Magnetoelastic dissipation in macroscopic magnetization tunneling

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The tunneling of the total magnetization of a small ($\sim$100-Å) magnetic particle out of a metastable easy direction is a promising candidate for macroscopic quantum tunneling. It is shown that the coupling of the magnetization to the phonons is an extremely weak source of dissipation that does not reduce the tunneling rate. A model in which the magnetic particle is embedded in a solid medium is considered, and the spectral density is derived for various crystal symmetries.

I. INTRODUCTION

A few years ago, Chudnovsky and Gunther[1] argued that small ferromagnetic particles were viable candidates in which the quantum tunneling of a macroscopic degree of freedom (henceforth called macroscopic quantum tunneling, or MQT) could be observed. They calculated the rate for the total magnetization, **M**, to tunnel out of a metastable easy direction or between two easy directions for a few simple configurations. The most relevant of these from the experimental standpoint is one where an external demagnetizing field, **H**, less than the coercive field, H_c, is applied opposite to the initial easy axis. The barrier to and the driving force for tunneling both arise from the magnetic anisotropy energy. The surprising result is that the tunneling rate is quite large: for a 100-Å particle, and typical values of the saturation magnetization M_0 and anisotropy energies, this can be made as large as 10^6–10^{10} sec^{-1} by adjusting H to be within $\sim 0.1\%$ of H_c. The largeness of this rate makes this system very attractive for the study of MQT, which until now has been largely confined to current biased Josephson junctions and rf superconducting quantum interference devices (SQUID's).[2,3]

A *macrovariable* by its very nature is inevitably subject to coupling to other (environmental) degrees of freedom, often very strongly. It is known from previous theoretical work that such coupling, or dissipation, can be extremely important in MQT. For example, if a rf SQUID is biased at exactly half a flux quantum so as to produce two degenerate wells, and if the junction is described by the resistively shunted junction model, then treating the dissipation within the Caldeira-Leggett scheme[4] leads to the prediction that the flux will be completely trapped in the initial well and never tunnel out of it if the shunt resistance is less than a certain critical value of the order of the resistance quantum h/e^2.[5–7]

Within the Caldeira-Leggett model of treating the environment as a set of harmonic oscillators, the dynamical influence of the environment on the system is completely determined by a certain combination of oscillator parameters and couplings known as the spectral density $J(\omega)$, which we define below. Leggett has shown that $J(\omega)$ is proportional to the imaginary part of the dynamical susceptibility of the linear combination of oscillator coordinates to which the macrovariable or system variable is coupled. In particular, the ω dependence of $J(\omega)$ at frequencies smaller than the characteristic frequency of the macrovariable is very important. If we write $J(\omega)\sim\omega^s$, then qualitatively different behavior is obtained depending on the value of the exponent s.[8] The trapping phenomenon mentioned earlier, for example, occurs only if $s\leq 1$.

In a previous paper,[9] we presented the results of a calculation of the effect of magnetoelastic dissipation when the magnetic particle is embedded in a nonmagnetic background, as is likely to be the case in any experiment. For typical values of the magnetoelastic coupling, the Wentzel-Kramers-Brillouin (WKB) exponent in the $T=0$ tunneling rate is increased by 10^{-4}–10^{-6} over its value without dissipation. The smallness of this effect makes it very encouraging to look for MQT in magnetic particles. Furthermore, we do not believe that the form of the spectral density is obvious, so in this paper we present the details of our calculation of $J(\omega)$ for various crystal symmetries. A knowledge of the spectral density would also be necessary to calculate the temperature dependence of the tunneling rate.

The plan of the paper is as follows. In Sec. II, we set up the standard instanton calculation of the tunneling rate[10,11] and present the action for **M** and the phonons, including the coupling. In Sec. III, we generalize $J(\omega)$ and Leggett's method[12] for calculating it to the case where the coupling is more conveniently described as a sum of couplings. We implement this method by obtaining an integral equation for the elastic displacement field and solving it in the long-wavelength limit to obtain $J(\omega)$ as $\omega\rightarrow 0$. In Sec. IV, we find an approximate formula for the effective action that is valid for all major crystal symmetries, and we illustrate the reduction of the tunneling rate for the case of cubic symmetry with a [111] easy axis.

II. FORMULATION OF THE PROBLEM

The system we shall consider is a small ($\sim$100-Å), single-domain ferromagnetic particle at temperatures so low that spin waves are frozen out, and the total magnetization of the particle **M** has a magnitude very close to the saturation value M_0. We shall take the particle to be spherical so that the easy axis for **M**, denoted $\hat{\mathbf{n}}$, is not influenced by shape anisotropy but determined solely by the magnetocrystalline anisotropy energy density $E_{\rm aniso}(\hat{\mathbf{M}})$.[13] We assume that this energy is experimentally known. We also apply an external field $\mathbf{H}$ opposite to the initial easy axis $\hat{\mathbf{n}}$, so that this direction becomes metastable and the possibility arises that $\mathbf{M}$ can tunnel out of it into a direction that is lower in energy. We shall assume, as in earlier work,[1,9] that because the particle size is smaller than the critical single-domain size *and* the typical width of a domain wall, the tunneling takes place via subbarrier rotation in unison of all the individual moments.[14] A qualitative justification for excluding nonuniform tunneling configurations is as follows. Consider a nonuniform distribution of the magnetization that is a candidate escape configuration, i.e., the analog of the escape point or the turning point in a one-dimensional problem. In order that the barriers to reaching this configuration not be too large, it must consist of small deviations from the uniform state. We can then apply the stability analysis that leads to the single-domain critical radius.[15,16] This suggests that, for particles smaller than this radius, the candidate configuration has a higher energy than the initial state and is thus inadmissible.

With the above assumption, the only dynamical variable left in the problem is the direction $\hat{\mathbf{M}}$ of $\mathbf{M}$. Ignoring coupling to other degrees of freedom for the moment, the relevant part of the total energy *density* of the particle is given by

$$E(\theta,\phi)=E_{\rm aniso}(\hat{\mathbf{M}})+H\mathbf{M}\cdot\hat{\mathbf{n}} \ , \tag{2.1}$$

where θ and ϕ are the polar coordinates of $\hat{\mathbf{M}}$. Since this energy generates the correct semiclassical dynamics of $\hat{\mathbf{M}}$, we can take it to be the Hamiltonian for the quantum-mechanical dynamics as well. This is completely analogous to using the experimentally determined capacitance and critical current of the junction in writing down the Hamiltonian for the flux in a rf SQUID.[17]

In the absence of dissipation, the tunneling rate Γ is given (up to factors of order unity) by[10,11]

$$\Gamma\sim\omega_p(S_0^{\rm cl}/2\pi\hbar)^{1/2}\exp(-S_0^{\rm cl}/\hbar) \ , \tag{2.2}$$

where ω_p is the classical small precession frequency for $\hat{\mathbf{M}}$ about $\hat{\mathbf{n}}$, and $S_0^{\rm cl}$ is the least value of the Euclidean action

$$S_0[\hat{\mathbf{M}}(\tau)]=v_0\int[E(\theta,\phi)-i\gamma^{-1}M_0\cos\theta\,\dot{\phi}(\tau)]d\tau \ . \tag{2.3}$$

Here, v_0 is the volume of the particle, $\gamma=g\mu_B/\hbar$, and g is a g factor.

Perhaps the most obvious coupling of $\mathbf{M}$ to the environment is that to phonons. (This is also mentioned briefly in Ref. 1.) As discussed in Ref. 9, the vibrational modes of an isolated particle of 100-Å size are at a much higher frequency than ω_p and, so, do not couple effectively to $\mathbf{M}$. In any real experiment, there must exist some means of physically anchoring the particle. We shall therefore consider a situation where it is embedded in a nonmagnetic solid medium. The low frequency, coupled phonons of the medium, and the particle can now couple to $\mathbf{M}$. This coupling can be adequately described by continuum elasticity theory. The total action now is

$$S(\hat{\mathbf{M}},\mathbf{u})=S_0(\hat{\mathbf{M}})-\int\int d\tau\, d^3x[q_{\alpha\beta}u_{\alpha\beta}\theta(a-r)-L_E^{\rm elas}] \ , \tag{2.4}$$

where

$$q_{\alpha\beta}=a_{\alpha\beta\gamma\delta}(M_\gamma M_\delta-M_0^2\hat{n}_\gamma\hat{n}_\delta) \ , \tag{2.5}$$

and $\mathbf{u}$ and $u_{\alpha\beta}$ are the displacement and strain fields, a is the radius of the particle, $a_{\alpha\beta\gamma\delta}$ is the magnetoelastic tensor, and $L_E^{\rm elas}$ is the Euclidean Lagrangian for the phonons alone. Rather than write this explicitly, we simply state that it will be chosen so as to produce freely propagating waves in each solid (particle and background) and Newton's third law at the interface. Further, we will take the two media to be isotropic as far as their elastic properties are concerned. This does not alter the qualitative aspects of the dissipation and makes the analysis simpler. The term $M_0^2\hat{n}_\alpha\hat{n}_\beta$ is subtracted from the coupling because we assume that the phonons have come to equilibrium with the magnetization in its initial state before the demagnetizing field is applied. This term then ensures that $\mathbf{u}=\mathbf{0}$ in the initial state, i.e., that it is taken to be the unstrained state.

We note that we have not explicitly included a counterterm involving $\hat{\mathbf{M}}$ alone in Eq. (2.4). Without such a term, the phonon coupling results in a renormalization of the potential energy for $\hat{\mathbf{M}}$ when the phonons are integrated out. The counterterm cancels this renormalization. As discussed at length by Caldeira and Leggett,[4(b)] this is a purely classical effect equivalent to the reduction of the resonance frequency of a pendulum because of friction. In the present case, including it has the effect of changing the anisotropy energy appearing in S_0 from one measured at constant strains to one measured at constant external stresses. The difference is precisely the stress-induced anisotropy energy, which is 10^{-2}–10^{-4} times the anisotropy energy itself.[13] Since measurements of $E_{\rm aniso}$ are rarely done to this precision, this point is largely academic. We mention it nevertheless because it has created confusion in the past about whether dissipation increases or decreases the tunneling rate in MQT.[18] One can assume, if one wishes, that the $E_{\rm aniso}$ that appears in S_0 is measured at constant stress.

Since the action (2.4) is quadratic in the phonons, these can be integrated out, leaving an effective action $S_{\rm eff}(\hat{\mathbf{M}})$ for $\hat{\mathbf{M}}$ alone, which we will write as

$$S_{\rm eff}(\hat{\mathbf{M}})=S_0(\hat{\mathbf{M}})+S_1(\hat{\mathbf{M}}) \ , \tag{2.6}$$

where S_0 is given by Eq. (2.3), and S_1 is an expression involving a double time integral, which we shall calculate in the next section. The tunneling rate continues to be given by an expression like Eq. (2.2), except that $S_0^{\rm cl}$ is replaced by $S_{\rm eff}^{\rm cl}$, the least value of the effective action, and ω_p is replaced by a damped small precession frequency.[4]

We shall obtain a general formula for $S_1(\hat{\mathbf{M}})$ in Sec. III. We shall apply it in Sec. IV to orthorhombic, tetragonal, hexagonal, and cubic crystal symmetries.[19] We present here for each of these cases the form of $E_{\rm aniso}$ and the nonzero components of the magnetoelastic tensor.

Orthorhombic symmetry:

$$E_{\text{aniso}}(\hat{\mathbf{M}})=-K_1\hat{M}_z^2+K_2\hat{M}_y^2 , \tag{2.7}$$

where the anisotropy coefficients K_1 and K_2 are both positive, and it is assumed that higher anisotropy coefficients are negligible. The easy axis is then $\hat{\mathbf{z}}$, and the hard axis is $\hat{\mathbf{y}}$. There are 12 different nonzero tensor components: a_{xxxx}, etc., a_{xxyy}, etc., and a_{xyxy}, etc.

Tetragonal symmetry:

$$E_{\text{aniso}}=K_1\sin^2\theta+(K_2-K_2'\cos 4\phi)\sin^4\theta . \tag{2.8}$$

We shall take the easy axis to be the fourfold axis, denoted $\hat{\mathbf{z}}$. The distinct tensor components are *zzzz*, *zzxx* = *zzyy*, *xxzz* = *yyzz*, *xxyy* = *yyxx*, *xxxx* = *yyyy*, *xyxy*, and *xzxz* = *yzyz*.

Hexagonal symmetry:

$$E_{\text{aniso}}=K_1\sin^2\theta+K_2\sin^4\theta+(K_3-K_3'\cos 6\phi)\sin^6\theta . \tag{2.9}$$

Once again we shall take the easy axis to be the sixfold axis (denoted $\hat{\mathbf{z}}$). The tensor components are the same as in the tetragonal case, except that $a_{xxxx}=2a_{xyxy}+a_{xxyy}$.

Cubic symmetry:

$$E_{\text{aniso}}=K_1(\hat{M}_x^2\hat{M}_y^2+\hat{M}_y^2\hat{M}_z^2+\hat{M}_z^2\hat{M}_x^2) . \tag{2.10}$$

The easy axis is of type [100] if $K_1>0$, and of type [111] if $K_1<0$. The magnetoelastic tensor components are *xxxx*, *xxyy*, and *xyxy*.

III. CALCULATION OF THE SPECTRAL DENSITY

A. General formulas

The general Lagrangian for a set of coordinates q_μ linearly coupled to a bath of harmonic oscillators can be written as

$$L=L_0(q_\mu,\dot{q}_\mu)+\tfrac{1}{2}\sum_i m_i(\dot{x}_i^2+\omega_i^2x_i^2)+\sum_{i,\mu}c_{i\mu}q_\mu x_i . \tag{3.1}$$

By a straightforward extension of Ref. 4, we find that the extra term S_1 in the effective action obtained after integrating out the phonons can be written as

$$S_1[q_\mu(\omega)]=\frac{1}{2\pi^2}\sum_{\mu,\nu}\int_{-\infty}^{\infty}d\omega\int_0^\infty d\varpi\frac{\omega^2}{\varpi(\omega^2+\varpi^2)}\times q_\mu(\omega)q_\nu^*(\omega)J_{\mu\nu}(\varpi) . \tag{3.2}$$

Here, $J_{\mu\nu}$ is the spectral density, which is given by

$$J_{\mu\nu}(\omega)=\frac{\pi}{2}\sum_i\frac{c_{i\mu}c_{i\nu}}{m_i\omega_i}\delta(\omega-\omega_i) . \tag{3.3}$$

For the problem at hand, it is clear that the action is indeed of the type discussed above. It is, however, very cumbersome to obtain the spectral density by finding the normal modes of the phonon system and their couplings to the magnetization and using Eq. (3.3) directly. It is better in such cases to use a method due to Leggett.[12] One solves the (linear) equations of motion for the oscillator coordinates x_i with the q_μ as arbitrary driving forces. After substituting these solutions and assuming that m_μ is the mass for q_μ, the equations of motion for q_μ can be written as

$$-m_\mu\zeta^2q_\mu(\zeta)+\sum_\nu Q_{\mu\nu}(\zeta)q_\nu(\zeta)=-\left[\frac{\partial U(\{q\})}{\partial q_\mu}\right](\zeta) , \tag{3.4}$$

where $q_\mu(\zeta)$ is the Fourier transform of $q_\mu(\tau)$, defined by

$$q_\mu(\zeta)=\int_{-\infty}^{\infty}d\tau\, q_\mu(\tau)e^{-i\zeta\tau} , \tag{3.5}$$

and $Q_{\mu\nu}(\zeta)$ is given by[20]

$$Q_{\mu\nu}(\zeta)=-\frac{2\zeta^2}{\pi}\int_0^\infty d\omega\frac{J_{\mu\nu}(\omega)}{\omega(\omega^2-\zeta^2)} . \tag{3.6}$$

It follows that

$$J_{\mu\nu}(\omega)=\lim_{\epsilon\to 0+}\operatorname{Im}Q_{\mu\nu}(\omega-i\epsilon) . \tag{3.7}$$

The problem is thus reduced to the calculation of $Q_{\mu\nu}(\zeta)$.

B. Solution of the coupled elastic wave equations

Before presenting the solution for the spectral density for magnetoelastic dissipation, we give a simple argument for its frequency dependence. It is apparent from the above discussion that $J_{\mu\nu}(\omega)$ is proportional to the imaginary part of a dynamic susceptibility, and that the power dissipated at frequency ω is proportional to $\sum_{\mu\nu}\omega J_{\mu\nu}(\omega)q_\mu(\omega)q_\nu^*(\omega)$. In the present case, if $q_{\alpha\beta}$ (we must now use the tensor component $\alpha\beta$ for the label μ) is regarded as a driving force, it results in the radiation of elastic waves. At low frequencies, we can ignore all the higher multipoles and keep only dipole radiation, for which the power dissipated varies as ω^4. It follows that $J_{\alpha\beta,\gamma\delta}$ varies as ω^3 as $\omega\to 0$. The rest of this section can be regarded as a confirmation of this argument and as a calculation of the strength of the elastic dipole moments. As mentioned in Sec. II, we shall take the two elastic media to be isotropic but different. This simplification does not change the form of $J_{\alpha\beta,\gamma\delta}$ and gives a good estimate of its size.

The equation of motion for $\mathbf{u}(\mathbf{x},\tau)$ is

$$\rho(r)\ddot{u}_\alpha-\frac{\partial}{\partial x_\beta}\sigma_{\alpha\beta}=-q_{\alpha\beta}\frac{\partial}{\partial x_\beta}\theta(a-r) , \tag{3.8}$$

where $\sigma_{\alpha\beta}$ is the stress tensor, given by

$$\sigma_{\alpha\beta}=\lambda(r)u_{\gamma\gamma}\delta_{\alpha\beta}+2\mu(r)u_{\alpha\beta} , \tag{3.9}$$

and $\rho(r)$, $\lambda(r)$, and $\mu(r)$ are the density, and the Lamé coefficients. We shall use subscripts 1 and 2 for their values inside and outside the particle, respectively. Thus,

$$\rho(r)=\begin{cases}\rho_1 & \text{for } r<a\\ \rho_2 & \text{for } r>a ,\end{cases} \tag{3.10}$$

and likewise for $\lambda(r)$ and $\mu(r)$. In terms of these, the longitudinal and transverse sound speeds in medium 2, for example, are

$$c_{l2}^2=(\lambda_2+2\mu_2)/\rho_2,\quad c_{t2}^2=\mu_2/\rho_2\ . \tag{3.11}$$

We also define $\Delta\rho=\rho_1-\rho_2$, and similarly $\Delta\lambda$ and $\Delta\mu$. Further, $\mathbf{u}(\mathbf{x},\tau)$ must be continuous at the interface, and the normal component of the stress tensor must have a discontinuity obtainable by integrating Eq. (3.8).

To solve Eq. (3.8), we Fourier transform it in both space and time variables to obtain

$$\zeta^2\rho*u_\alpha+ik_\beta\sigma_{\alpha\beta}=iq_{\alpha\beta}k_\beta s(k)\ , \tag{3.12}$$

where $\rho*u_\alpha$ denotes the convolution of $\rho(\mathbf{k})$ and $u_\alpha(\mathbf{k})$, and

$$s(k)\equiv\int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}\theta(a-r)$$
$$=\frac{4\pi}{k^3}(\sin ka-ka\cos ka)\ . \tag{3.13}$$

Note that $s(k)\to v_0$ as $k\to 0$, and that $\rho(\mathbf{k})$ can be written as

$$\rho(\mathbf{k})=\rho_2(2\pi)^3\delta(\mathbf{k})+\Delta\rho\, s(k)\ . \tag{3.14}$$

When written out in full, the left-hand side of Eq. (3.12) reads

$$\rho_2\zeta^2u_\alpha(\mathbf{k})-[(\lambda_2+\mu_2)k_\alpha k_\gamma+\mu_2k^2\delta_{\alpha\gamma}]u_\gamma(\mathbf{k})+\int_{\mathbf{k}'}s(\mathbf{k}-\mathbf{k}')[\Delta\rho\zeta^2\delta_{\alpha\beta}-\Delta\lambda\, k_\alpha k'_\beta-\Delta\mu(k_\beta k'_\alpha+k_\gamma k'_\gamma\delta_{\alpha\beta})]u_\beta(\mathbf{k}')\ , \tag{3.15}$$

where we have introduced the shorthand

$$\int_{\mathbf{k}}=\int\frac{d^3k}{(2\pi)^3}\ . \tag{3.16}$$

Since we only want the solution as $k,\omega\to 0$, we can approximate the integral over $\mathbf{k}'$ as follows. We replace $s(\mathbf{k}-\mathbf{k}')$ by $s(k')$, and drop the term proportional to $\Delta\rho$ as it is of higher order in k,ω. It can then be written as $-T_{\alpha\gamma}k_\gamma$, where $T_{\alpha\gamma}$ is given by

$$T_{\alpha\gamma}=\int_{\mathbf{k}'}s(k')\{\Delta\lambda k'_\beta u_\beta(\mathbf{k}')\delta_{\alpha\gamma}+\Delta\mu[k'_\alpha u_\gamma(\mathbf{k}')+k'_\gamma u_\alpha(\mathbf{k}')]\}\ . \tag{3.17}$$

Note that $T_{\alpha\gamma}=T_{\gamma\alpha}$. The solution to Eq. (3.12) can then be written as

$$u_\alpha(\mathbf{k})=G_{\alpha\beta}(\mathbf{k},\zeta)[iq_{\beta\gamma}s(k)+T_{\beta\gamma}]k_\gamma\ , \tag{3.18}$$

where $G_{\alpha\beta}$ is given by

$$G_{\alpha\beta}=\frac{\delta^T_{\alpha\beta}}{\rho_2(\zeta^2-c_{t2}^2k^2)}+\frac{\hat{k}_\alpha\hat{k}_\beta}{\rho_2(\zeta^2-c_{l2}^2k^2)}\ , \tag{3.19}$$

with $\delta^T_{\alpha\beta}=\delta_{\alpha\beta}-\hat{k}_\alpha\hat{k}_\beta$, and the carets denote unit vectors.

We now simply solve for the tensor $T_{\alpha\beta}$ self-consistently by substituting Eq. (3.18) in Eq. (3.17). It is useful to define two tensors

$$\begin{aligned}H^L_{\alpha\beta\gamma\delta}&=15\int\frac{d^2\hat{\mathbf{k}}}{4\pi}\hat{k}_\alpha\hat{k}_\beta\hat{k}_\gamma\hat{k}_\delta=(\delta_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})\ ,\\ H^T_{\alpha\beta\gamma\delta}&=\frac{15}{2}\int\frac{d^2\hat{\mathbf{k}}}{4\pi}(\delta^T_{\alpha\beta}\hat{k}_\gamma\hat{k}_\delta+\delta^T_{\gamma\beta}\hat{k}_\alpha\hat{k}_\delta)=\frac{1}{2}(3\delta_{\alpha\beta}\delta_{\gamma\delta}-2\delta_{\alpha\gamma}\delta_{\beta\delta}+3\delta_{\alpha\delta}\delta_{\beta\gamma})\ .\end{aligned} \tag{3.20}$$

We thus obtain

$$\begin{aligned}T_{\alpha\gamma}&=\int_{\mathbf{k}}s(k)[\Delta\lambda\delta_{\alpha\gamma}k_\beta+\Delta\mu(k_\alpha\delta_{\beta\gamma}+k_\gamma\delta_{\alpha\beta})]G_{\beta\sigma}[T_{\sigma\tau}+iq_{\sigma\tau}(\zeta)s(k)]k_\tau\\ &=\frac{1}{30\pi^2\rho_2}\int_0^\infty dk\left[\frac{5\Delta\lambda\delta_{\alpha\gamma}\delta_{\sigma\tau}}{\zeta^2-c_{l2}^2k^2}+2\Delta\mu\left[\frac{H^T_{\alpha\sigma\gamma\tau}}{\zeta^2-c_{t2}^2k^2}+\frac{H^L_{\alpha\gamma\sigma\tau}}{\zeta^2-c_{l2}^2k^2}\right]\right][T_{\sigma\tau}+iq_{\sigma\tau}(\zeta)s(k)]k^4s(k)\ .\end{aligned} \tag{3.21}$$

We now define two integrals:

$$I^{(n)}(c,\zeta)=\frac{1}{v_0^{n-1}}\int_0^\infty\frac{k^4}{\zeta^2-c^2k^2}s^n(k)dk\ , \tag{3.22}$$

for $n=1,2$, and two new tensors

$$M^{(n)}_{\alpha\gamma\beta\delta}=\frac{1}{30\pi^2\rho_2}[(5\Delta\lambda\delta_{\alpha\gamma}\delta_{\beta\delta}+2\Delta\mu H^L_{\alpha\gamma\beta\delta})I^{(n)}(c_{l2})+2\Delta\mu H^T_{\alpha\beta\gamma\delta}I^{(n)}(c_{t2})]\ . \tag{3.23}$$

In terms of these, Eq. (3.21) can be written as

$$T_{\alpha\gamma}=M^{(1)}_{\alpha\gamma\beta\delta}T_{\beta\delta}+iv_0M^{(2)}_{\alpha\gamma\beta\delta}q_{\beta\delta}(\zeta) . \quad (3.24)$$

To solve Eq. (3.24), we first take its trace. This yields

$$T_{\alpha\alpha}=i\frac{\Delta BI^{(2)}(c_{l2})}{2\pi^2\rho_2-\Delta BI^{(1)}(c_{l2})}v_0q_{\alpha\alpha}(\zeta) , \quad (3.25)$$

where $\Delta B=\Delta\lambda+(2/3)\Delta\mu$ is the jump in the bulk modulus. We now construct the traceless part of $T_{\alpha\gamma}$,

$$T'_{\alpha\gamma}=T_{\alpha\gamma}-\tfrac{1}{3}T_{\beta\beta}\delta_{\alpha\gamma} , \quad (3.26)$$

and similarly $q'_{\alpha\gamma}$. It then follows that

$$T'_{\alpha\gamma}=i\frac{\Delta\mu\langle I^{(2)}(c_2)\rangle}{3\pi^2\rho_2-\Delta\mu\langle I^{(1)}(c_2)\rangle}v_0q'_{\alpha\gamma}(\zeta) , \quad (3.27)$$

where the angular brackets denote an average,

$$\langle f(c_2)\rangle=\tfrac{1}{5}[3f(c_{t2})+2f(c_{l2})] , \quad (3.28)$$

for a general function f.

The explicit solution for $\mathbf{u}(\mathbf{k},\zeta)$ can now be obtained by substituting Eqs. (3.25)–(3.27) into Eq. (3.18). We only want the solution, however, in the limit $k,\zeta\to 0$. In this limit, it is not difficult to show that

$$I^{(n)}(c,\zeta)\approx-\frac{2\pi^2}{c^2}\left[1+k^{(n)}\left[\frac{\zeta a}{c}\right]^2-\frac{i}{3}\left[\frac{\zeta a}{c}\right]^3+O(\zeta^4)\right] , \quad (3.29)$$

where $k^{(1)}=\frac{1}{2}$ and $k^{(2)}=\frac{2}{5}$. Since we only need the leading terms in either the real or the imaginary parts of these integrals, we can ignore the difference in the terms of order ζ^2 and write $I^{(1)}(c)=I^{(2)}(c)=I(c)$ henceforth. With this and the fact that $s(k)\approx v_0$, we can write the solution (3.18) as

$$u_\alpha\approx iv_0G_{\alpha\beta}[F'(\zeta)q'_{\beta\gamma}+\tfrac{1}{3}F(\zeta)q_{\delta\delta}\delta_{\beta\gamma}]k_\gamma , \quad (3.30)$$

where F and F' are given by

$$F(\zeta)=\frac{2\pi^2\rho_2}{2\pi^2\rho_2-\Delta BI(c_{l2},\zeta)} ,$$
$$F'(\zeta)=\frac{3\pi^2\rho_2}{3\pi^2\rho_2-\Delta\mu\langle I(c_2,\zeta)\rangle} . \quad (3.31)$$

The next step is to identify the kernel $Q_{\alpha\beta,\gamma\delta}(\zeta)$ by substituting the solution obtained above in the equation of motion for $\mathbf{u}$. Since $Q_{\alpha\beta,\gamma\delta}(\zeta)$ only depends on the couping term in the equation of motion, we have

$$Q_{\alpha\beta,\gamma\delta}(\zeta)q_{\gamma\delta}(\zeta)=-\int_{\mathbf{k}}u_{\alpha\beta}(\mathbf{k},\zeta)s(k) . \quad (3.32)$$

If we now use the relation $u_{\alpha\beta}=(i/2)(u_\alpha k_\beta+u_\beta k_\alpha)$, and combine Eqs. (3.30)–(3.32), it is straightforward to perform the angular $\mathbf{k}$ integrals using Eq. (3.20). The radial $\mathbf{k}$ integrals are seen to involve only $I^{(1)}(c)$. After some simple algebra, we obtain

$$Q_{\alpha\beta,\gamma\delta}(\zeta)=\frac{v_0}{12\pi^2\rho_2}F'(\zeta)\langle I(c_2,\zeta)\rangle(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})+\frac{v_0}{18\pi^2\rho_2}[F(\zeta)I(c_{l2},\zeta)-F'(\zeta)\langle I(c_2,\zeta)\rangle]\delta_{\alpha\beta}\delta_{\gamma\delta} . \quad (3.33)$$

Taking the limit as $\zeta\to\omega-i\epsilon$, and using Eqs. (3.31) and (3.29), we finally obtain the desired spectral density as $\omega\to 0$:

$$J_{\alpha\beta,\gamma\delta}(\omega)=\frac{v_0^2}{24\pi\rho_2}\frac{\langle c_2^{-5}\rangle}{(1+2\eta)^2}\omega^3(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})+\frac{v_0^2}{36\pi\rho_2}\left[\frac{c_{l2}^{-5}}{(1+2\chi)^2}-\frac{\langle c_2^{-5}\rangle}{(1+2\eta)^2}\right]\omega^3\delta_{\alpha\beta}\delta_{\gamma\delta} , \quad (3.34)$$

where

$$\eta=(\Delta\mu/3\rho_2)\langle c_2^{-2}\rangle ,$$
$$\chi=(\Delta B/2\rho_2)c_{l2}^{-2} . \quad (3.35)$$

In terms of this, the correction to the effective action is given by

$$S_1[\hat{\mathbf{M}}(\omega)]=\frac{1}{2\pi^2}\int_{-\infty}^{\infty}d\omega\int_0^\infty d\varpi\frac{\omega^2}{\varpi(\omega^2+\varpi^2)}q_{\alpha\beta}(\omega)q^*_{\gamma\delta}(\omega)J_{\alpha\beta,\gamma\delta}(\varpi) , \quad (3.36)$$

with $q_{\alpha\beta}$ given by Eq. (2.5).

We conclude this section by noting that the spectral density must vanish for frequencies much larger than $\langle c_2\rangle/a$. The precise form for how it gets cut off is unimportant, so we shall simply multiply Eq. (3.34) by $e^{-\omega/\omega_c}$, where $\omega_c=b\langle c_2\rangle/a$, and b is some number of order unity.

IV. TUNNELING RATE FOR CERTAIN CRYSTAL SYMMETRIES

We can simplify the expression (3.36) considerably if the initial easy axis is one of high symmetry and if the point of escape is close to this direction. In that case, we can assume that $\hat{\mathbf{M}}\cdot\hat{\mathbf{n}}\approx M_0$ throughout the tunneling process, and that the transverse components $\hat{\mathbf{M}}_\perp$ are small. In that case, we can retain only those terms in $q_{\alpha\beta}$ that involve one factor of $\hat{\mathbf{M}}\cdot\hat{\mathbf{n}}$ and one factor of $\hat{\mathbf{M}}_\perp$. It is then easy to see that in each of the symmetries we listed in Sec. II there is no contribution to S_1 from the $\delta_{\alpha\beta}\delta_{\gamma\delta}$ part of $J_{\alpha\beta,\gamma\delta}$. For, in all but the cubic case with easy axis [111], the nonvanishing components of $a_{\alpha\beta\gamma\delta}$ are such that

$$\delta_{\alpha\beta}q_{\alpha\beta}(\omega)\approx\sum_{\beta=x,y}a_{\alpha\alpha\beta\beta}(M_\beta * M_\beta)\ , \tag{4.1}$$

which is of second order in smallness. [$M_\alpha * M_\beta$ again denotes a convolution of $M_\alpha(\omega)$ and $M_\beta(\omega)$.] In the cubic case with easy axis [111],

$$\begin{aligned}\delta_{\alpha\beta}q_{\alpha\beta}(\tau)&=a_{\alpha\alpha\mu\nu}(M_\mu M_\nu - M_0^2\,\hat{n}_\mu\hat{n}_\nu)\\&=M_0^2 a_{\alpha\alpha\mu\nu}(\tfrac{1}{3}\delta_{\mu\nu}-\hat{n}_\mu\hat{n}_\nu)\ ,\end{aligned} \tag{4.2}$$

which is constant with time. The contribution to S_1 is thus part of the potential renormalization that has already been absorbed in the definition of the anisotropy energy.

For the remaining component of $J_{\alpha\beta,\gamma\delta}$, we have

$$\begin{aligned}(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})&q_{\alpha\beta}(\omega)q^*_{\gamma\delta}(\omega)\\&=2a_{\alpha\beta\mu\nu}a_{\alpha\beta\sigma\tau}(M_\mu * M_\nu)(M_\sigma * M_\tau)^*\ ,\end{aligned} \tag{4.3}$$

with the understanding that the $M_0^2\,\hat{\mathbf{n}}_\alpha\hat{\mathbf{n}}_\beta$ part of $q_{\alpha\beta}$ has been removed. For the orthorhombic case, the component M_x is much larger than M_y because the tunneling takes place essentially in the x-z plane. Thus, to leading order the expression (4.3) can be written as $g_c^2|\hat{\mathbf{M}}_x(\omega)|^2$, where $g_c^2=16a_{xzxz}^2M_0^4$. For all the other symmetries, it can be written as

$$g_c^2\,\hat{\mathbf{M}}_\perp(\omega)\cdot\hat{\mathbf{M}}_\perp^*(\omega)\ , \tag{4.4}$$

where $g_c^2=16a_{xzxz}^2M_0^4$ once again for the tetragonal, hexagonal, and cubic [100] easy axis cases, while for the cubic system with easy axis [111],

$$g_c^2=\tfrac{8}{3}[(A-B)^2+2C^2]M_0^4\ , \tag{4.5}$$

with $A=a_{xxxx}$, $B=a_{xxyy}$, and $C=a_{xyxy}$.

We can thus write down a single unified expression for S_1 for the symmetries we are considering:

$$\begin{aligned}S_1(\hat{\mathbf{M}})=\frac{1}{2\pi^2}\int_{-\infty}^{\infty}d\omega\int_0^\infty d\varpi\frac{\omega^2}{\varpi(\omega^2+\varpi^2)}\\\times\hat{\mathbf{M}}_\perp(\omega)\cdot\hat{\mathbf{M}}_\perp^*(\omega)J(\varpi)\ ,\end{aligned} \tag{4.6}$$

with

$$J(\omega)=\frac{v_0^2g_c^2}{24\pi\rho_2}\frac{\langle c_2^{-5}\rangle}{(1+2\eta)^2}\omega^3\ , \tag{4.7}$$

provided we retain only the x components in the dot product in Eq. (4.6) for the orthorhombic case. In fact, with this understanding Eqs. (4.6) and (4.7) apply to all lower symmetries as well—the coupling constant g_c^2 is given by a more complicated expression, but is still of order $(a_{\alpha\beta\gamma\delta})^2M_0^4$.

As an illustration of the claim that magnetoelastic dissipation is small, we conclude with a calculation of the WKB exponent and the dissipative correction to it for the case of cubic anistropy with an easy axis along [111].[19] Let us first ignore dissipation. Choosing the polar axis to be [111], the energy (2.10) can be written as

$$\begin{aligned}E(\theta,\phi)=\tfrac{2}{3}\epsilon|K_1|\theta^2+\frac{2^{1/2}}{3}|K_1|\theta^3\cos3\phi\\-\tfrac{3}{4}|K_1|\theta^4+\cdots\ ,\end{aligned} \tag{4.8}$$

where $\epsilon=(1-H/H_c)$, $H_c=4|K_1|/3M_0$, and we have dropped an additive constant. To get an appreciable rate, we need $\epsilon\ll1$. An expansion in powers of θ is then valid, since the escape point is located at $\theta\approx\sqrt{2}\epsilon$. There are three equivalent least action trajectories:

$$\begin{aligned}\bar{\phi}(\tau)&=(2n+1)\frac{\pi}{3}-i\omega_p\tau,\quad n=0,1,2\ ,\\\bar{\theta}(\tau)&=2^{1/2}\epsilon\,\mathrm{sech}(3\omega_p\tau)\ ,\end{aligned} \tag{4.9}$$

where $\omega_p=\epsilon\gamma H_c/2$ is the small precession frequency. The classical or least action is found to be

$$S_0^{\rm cl}/\hbar=(2M_0/3\hbar\gamma)v_0\epsilon^2\ . \tag{4.10}$$

Taking typical values $M_0\sim500$ G, $|K_1|\sim5\times10^4$ erg/cm^3, we obtain $\omega_p\sim10^8$ sec^{-1}. By tuning ϵ to be about 0.01, we can obtain "bare" tunneling rates [see Eq. (2.2)] of the order of 10^6 sec^{-1}.

To calculate the dissipative correction, we assume that it is small so that the uncorrected classical path (4.9) can be used in Eq. (4.6). The Fourier transforms of the components $\hat{M}_x$ and $\hat{M}_y$ are [for the $n=0$ case in Eq. (4.9)]

$$\begin{aligned}\hat{M}_x(\omega)&=\frac{\sqrt{2}\pi\epsilon}{3\omega_p}(\sqrt{3}\cosh\alpha+\sqrt{3}\sinh\alpha)/(3+4\sinh^2\alpha)\ ,\\\hat{M}_y(\omega)&=\frac{\sqrt{2}\pi\epsilon}{3\omega_p}(3\cosh\alpha-\sinh\alpha)/(3+4\sinh^2\alpha)\ ,\end{aligned} \tag{4.11}$$

where $\alpha\equiv\pi\omega/6\omega_p$. We thus find

$$\hat{\mathbf{M}}_\perp\cdot\hat{\mathbf{M}}_\perp^*=2\left[\frac{2\pi\epsilon}{3\omega_p}\right]^2\Big/(3+4\sinh^2\alpha)\ . \tag{4.12}$$

The integrals in Eq. (4.6) can then be done and we obtain

$$S_1=\frac{8(\epsilon v_0g_c)^2}{81\sqrt{3}\pi\rho_2}\frac{\langle c_2^{-5}\rangle}{(1+2\eta)^2}\omega_p\omega_c\ . \tag{4.13}$$

The quantity $g_c^2/\rho_2\bar{c}^2$, where $\bar{c}$ is an average sound speed, is of the order of $K_{\rm ME}$, the strain-induced change in the anisotropy coefficients; typically, this is 10^{-2}–10^{-4} times smaller than the anisotropy coefficients themselves.[13] The ratio S_1/S_0 is thus approximately

$$\frac{S_1}{S_0} = (\mathrm{const})\epsilon \frac{K_{\mathrm{ME}}}{|K_1|} \left[\frac{a\omega_H}{\bar{c}} \right]^2 , \tag{4.14}$$

where a is the radius of the particle, $\omega_H = \gamma H_c$, and the constant is of order unity. Each one of the remaining three factors in this expression is of the order 10^{-2}, giving a relative correction to the WKB exponent of order 10^{-6}.

We have shown that magnetoelastic dissipation causes practically no reduction in the tunneling rate for macroscopic magnetization tunneling. This is in contrast to the case of junction-based systems where dissipation is believed to be quite large. Barring other sources of dissipation, the smallness of this effect makes it very promising to look for MQT in magnetic particles.

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