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# Experiments on scaling in $Al_xGa_{1-x}As/GaAs$ heterostructures under quantum Hall conditions

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The influence of repulsive (Be) and attractive (Si) scattering centers on the temperature-dependent half-width of the Shubnikov–de Haas peaks $\Delta B(T)$ and the maximum of the slope of the Hall resistance $\partial\rho_{xy}/\partial B$ has been investigated at millikelvin temperatures for the two-dimensional electron gas in $Al_xGa_{1-x}As/GaAs$ heterostructures. Both sheet-doped and homogeneously doped samples were studied. In contrast to results reported for $In_xGa_{1-x}As/InP$ heterostructures, where a scaling exponent $\kappa=0.42\pm0.04$ with $\Delta B\propto T^{\kappa}$ and $(\partial\rho_{xy}/\partial B)^{\max}\propto T^{-\kappa}$ is found and suggested to be universal, our data lead to scaling exponents that systematically increase with decreasing mobility. The transition between the integer filling factor 1 and the fractional filling factor $\frac{2}{3}$ has been studied and a scaling behavior observed.

In two recent publications by Wei *et al.*, the critical behavior of transport coefficients that are characteristic of the quantum Hall effect (QHE) has been studied.[1,2] It was shown that in the integral QHE (IQHE) studied on $In_xGa_{1-x}As/InP$ heterostructures[1] the half-width $\Delta B$ of the peaks in $\rho_{xx}$ between adjacent integer filling factors (FF's) exhibited a scaling behavior of the form $\Delta B(T)\propto T^{\kappa}$ with $\kappa=0.42\pm0.04$ as the exponent of the temperature $T$. Similarly, the maxima of the derivative of the Hall resistance $\rho_{xy}$ with respect to the magnetic field $B$ followed the law $(\partial\rho_{xy}/\partial B)^{\max}\propto T^{-\kappa}$ with the same exponent $\kappa$. These experimental results follow the original scaling and universality predictions of Pruisken.[3] Engel *et al.*[4] have extended the experimental methodology to include the fractional QHE (FQHE) regime (in *high-mobility* $Al_xGa_{1-x}As/GaAs$ heterostructures). The same value for $\kappa$ was found for the transition between FF's $\nu=\frac{1}{3}$ and $\frac{2}{5}$. An important question to ask is to what extent differences in the sample characteristics (i.e., impurity type, distribution, and sample geometry) will influence scaling.

In this paper we report on results obtained on *low-mobility* $Al_xGa_{1-x}As/GaAs$ heterostructures with intentionally built-in scatterers. The scatterer distribution was either homogeneous or a sheet of about one monolayer thickness (sheet-doping[5]). Attractive and repulsive Coulomb scatterers were used, realized by silicon and beryllium dopants, respectively. The doping concentrations are given in Table I. We define "scaling behavior" as a power-law dependence on temperature as denoted above. (When speaking of the phenomenon reported by Wei *et al.*,[1] we will refer to "universal scaling.") All the samples show a scaling behavior, in the temperature range from 40 mK to 1.1 K. We find that the exponent $\kappa$ depends on both type and strength of the doping and may be different in different Landau levels (LL's); $\kappa$ increases with decreasing mobility.

The samples were grown using the technique of molecular-beam epitaxy. On top of 2 $\mu$m nominally GaAs, a 21-nm undoped $Al_xGa_{1-x}As$ spacer layer was grown followed by 40 nm of Si-doped $Al_xGa_{1-x}As$. During the growth process, Si or Be impurities were introduced into the GaAs, either homogeneously or at a fixed distance (2 nm) from the interface (sheet-doping[5]). The mobilities of the doped heterostructures were drastically lower than those of the reference structures grown before

TABLE I. Type and strength of doping, carrier concentration, and mobility at $T=1.5$ K for the samples used. Temperature exponents $\kappa$, given for Landau levels $N=0$ and 1. NM represents transitions not measurable, NR represents spin splittings not resolved.

| | | | $n_s/n_d$ | $n_e$ | $\mu_e$ | Landau level, transition | | |
|---|---|---|---|---|---|---|---|---|
| Sample | Dopant | Distribution | $10^{10}$ cm$^{-2}$ | $10^{11}$ cm$^{-2}$ | $10^5$ cm$^2$/Vs | $0\uparrow$, $1-\frac{2}{3}$ | $0\downarrow$, 1–2 | 1, 2–3,3–4 |
| BeS1 | Be | Sheet | 0 | 2.52 | 3.02 | $0.77\pm0.02$ | $0.36\pm0.04$ | $0.51\pm0.03$ |
| BeS2 | Be | Sheet | 0.5 | 2.40 | 0.903 | $0.63\pm0.07$ | $0.56\pm0.05$ | $0.51\pm0.03$ |
| BeS3 | Be | Sheet | 2.0 | 2.10 | 0.225 | NM | $0.81\pm0.04$ | NR |
| SiS1 | Si | Sheet | 0.3 | 3.06 | 0.984 | $0.56\pm0.02$ | $0.44\pm0.02$ | $0.45\pm0.05$ |
| SiS2 | Si | Sheet | 0.6 | 3.10 | 0.839 | $0.68\pm0.05$ | $x0.53\pm0.07$<br>$y0.43\pm0.10$ | $0.45\pm0.05$ |
| SiS3 | Si | Sheet | 2.0 | 3.28 | 0.340 | NM | $0.62\pm0.03$ | NR |
| | | | $10^{15}$ cm$^{-3}$ | | | | | |
| SiH1 | Si | Homogeneous | 1.5 | 2.94 | 1.77 | NM | $0.28\pm0.06$ | $0.52\pm0.03$ |
| SiH2 | Si | Homogeneous | 5.0 | 3.12 | 0.841 | NM | $x0.53\pm0.06$<br>$y0.43\pm0.10$ | $0.63\pm0.03$ |

 

the doped structures, whereas the carrier concentration changed only slightly (see Table I). After the growth process the samples were etched into a standard Hall-bar geometry. The mobility $\mu$, electron and doping concentrations $n_e$ and $n_d$, and the type and distribution of the impurities of the samples used for the experiments are summarized in Table I. In order to measure $\rho_{xx}$ and $\rho_{xy}$ a standard ac lock-in technique was used, with frequencies of about 15 Hz. Using currents of 1 nA no current heating was observed.

In Fig. 1 the longitudinal resistivity $\rho_{xx}$ and the Hall resistivity $\rho_{xy}$ are shown for sample BeS2 as a function of magnetic field at temperatures of 40 mK and 0.95 K. The development of $\rho_{xx}$ and $\rho_{xy}$ in LL 0↓ is remarkable. At $T=0.95$ K, $\rho_{xx}$ shows a saddle-shaped structure and $\rho_{xy}$ a minimum in slope. This structure and the related phenomenon of asymmetric Shubnikov–de Haas oscillations have already been studied by Haug, Klitzing, and Ploog.[6] The minimum in $\rho_{xx}$ at a magnetic field of $B \approx 7.1$ T might suggest the development of a FQHE at lower temperatures. Instead, at 40 mK this structure has disappeared. Due to this irregular behavior (which is observed in samples with $\mu > 80000$ cm$^2$/Vs), in samples SiS2 and SiH2 we obtained different values of $\kappa$ from the half-width $\Delta B$ and the maxima in $\partial\rho_{xy}/\partial B$ (in this LL only, see Table I). In sample SiH1, these values coincide, but the two results have comparatively large errors.

The values of $\kappa$ taken from the half-width $\Delta B$ and the maxima of $\partial\rho_{xy}/\partial B$ give the same result within experimental error (with the two exceptions mentioned above). In the spin split LL $N=1$ the exponent $\kappa$ is the same for the transitions between FF's $\nu=2$ and 3 (LL 1↑) and between $\nu=3$ and 4 (LL 1↓). The temperature exponents of $(\partial^2\rho_{xx}/\partial B^2)$ and $(\partial^3\rho_{xy}/\partial B^3)$ at the critical point have also been analyzed, yielding results of $2\kappa$ and $3\kappa$, respectively (cf. Refs. 2 and 3). In a number of samples the transition between the *integer* FF $i=1$ and the *fractional* FF $i=\frac{2}{3}$ could also be analyzed, yielding a scaling behavior. The value of $\kappa$ for different samples and different LL's is not universal. This can be inferred from the results given in Table I; they will be discussed for each LL separately.

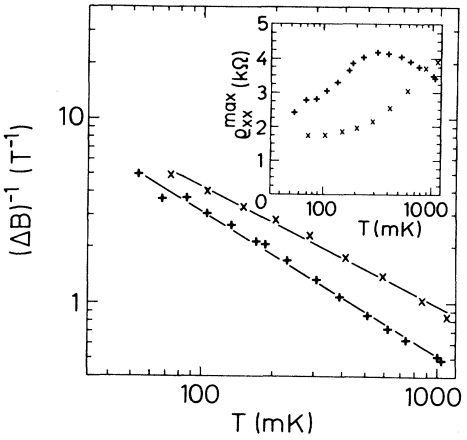


FIG. 2. Reciprocal half-width $(\Delta B)^{-1}$ as a function of temperature for Landau level 0↓ in samples BeS3 (+) and SiS3 (×). Inset: $\rho_{xx}^{max}$ for LL 0↓ for the two samples.

In LL 0↓ we generally found an increase of $\kappa$ with decreasing mobility, up to a value of $0.81 \pm 0.04$ in the strongly beryllium-doped sample BeS3. This increase is weaker for the silicon sheet-doped samples (see Table I and Fig. 2). In the inset of this figure we show the temperature dependence of the maxima in $\rho_{xx}$ for these two samples. In both samples the maxima in $\rho_{xx}$ show a decrease with decreasing temperature; for sample BeS3 this is the case for temperatures lower than 300 mK, while for sample SiS3 $\rho_{xx}^{max}$ decreases over the whole temperature range. This decrease of $\rho_{xx}^{max}$ with decreasing temperature has been emphasized as the characteristic starting point for universal scaling.[1]

For LL $N=1$ we find that for homogeneously silicon-doped samples there is an increase of the value of $\kappa$ with doping. While in the moderately doped sample SiH1 we find the value $\kappa=0.52 \pm 0.03$, in the strongly doped sample SiH2 we get the result $\kappa=0.63 \pm 0.03$ (see Fig. 3). The tendency towards a slightly lower $\kappa$ at the lowest tem-

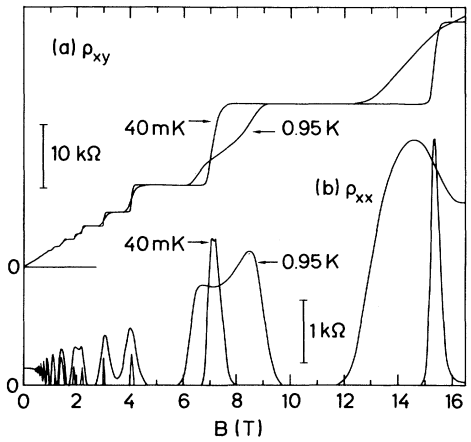


FIG. 1. (a) Hall resistivity $\rho_{xy}$ and (b) longitudinal resistivity $\rho_{xx}$ at 40 mK and 0.95 K for sample BeS2.

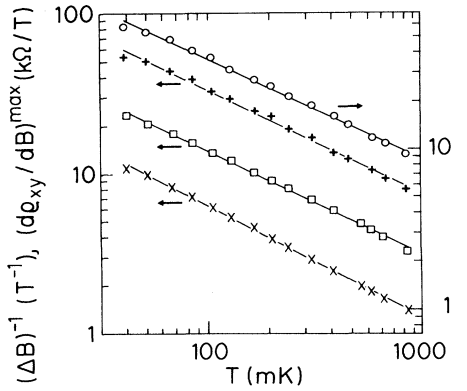


FIG. 3. $(\Delta B)^{-1}$ and maxima of the derivative $\partial\rho_{xy}/\partial B$ as a function of temperature for sample SiH2. Shown are the values for Landau levels 1↑ [×: $(\Delta B)^{-1}$; +: $\partial\rho_{xy}/\partial B$] and 1↓ (□,○). ×,□,+: left scale, ○: right scale, as indicated by the arrows.

peratures can be explained by a variation of the carrier concentration of about one percent. In the case of sheet-doping the samples with the same type of scatterers show the same value of $\kappa$ (for LL $N=1$). The fact that here we do not find an increase of $\kappa$ with doping is not astonishing in the Si-sheet-doped samples since the mobilities of samples SiS1 and SiS2 do not differ much. In the Be-sheet-doped samples, although the difference in the zero-field mobility is rather large, the effect of the doping on the spin splitting at temperatures of about 1 K is small, thus the difference in "effective" mobility (in this LL) is small as well. It was not possible to study the effect of higher sheet-doping concentrations on the scaling behavior due to the fact that in this case the spin splitting of LL $N=1$ is no longer well resolved.

To check a possible influence of geometry we determined the value of $\kappa$ for different length to width ratios on the given sample, yielding the same result in all cases. Further, one of the samples was measured in two different Hall-bar geometries, with channel widths of 100 and 400 $\mu$m. In both geometries we found the same values for $\kappa$ so that we conclude that these values do not depend upon geometry. The value of $\kappa$ thus seems to be determined by bulk properties, without a significant role of edge states.

Up to now, only transitions between *two integer* FF's (Refs. 1 and 2) and between *two fractional* FF's (Ref. 4) have been studied experimentally in terms of scaling in the literature. On four of our samples it was possible to study the transition from the *integer* FF 1 to the *fractional* FF $\frac{2}{3}$. The experimental results are given in Fig. 4 for samples BeS1 and SiS1. On two of the silicon-sheet-doped samples we found the same phenomenon as in the integer case: an increase of $\kappa$ with decreasing mobility. Opposite to this, we find that in the Be-doped series there is a *decrease* of $\kappa$ from a value of $0.77\pm0.02$ in sample BeS1 to $\kappa=0.63\pm0.07$ in sample BeS2. This means that there is a qualitative difference between transitions between integer FF's and the transitions from integer to fractional FF's. This will be discussed below.

As a summary of these experimental results we state that, in general, universal scaling is not observed in $Al_xGa_{1-x}As/GaAs$ heterostructures. For the IQHE transitions, we find an increase of $\kappa$ with decreasing mobility. This effect is most pronounced in the case of Be-sheet-doped samples in LL $N=0\downarrow$ where an increase of $\kappa$ of a factor of more than two results. The same effect occurs in Si-sheet-doped samples, although the change is smaller. In LL $N=1$ we see the increase of $\kappa$ with decreasing mobility in the homogeneously doped samples. The transition between FF's 1 and $\frac{2}{3}$ behaves qualitatively different; in this case there is no correlation between the value of $\kappa$ and the mobility.

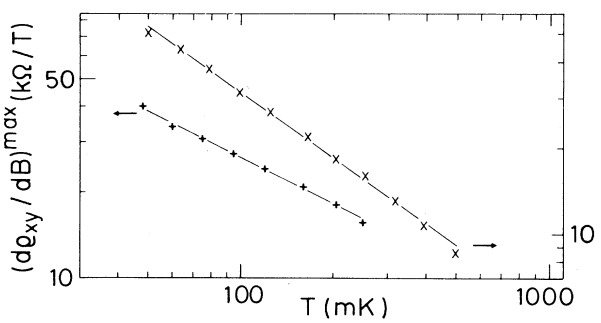


FIG. 4. $(\partial\rho_{xy}/\partial B)^{max}$ as a function of temperature for the transition between filling factors 1 and $\frac{2}{3}$ in samples SiS1 (+) and BeS1 (×).

A theory describing the universal scaling reported by Wei *et al.*[1] has been given by Pruisken.[3] He uses the inelastic-scattering length $L_{in}$ (Thouless length[7]) as an effective sample size which behaves like $L_{in}\propto T^{-p/2}$ with $T$ the temperature and an exponent $p$ of order 1. The localization length $\xi$ describes the transition from delocalized to localized states near the Fermi energy as $\xi\propto|B-B^*|^{-\nu}$ with $B$ the magnetic field and an exponent $\nu$ of order 1. $B^*$ is the critical field. The following relation for the scaling exponent $\kappa$ is derived:

$$\kappa=p/2\nu. \tag{1}$$

For *zero*-magnetic field, values of $p=1$ (Ref. 8) ("dirty metal limit") and $p=2$ (Fermi-liquid theory) have been given in the literature. Numerical studies on the localization length exponent $\nu$ (which has been predicted to be universal[3]) lead to a value of $\nu\approx2.3$ (Refs. 9–11) in the lowest LL for a potential with short-range correlations. This is in accordance with the value for the percolation limit[12] ($\nu=\frac{4}{3}$) including quantum tunneling[13] ($\nu=\frac{7}{3}$) (which corresponds to a potential with long-range correlation). In LL $N=1$ the situation seems less clear; according to several numerical studies,[9,11,14] a value of $\nu\approx4$ results, which is in disagreement with the universality prediction.[3] It seems difficult to relate our measured values of $\kappa$ to these values of $p$ and $\nu$ according to (1). This is mainly due to the lack of information about $p$ in high magnetic fields, but also due to the unclear situation concerning $\nu$ in LL $N=1$.

Obviously, universal scaling (i.e., a power law with the same exponent as in Ref. 1) is, in general, not observed in our samples. The question arises whether this might be the case at even lower temperatures. From our experimental studies, we cannot answer this problem in detail since we do not know how, in general, the microscopic details of the system affect the temperature scale for observing universal behavior. Nevertheless we do know that our samples are different from the $In_xGa_{1-x}As/InP$ heterostructures in Ref. 1. In the latter case, alloy scattering is dominant which corresponds to short-range potential fluctuations (compared to the cyclotron radius). In our samples there are Coulomb scatterers[15] separated by approximately 100 nm creating slowly varying potential fluctuations. Under these conditions, we may observe a crossover between two fixed points in the flow diagrams in the $\sigma_{xx}$-$\sigma_{xy}$ plane[3] (e.g., from percolation to localization, corresponding to different degrees of quantum tunneling). This interpretation would be consistent with the universality concept. Another possibility might be the existence of a third dimension in the flow diagrams due to Coulomb interaction, with a line of fixed points. These considerations are of a speculative character. In any case, the presence of strong Coulomb scattering universal scaling in general is not observed at temperatures available in the laborato-

ry. Nevertheless, we observe a scaling behavior over more than a decade in temperature in the experimentally accessible range.

A different approach for the explanation of the observed increase of $\kappa$ with decreasing mobility might be a systematically varying value of the scattering rate exponent $p$. In a paper by Haug *et al.*,[15] the effect of sheet doping on the position of the Hall plateaus has been studied, in the temperature range from 1.4 K to 15 K. The samples were essentially the same as those used in this work. It was reported that the center of the Hall plateaus shifts to higher (lower) magnetic fields for repulsive (attractive) scatterers, when compared with the position expected from the classical Hall line. In a model calculation, this effect could be explained by strong, non-Born scattering of electrons by individual impurities due to the presence of Coulomb scatterers in the range of the two-dimensional electron gas. This non-Born scattering may also influence the inelastic-scattering rate exponent $p$ and thus the observed increase of $\kappa$ with decreasing mobility; however, the microscopic mechanism is not understood at present.

The transition between the integer FF $\nu=1$ and the fractional FF $\nu=\frac{2}{3}$ has recently been discussed by Jain, Kivelson, and Trivedi.[16] They propose a law of corresponding states between the integer and fractional quantum Hall regimes. A "family" of fractional states is given by states with the same odd integer $m$ in the equation

$$\nu=n/[n(m-1)+1]\,, \qquad (2)$$

where $n$ is an integer. $m=1$ corresponds to integer FF's while $m=3$, e.g., describes the states $\nu=\frac{1}{3}$ and $\frac{2}{5}$. Jain *et al.* argue that transitions between states of the same family should exhibit universal scaling, while transitions between states of different families, e.g., from $\nu=\frac{2}{3}$ to $\nu=1$ should *not* exhibit scaling behavior, but a strong sample dependence.[17] In fact, we found a scaling behavior. However, a sample dependence was found which is entirely different from the results obtained for the IQHE. For the $1\rightarrow\frac{2}{3}$ transition there is no correlation to the mobility, in contrast to the IQHE transitions.

In summary, in specifically doped $Al_xGa_{1-x}As/GaAs$ heterostructures we observe scaling behavior, but in general we do not observe universal scaling. Instead, we find systematic changes of the exponent $\kappa$ which describes the temperature dependence of the half-width of the $\rho_{xx}$ peaks and the maximum of the slope of the Hall resistance. $\kappa$ increases with decreasing mobility, depending on the Landau level and the distribution of scattering centers. This can be ascribed to the presence of strong Coulomb scattering. However, our results do not necessarily violate the concept of universality as such. On the other hand, the transition from filling factor $\nu=1$ to $\nu=\frac{2}{3}$ behaves qualitatively different from the transitions between integer filling factors.

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[17]In an erratum to Ref. 16 [Phys. Rev. Lett. **64**, 1993(E) (1990)] the authors propose the same exponent $\kappa=0.42$ also for the transition from $\nu=\frac{2}{3}$ to $\nu=1$. Our results, however, favor the original proposal.