{\omega}\end{bmatrix}[W_{a',k-q,a,k}f_{a',k-q}(1-f_{ak})-W_{a,k,a',k-q}f_{ak}(1-f_{a',k-q})] \ , \tag{56}$$

where $\tilde{\omega}=\hbar[k^2-(k-q)^2]/2m^*+\omega_{aa'}$. For simplicity we model the diagonal part of the 1DEG density matrix, f_{ak}, by a drifted Fermi-Dirac distribution

$$f_{ak}=f_{\mathrm{eq}}[\hbar\omega_a+\hbar^2(k-m^*u/\hbar)^2/2m^*] \ ; \tag{57}$$

$f_{\mathrm{eq}}(\epsilon)=\{1+\exp[(\epsilon-\mu)/k_BT]\}^{-1}$, μ and T are the chemical potential and the effective temperature of the 1DEG, respectively, k_B is Boltzmann's constant, and $u=j/en$ is the mean drift velocity of its particles. The parameters u and T are to be found from the balance equations (52) and (53). We now substitute Eq. (57) into Eq. (56) and assume that the external system is in equilibrium at temperature T_s. Using Eq. (39) for W^p_{AB}, we obtain

$$\begin{bmatrix}R\\P\end{bmatrix}=\frac{\hbar}{4\pi^2n}\int\int d\omega\, dq\begin{bmatrix}q\\ \omega\end{bmatrix}\{\coth[\hbar(\omega-uq)/2k_BT]-\coth(\hbar\omega/2k_BT_s)\}\sum_K\Pi_K(\omega,q)\mathrm{Im}\Delta\epsilon^{eq}_K(\omega-qu,q) \ . \tag{58}$$

Equation (58) is convenient for applications because the quantities R and P are expressed in terms of the usually known susceptibilities of the 1DEG and of the external system.

When the external electric field E is small, Eq. (58) can be linearized for R over u and for P over $T-T_s$, if $T-T_s\ll k_BT^2/\hbar\omega_0, T$ and $\hbar q_0u\ll k_BT$. Here q_0 and ω_0 are the values of the wave vector and of the frequency which give the maximal contribution to the integrals over q and ω, respectively. In this case, the mean frequencies ν^m,ν^T of momentum and temperature relaxation, respectively, are given with the help of the definition

$$P=\nu^TK_B(T-T_s), \quad R=-m^*\nu^mu \ , \tag{59}$$

by

$$\begin{bmatrix}\nu^m\\ \nu^T\end{bmatrix}=\frac{1}{2\pi^2n}\int\int d\omega\, dq\frac{1}{\omega^2}\begin{bmatrix}q^2k_BT/m^*\\ \omega^2\end{bmatrix}\frac{\omega_T\omega_{Ts}}{\sinh(\omega_T)\sinh(\omega_{Ts})}\sum_K\Pi^{\mathrm{eq}}_K(\omega,q)\mathrm{Im}\Delta\epsilon^{\mathrm{eq}}_K(\omega,q) \ , \tag{60}$$

where $\omega_T=\hbar\omega/2k_BT$ and $\omega_{Ts}=\hbar\omega/2k_BT_s$. We notice that for a homogeneous system in the absence of a magnetic field, the energy balance equation takes the form $jE=\nu^Tk_B(T-T_s)$. Hence, it follows that $T=T_s(1+e^2E^2/k_BT_sm^*\nu^m\nu^T)$.

For the results (58)–(60) we employed the two-parameter model in which the drift velocities u_a and temperatures T_a are the same for all subbands: $u_a=u_a=\cdots=u$ and $T_a=T_{a'}=\cdots=T$. The same assumption was made for the effective masses. In a more sophisticate model the drift velocity and temperature depend on the subband index a. Replacing u by u_a, m^* by m^*_a, and T by T_a in Eq. (57) we obtain a set of balance equations for all subbands:

$$\frac{\partial n_a}{\partial t}\equiv(2/L)\sum_k\mathrm{St}f_{ak} \ , \tag{61}$$

$$\frac{\partial j_a}{\partial t}=\frac{en_a}{m^*_a}(eE-R_a), \quad \frac{\partial \epsilon_a}{\partial t}=j_aE-P_an_a \ , \tag{62}$$

$$\begin{bmatrix}R_a\\P_a\end{bmatrix}=-(2/n_aL)\sum_k\begin{bmatrix}\hbar k\\ \hbar^2k^2/2m^*_a\end{bmatrix}\mathrm{St}f_{ak} \ . \tag{63}$$

If intersubband transitions are forbidden or neglected, Eq. (63) is equivalent to the following equations:

$$R_a = R_{as} + \sum_{a'} R_{aa'}, \quad R_{aa} = 0 , \tag{64}$$

$$P_a = P_{as} + \sum_{a'} P_{aa'}, \quad P_{aa} = 0 , \tag{65}$$

where

$$\begin{bmatrix} R_{as} \\ P_{as} \end{bmatrix} = \frac{\hbar}{4\pi^2 n_a} \int\int d\omega\, dq \begin{bmatrix} q \\ \omega \end{bmatrix} [\coth(\hbar\bar{\omega}_a/2k_B T_a) - \coth(\hbar\omega/2k_B T_s)] \mathrm{Im}\Delta\epsilon_{aa}^{eq}(\bar{\omega}_a, q)\Pi_{aa}(\omega, q) , \tag{66}$$

$$\begin{bmatrix} R_{aa'} \\ P_{aa'} \end{bmatrix} = (\hbar/4\pi^2 n_a) \int\int d\omega\, dq \begin{bmatrix} q \\ \omega \end{bmatrix} [\coth(\hbar\bar{\omega}_a/2k_B T_a) - \coth(\hbar\bar{\omega}_{a'}/2k_B T_{a'})] \frac{\mathrm{Im}\Delta\epsilon_{aa}^{eq}(\bar{\omega}_a, q)\mathrm{Im}\Delta\epsilon_{a'a'}^{eq}(\bar{\omega}_{a'}, q)}{|\epsilon_{aa,a'a'}(\omega, q)|^2} , \tag{67}$$

with $\bar{\omega}_a = \omega - q u_a$.

Linearizing Eq. (66) over u_a and $T_a - T_s$, and Eq. (67) over $u_a - u_{a'}$ and $T_a - T_{a'}$, defines the momentum and temperature relaxation frequencies by the following equations:

$$R_{as} = -m_a^* \nu_{as}^m u_a, \quad P_{as} = \nu_{as}^T k_B (T_a - T_s), \quad R_{aa'} = m_a^* \nu_{aa'}^m (u_a - u_{a'}), \quad P_{aa'} = \nu_{aa'}^T k_B (T_a - T_{a'}) . \tag{68}$$

Notice that

$$\nu_{aa'}^m n_a m_a^* = \nu_{a'a}^m n_{a'} m_{a'}^* \quad \text{and} \quad \nu_{aa'}^T n_a = \nu_{aa'}^T n_{a'} . \tag{68'}$$

The relaxation frequencies are given by

$$\begin{bmatrix} \nu_a^m \\ \nu_{as}^T \end{bmatrix} = \frac{1}{2\pi^2 n_a} \int\int d\omega\, dq \frac{1}{\omega^2} \begin{bmatrix} k_B T q^2/m_a \\ \omega^2 \end{bmatrix} \frac{\omega_{Ta}\omega_{Ts}}{\sinh(\omega_{Ta})\sinh(\omega_{Ts})} \Pi_{aa}^{eq}(\omega, q)\mathrm{Im}\Delta\epsilon_{aa}^{eq}(\omega, q) \tag{69}$$

and

$$\begin{bmatrix} \nu_{aa'}^m \\ \nu_{aa'}^T \end{bmatrix} = \frac{1}{2\pi^2 n_a} \int\int d\omega\, dq \frac{1}{\omega^2} \begin{bmatrix} k_B T q^2/m_a^* \\ \omega^2 \end{bmatrix} \frac{\omega_{Ta}\omega_{Ta'}}{\sinh(\omega_{Ta})\sinh(\omega_{Ta'})} \frac{\mathrm{Im}\Delta\epsilon_{aa}^{eq}(\omega, q)\mathrm{Im}\Delta\epsilon_{a'a'}^{eq}(\omega, q)}{|\epsilon_{aa,a'a'}^{eq}(\omega, q)|^2} . \tag{70}$$

Notice that in calculating ν^m one has to neglect the difference between T_s and $T_{a'}$, i.e., one has $T_a = T_{a'} = T$.

IV. SUMMARY AND DISCUSSION

Equations (33)–(40) for the collision integrals, appropriate to a Q1DEG occupying several subbands and interacting with an arbitrary fluctuating potential, were derived using the dielectric formalism. The relevant dielectric functions and potential correlators are evaluated in the next paper.

Concerning the kinetic equations, we see that the screening of the relevant scattering potentials, as expressed by the functions $\epsilon_{KK'}(\omega, q)$ and $\Pi_K(\omega, q)$, is a rather complicated effect since it involves the dielectric function of the external system $\epsilon_s(\omega, q)$ and that of the electron system $\Delta\epsilon_K(\omega, q)$ in a complicated manner, cf. Eqs. (25) and (40). Only in the case of one sort of carrier, in the lowest subband, do the Q1DEG and the external system contribute to the total screening additively, cf. Eq. (42). We notice that the present rigorous treatment of screening, from first principles, avoids the usual drawback of other approaches where screening is introduced, in the quantum kinetic (or Boltzmann) equation *a posteriori* in a rather heuristic manner. The treatment also shows that in systems of low dimensionality one cannot, as a rule, use the usual, from 3D systems, separation $\epsilon(\omega, \mathbf{q}) = \epsilon_L + \Delta\epsilon(\omega, \mathbf{q})$ for the dielectric constant and $\varphi(\omega, q)/\epsilon(\omega, \mathbf{q})$ for the effective potential; instead one must find the effective scattering potential in a rather sophisticated manner.

The kinetic equations obtained in this paper can be written in the usual Boltzmann form, i.e., the collision integrals are expressed in terms of the transition probabilities. The main difference of Eqs. (33)–(39) from the usual form involving static screening is that they involve dynamical screening of the scattering potential both by electrons and the external system.

An essential feature of the present formalism is its generality: one does not have to specify, from the beginning, the scattering mechanism or the external system. One can simply take the final expression, e.g., for the relaxation frequencies ν^m and ν^T, and substitute in them the relevant quantities appropriate for scattering by, e.g., impurities, phonons, etc.

Finally, we remark that we have not, in this paper, attempted to solve the Boltzmann-type kinetic equations. Instead we have used the balance equations and the drifted Fermi-Dirac distribution function (57) to obtain, upon linearizing the former, relatively simple expressions for

the relaxation frequencies in terms only of the dielectric functions. The latter determine both the dissipation and screening in the system. From these frequencies we can evaluate the conductivity and the effective temperature of the Q1DEG. Specific applications will be published elsewhere.

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APPENDIX A

Below we give several relations we used in obtaining the results of the text.

A. Symmetry properties

From Eqs. (20), (22), and (29) we deduce [$\bar{K}=(p,b,a)$]

$$\epsilon^s_{KK'}(\omega,q)=[\epsilon^s_{\bar{K}\bar{K}'}(-\omega,{}^{-}q)]^* , \tag{A1}$$

$$\Delta\epsilon_K(\omega,q)=\Delta\epsilon^*_{\bar{K}}(-\omega,-q) , \tag{A2}$$

and

$$\langle\delta\varphi^2_s\rangle^{KK'}_{\omega,q}=\langle\delta\varphi^2\rangle^{\bar{K}'\bar{K}}_{-\omega,-q}=(\langle\delta\varphi^2_s\rangle^{K'K}_{\omega,q})^* . \tag{A3}$$

B. Connection between potential correlator and dielectric function

A test charge, with density $e\delta N(\omega,q,\rho)$, placed in a polar medium induces a potential $\delta\varphi(\omega,q,\rho)$ given by Eq. (15) of the text (with $q_x\equiv q$).

The interaction energy of the test particles $V_{\rm int}$ with the induced field has the general form

$$V_{\rm int}=e\int\varphi(\mathbf{r},t)N(\mathbf{r},t)d^3r . \tag{A4}$$

Within linear response theory we can write[13]

$$V_{\rm int}=-\int f(t,\mathbf{r})X(t,\mathbf{r})d^3r , \tag{A5}$$

where $f(t,\mathbf{r})$ is the external force acting on the system and $X(t,\mathbf{r})$ is the system's linear response to it. In the frequency domain we have

$$X(\omega,\mathbf{r})=\int\alpha(\omega,\mathbf{r},\mathbf{r}')f(\omega,\mathbf{r}')d^3r' . \tag{A6}$$

Then $\alpha(\omega,\mathbf{r},\mathbf{r}')$ is the generalized susceptibility given by the Kubo formula[13]

$$\alpha(\omega,\mathbf{r},\mathbf{r}')=(i/\hbar)\int_0^\infty e^{i\omega t}\langle X(t,\mathbf{r})X(0,\mathbf{r}') - X(0,\mathbf{r}')X(t,\mathbf{r})\rangle dt . \tag{A7}$$

Comparing Eqs. (15) of the text and (A4) with Eqs. (A5) and (A6) leads to the correspondence $X(t,\mathbf{r})\rightarrow-\delta\varphi(\mathbf{r},t)$, $f(t,\mathbf{r})\rightarrow e\delta N(\mathbf{r},t)$, and $\alpha(\omega,\mathbf{r},\mathbf{r}')\rightarrow-\epsilon_s^{-1}(\omega,\mathbf{r},\mathbf{r}')$. Fourier transforming the correlator $\langle\delta\varphi(\mathbf{r},t)\delta\varphi(\mathbf{r}',0)\rangle$ leads to the identity [$\rho=(y,z)$, $q_x\equiv q$]

$$i\hbar[\epsilon_s^{-1}(\omega,q,\rho,\rho')-\epsilon_s^{*-1}(\omega,q,\rho,\rho')] =\langle\delta\varphi(\rho)\delta\varphi(\rho')\rangle_{\omega,q}-\langle\delta\varphi(\rho')\delta\varphi(\rho)\rangle_{-\omega,-q} . \tag{A8}$$

Integrating Eq. (A8) over ρ and ρ' and using Eqs. (21) and (29) of the text we obtain

$$i\hbar\left[\frac{1}{\epsilon^s_{KK'}(\omega,q)}-\frac{1}{\epsilon^{*S}_{K'K}(\omega,q)}\right] =\langle\{\delta\varphi_K,\delta\varphi_{\bar{K}'}\}\rangle^s_{\omega,q}-\langle\{\delta\varphi_{\bar{K}'},\delta\varphi_K\}\rangle^s_{-\omega-q} . \tag{A9}$$

If the external system is in equilibrium at temperature T_s, with the help of the fluctuation-dissipation theorem[10] we obtain

$$\langle\delta\varphi^2\rangle^{\rho,\rho'}_{\omega,q}=(i\hbar/2)[\epsilon_s^{-1}(\omega,q,\rho,\rho') -\epsilon_s^{*-1}(\omega,q,\rho,\rho')]\coth(\hbar\omega/2k_BT_s) . \tag{A10}$$

Integrating over ρ and ρ' gives

$$\langle\delta\varphi^2_s\rangle^{K\bar{K}'}_{\omega,q}=(i\hbar/2)\coth(\hbar\omega/2k_BT_s) \times[1/\epsilon^s_{KK'}(\omega,q)-1/\epsilon^{*s}_{K'K}(\omega,q)] . \tag{A11}$$

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