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Spin splitting and anomalous Hall resistivity in three-dimensional disordered systems

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We comment on recently observed oscillations of the Hall resistivity around the classical value with particular emphasis on the coexistence of localized and extended states in the case of a spin-split lowest Landau level. We prove the existence of localization at the bottom of the 0↓ level, thus explaining the observed plateaulike feature of the Hall resistivity.

Although the Shubnikov–de Haas (SdH) oscillations of the magnetoresistance are by now well understood, no convincing theoretical explanation for the oscillatory deviations of the Hall resistivity from the classical behavior has been presented so far. In recent experiments[1–3] the Hall resistivity ρ_{xy} was investigated in narrow-gap, bulk semiconductors such as InAs, InSb, and $Hg_{1-x}Cd_xTe$ with particular emphasis on the magnetic-field region where the last SdH minimum of the longitudinal resistivity ρ_{xx} is observed. The interesting feature was a distinct decrease of the slope of the ρ_{xy} versus B graph in the same region. Murzin[1] was the first to suggest a relation to the quantum Hall effect (QHE) in two-dimensional (2D) systems and conjectured that electron-electron interaction might account for an additional phase—a *Hall insulator*—in three dimensions. Mani[3] tried to reproduce the essential experimental features in a simple model by assuming the existence of localized states at the bottom of each 3D Landau level (LL). His model suggested real plateaus at $T=0$ coinciding with the SdH minima. In Refs. 1 and 2, however, only one plateaulike structure was observed together with the last SdH minimum. From the theoretical point of view three questions arise. (i) Does ρ_{xy} exhibit plateaus at $T=0$ or is only the slope decreasing? (ii) Can the existence of localized states be proven? How can the problem of coexistence of localized and extended states—which was put forward in Ref. 3—be solved in the presence of disorder? (iii) Does the magnetic-field range in which the last SdH minimum occurs play a distinguished role concerning the plateaulike feature of ρ_{xy}? Finally, we have also to comment on the observation of a Hall resistivity being smaller than the classical value for magnetic fields below the last SdH oscillation as well as before approaching the freezing-out Mott-insulator (MI) transition at high fields.

In the following we want to illustrate that the decreasing slope of ρ_{xy} can be understood within a model of 3D disordered systems studied previously,[4] provided spin-splitting is taken into account. Let us first recall that the g factor of InSb decreases from about 50 to 35 for a doping concentration varying from $n=10^{15}$ to 10^{17} cm^{-3} (cf., e.g., Ref. 5). With the effective mass $m \simeq 0.014 m_e$ this yields a spin splitting of one third of the LL separation so that at least for the lowest Landau level (LLL) the spin splitting is well resolved. Consequently, in the vicinity of the last observed SdH minimum the Fermi energy crosses the bottom of the 0↓ level. Since in the absence of magnetic impurities the one-particle states of the lowest spin-up and spin-down LL cannot mix, all results obtained previously for the LLL can now be applied to the 0↓ and the 0↑ level separately. We have demonstrated in Ref. 4 that in the lower exponential tails of the LLL where the one-instanton approximation applies all states are localized. The density of states (DOS) in this regime reads[6]

$$\rho(\varepsilon)=\frac{9}{\pi}\frac{1}{2\pi l^2}\left[\frac{2m}{\Gamma}\right]^{1/2}\left[\frac{|\varepsilon|}{\Gamma}\right]^{5/2}\exp\left[-\left[\frac{|\varepsilon|}{\Gamma}\right]^{3/2}\right], \quad (1)$$

where Γ denotes the disorder induced level broadening and $\varepsilon=E-\hbar\omega_c/2$ is the energy distance from the Landau energies. In the present case the DOS of the spin-up and spin-down bands $\rho_\uparrow, \rho_\downarrow$ are described by the same expression in the lower tails (see the shaded regions in Fig. 1). In particular, the DOS for the spin-down level in the tail region is given by

$$\rho_\downarrow(E)=\rho(\varepsilon-g_B B/2), \quad (2)$$

with ρ according to Eq. (1). Due to the independence of the two spin-split bands, localized states from the spin-

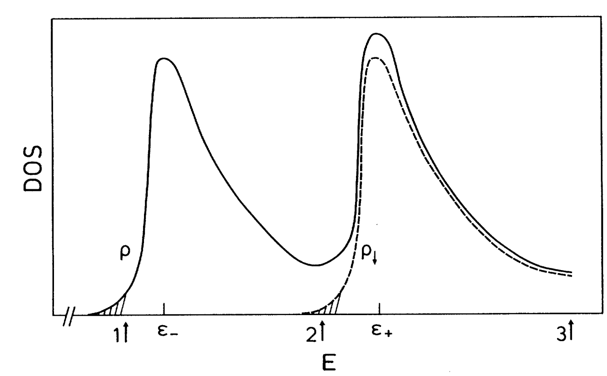

FIG. 1. Total density of states ρ (solid line) and DOS of the spin-down band (broken line) in the LLL; $\varepsilon_\pm=(\hbar\omega_c \pm g_B B)/2$. Impurity bands are omitted; shaded areas signify localization. Arrows 1,2,3 indicate spectral regions referred to in the text. In region 2 localized states belonging to the 0↓ level and extended states from the 0↑ level coexist causing a decrease of the slope of ρ_{xy} (see Fig. 2).

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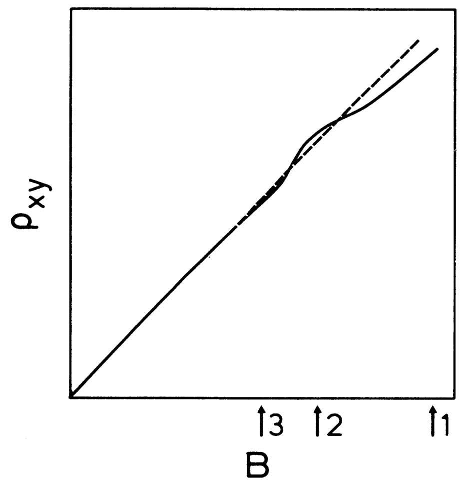

FIG. 2. Hall resistivity (solid line) oscillating around the classical straight line ρ_{xy}^{cl} (broken line) in the LLL as observed in Ref. 3. Arrows 1,2,3 correspond to those of Fig. 1. In region 3, ρ_{xy} approaches ρ_{xy}^{cl} from below.

down band and extended states belonging to the spin-up band coexist at the bottom of the $0\downarrow$ level (region 2 of Figs. 1-3). The localized states of the spin-down band do not contribute to the Hall conductivity. Assuming for simplicity that the contribution to the Hall resistivity coming from the spin-up band is described by the classical formula $\rho_{xy}^{cl}=B/en$, we obtain in region 2

$$\rho_{xy}=\frac{B}{en_\uparrow}=\frac{B}{e(n-n_\downarrow)} . \tag{3}$$

In analogy to the DOS [cf. Eq. (2)] the number of particles per unit volume in the spin-up and spin-down level is denoted by $n_\uparrow$ and $n_\downarrow$, respectively. The Hall resistivity exceeds the classical value and the slope is sublinear because $n_\downarrow$ increases with the magnetic field but no plateau is to be expected. Note that in the case of the quantized Hall effect (QHE) the number of extended states below

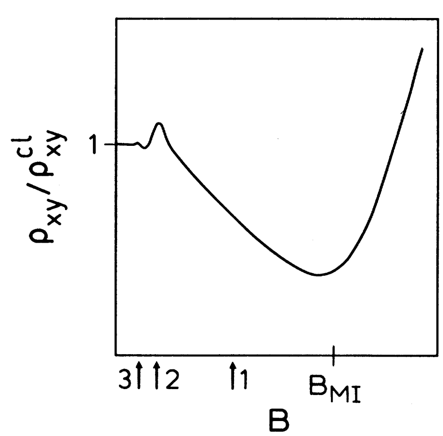

FIG. 3. Normalized Hall resistivity (cf. Ref. 8). For low doping region 1 is characterized by the Hall dip which is observed before reaching the freezing-out point B_{MI}.

the Fermi energy remains unchanged in the plateau region. This is not the case here.

We want to emphasize that in contrast to the discussion given in Ref. 3 localization is not to be assumed but can be proven. Spin splitting plays an essential role in that phenomenon. At the bottom of higher LL's the same argumentation does not apply any longer because mixing with extended states from the square-root tail of lower LL's cannot be excluded. This explains the peculiar role of the vicinity of the last SdH oscillation observed in experiment.

Let us now turn to a more quantitative analysis of the Hall resistivity. In view of the analogy which has been suggested between the present localization phenomenon and the QHE (Refs. 1-3) deviations of ρ_{xy} from the classical value are most natural even within a single band. It has been shown in Ref. 7 that the most general parametrization of the dc Kubo conductivities is given by

$$\begin{aligned}\sigma_{xx}&=\frac{en}{B}\frac{\tau\omega_c}{(1-\kappa\omega_c)^2+\tau^2\omega_c^2} ,\\ \sigma_{yx}&=\frac{en}{B}\left[1-\frac{1-\kappa\omega_c}{(1-\kappa\omega_c)^2+\tau^2\omega_c^2}\right] ,\end{aligned}\tag{4}$$

where τ denotes the scattering time and κ the *memory oscillation time.* From Eq. (4) we obtain the corresponding Hall resistivity

$$\rho_{xy}=\frac{B}{en}\left[1-\frac{\kappa\omega_c}{\kappa^2\omega_c^2+\tau^2\omega_c^2}\right] . \tag{5}$$

For $\kappa=0$ Eqs. (4) and (5) yield the Drude-Zener formula. However, this value of κ is incompatible with the QHE in 2D and there is no reason why it should hold in 3D in the quantum limit. Note that the Drude-Zener formula is the only approximation of Eq. (4) in disordered systems in which the classical result ρ_{xy}^{cl} can be obtained in Eq. (5). A general discussion of the properties of the memory oscillation time κ in 2D has been given in Ref. 7. In order to gain insight into the 3D behavior we suggest to the reader the following very illustrative toy model. Consider a system with random disorder depending on the (x,y) coordinates only. Integrating the 2D DOS at the Fermi energy over all allowed p_z values yields the 3D DOS as well as the particle density n. Let the 2D Hall conductivity be any function with particle-hole symmetry having quantized plateaus of finite width. The Hall conductivity in 3D can be obtained analogously from the 2D result. Within the above toy model we obtain the interesting result that for $\varepsilon_F \gg \Gamma$ the Hall conductivity in 3D approaches its classical value from above. A comparison with Eq. (4) yields that this is only possible if the square-root tails of the 3D LL's are characterized by $\tau\omega_c \gg 1$ and $\kappa\omega_c \simeq 1$. Thus, the Hall resistivity of Eq. (5) tends to be smaller than ρ_{xy}^{cl} in the absence of localization. However, if the Fermi energy is situated in region 2 where the spin-down states are localized, the Hall resistivity reads

$$\rho_{xy}=\frac{B}{en_\uparrow}\frac{1}{1+\tau_\uparrow^{-2}\omega_c^{-2}} . \tag{6}$$

As soon as the dependence on the particle concentration dominates, the Hall resistivity exceeds $\rho_{xy}^{\rm cl}$ as stated previously. The plateaulike feature of ρ_{xy} has already been obtained in Eq. (3) with simplified assumptions. However, the correction in Eq. (6) also explains that the Hall resistivity indeed crosses $\rho_{xy}^{\rm cl}$ at the bottom of the $0\downarrow$ level. Note that the SdH minimum of the longitudinal resistivity corresponding to Eq. (6), i.e., $\rho_{xx}=m(e^2 n_\uparrow \tau_\uparrow)^{-1}$, occurs when the decrease of $n_\uparrow$ with decreasing magnetic field becomes stronger than the increase of $\tau_\uparrow$. This happens already below the mobility edge of the $0\downarrow$ level. Thus, even in the presence of localization the minima of ρ_{xx} indicate approximately the position of the lower band edges of the LL's.

In region 3 of Figs. 1–3 the Fermi energy is situated in the metallic region of both spin-up and spin-down LLL and there are two additive conductivity contributions according to Eq. (4) with the corresponding spin-up and spin-down quantities, respectively. From $\kappa_\uparrow\omega_c \simeq \kappa_\downarrow\omega_c \simeq 1$ we obtain for the Hall resistivity

$$\rho_{xy}=\frac{B}{en}\frac{1}{1+\tau^{-2}\omega_c^{-2}},\quad \frac{1}{\tau}=\frac{n_\uparrow}{n}\frac{1}{\tau_\uparrow}+\frac{n_\downarrow}{n}\frac{1}{\tau_\downarrow}, \qquad (7)$$

i.e., the graph of ρ_{xy} approaches the classical straight line from below as the magnetic field decreases. This effect has been observed in Ref. 3 but not in Ref. 2.

To avoid confusion let τ^* denote the scattering time corresponding to the zero-field mobility, i.e., $\mu=e\tau^*/m$. However, we may safely assume in the following $\tau^* \propto \tau$ and thus $\tau\omega_c \propto \mu B$. In the region of validity of Eq. (6) the experimental parameters of Ref. 3 have been $\mu \simeq 3\times 10^5$ $\mathrm{cm^2/V\,s}$, $B \lesssim 0.7$ T whereas in Ref. 2 $\mu \simeq 8\times 10^4$ $\mathrm{cm^2/V\,s}$, $B \lesssim 7$ T. Obviously, $\tau^2\omega_c^2$ differs by 1 order of magnitude in the two cases giving rise to an observable effect in Ref. 3 but not in Ref. 2.

In region 1 of our figures we predicted in Ref. 4 a logarithmic decrease of $\rho_{xy}/\rho_{xy}^{\rm cl}$ with the magnetic field. However, this prediction referred to a situation in which freezing out is at most a competitive effect at the MI transition. In narrow-gap semiconductors at low doping concentrations this is not the case, and in Refs. 3 and 8 a different type of decrease has been observed. Shayegan, Goldman, and Drew[8] refer to it as the anomalous *Hall dip* in the graph of the normalized Hall coefficient and explain it by a picture of donor clusters (see Fig. 3 which contains the same information as Fig. 2 but stresses the Hall dip rather than the plateaulike feature of ρ_{xy}). When approaching the freezing-out transition electrons tend to be localized on isolated donor levels and the total area of the percolating cluster decreases. In Ref. 8 it has been shown that this decrease is faster than the decrease of the number of electrons moving within the cluster. Thus, the effective electron concentration increases with the magnetic field giving rise to the observed Hall dip. As mentioned above, this explanation applies at low doping concentrations where the magnetic-field induced MI transition takes place in the impurity band (freezing-out). On the other hand, for an effective donor binding energy being small compared to the disorder broadening localization becomes essential (cf. Ref. 4). This might be the reason why in Refs. 1 and 2—-i.e., for high impurity concentrations ($na_B^3 \simeq 16$)—no Hall dip has been reported.

Our aim was to explain that the decreasing slope of the Hall resistivity at the bottom of the $0\downarrow$ level can be understood within the already existing framework of localization without invoking the existence of an additional phase (Hall insulator). The larger the impurity broadening of the LL's the better our model describes the experiment. At very low doping the role of the localized states is taken over by bound hydrogenic donor levels. Nevertheless, the possibility of coexistence with extended states is again due to the fact that levels with opposite spin cannot mix in the absence of magnetic interaction.

Finally, we want to comment shortly on the possibility of the phase conjectured by Murzin due to the influence of electron-electron interaction. In analogy to the magnetoexcitons in 2D studied in Ref. 9 there might also exist excitonic coupling between the spin-up and spin-down level in the present case. Since the energy dispersion in the absence of disorder depends on the momentum p_z only, there exists another analogy to 1D systems known as *excitonic insulators* (cf. Ref. 10). Let us suppose that a ground state (spin wave in z direction) and a spectral gap Δ indeed exist. For a disorder broadening that is small compared to the spectral gap $\Gamma \ll \Delta$, a purely localized regime would appear giving rise to a real Hall plateau at $T=0$. In this sense a decreasing slope of the Hall resistivity and a real plateau are not quantitatively but qualitatively different phenomena. Screening effects being smaller at lower doping where also the disorder broadening decreases could favor the appearance of the above described effect.

In conclusion, the main features of oscillatory deviations of the Hall resistivity from its classical behavior have been explained. Particular emphasis has been placed on the coexistence of localized and extended states in the case of a spin-split LLL.

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