Temperature dependence of the cyclotron-resonance linewidth and effective mass in $GaAs/Ga_{1-x}Al_xAs$ square-well structures

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(Received 4 June 1990; revised manuscript received 19 October 1990)

The cyclotron-resonance linewidth (CRLW) is calculated in $GaAs/Ga_{1-x}Al_xAs$ single-quantum-well structures in the temperature range $1\ K < T < 300\ K$. Contributions to the CRLW due to electron–acoustic-phonon and electron–short-range-impurity interactions are treated on equal footing within the Green's-function formalism. Frequency-dependent self-energies of the above couplings are determined self-consistently with the density of states of two-dimensional electrons under strong magnetic field at low temperatures. The broadening due to the optical phonons is introduced through an elastic-scattering approximation at high temperatures. Our results are compared with available experiments in the low- and high-temperature regimes and good agreement is found. From the real part of the self-energy, we have calculated the electron effective mass as a function of temperature and found that the acoustic-phonon correction is not negligible.

I. INTRODUCTION

Magnetotransport properties of a two-dimensional (2D) electron gas confined in quantum-well semiconductor structures have attracted a lot of interest in recent years and their study still remains an active field. With the advent of high-quality (high-mobility) samples, it has been possible to measure the prominent new features that reflect the 2D or the quasi-2D character of electrons in semiconductor heterojunctions and superlattices. The cyclotron resonance technique in high magnetic fields applied in the direction normal to the 2D electron layer is a very useful tool for investigating the quantities such as effective mass, g factor, and various scattering mechanisms in these systems.

A considerable amount of work has been done both experimentally and theoretically to study the effects of various scattering mechanisms such as electron-phonon, electron-impurity interactions on the cyclotron resonance in 2D systems. There have been many experiments of cyclotron resonance in the 2D systems formed in $GaAs/Ga_{1-x}Al_xAs$ heterojunctions.[1–8] Most of them were performed at low temperatures and measured the linewidth and the magnetic field at which the cyclotron resonance line shape was investigated by Schlesinger *et al.*[4] Seidenbusch[5] performed cyclotron-resonance transmission and emission experiments at $T=4.2$ K and found that the polaron effect is reduced in comparison to the bulk GaAs. Brummell *et al.*[9] carried out cyclotron-resonance and magnetophonon experiments over a wide range of temperatures and indicated that the screening of the electron-phonon interactions is important and the dominant interaction is not with bulk GaAs optical phonons but with the ones associated with the interface. They have observed an anomalous increase with temperature in the cyclotron mass which was attributed to the screening of the electron-optical-phonon interaction. Hopkins *et al.*[10] measured the temperature dependence of the cyclotron-resonance linewidth (CRLW) in $GaAs/Ga_{1-x}Al_xAs$ up to room temperature. They found that the low-temperature linewidths varied slowly with electron concentration or inverse spacer thickness, suggesting that scattering by either impurities in the GaAs or alloy disorder in the $GaAs/Ga_{1-x}Al_xAs$ may be the dominant mechanism.

On the theoretical side, there have been numerous calculations of the Landau-level broadening, cyclotron- and magnetophonon-resonance linewidths in various approximations. Singh[11] and collaborators[12–13] studied the effects of phonons on the Landau-level broadening of 2D quantum-well structures using Green's-function techniques. The line shape of the cyclotron-resonance spectrum was investigated by several groups in the memory-function formalism.[14–17] The magnitude of the Landau-level broadening due to different types of scattering mechanisms, including their temperature and magnetic-field dependence, was estimated by Prasad and Singh,[12] Chaubey and Van Vliet,[18] and Mori *et al.*[19] The effects of screening in the CRLW were extensively studied by Ando and co-workers,[20,21] and other researchers.[22,23] We have recently reported[24] on the phonon-assisted cyclotron resonance in n-type 2D systems, where an additional peak in the cyclotron resonance line shape due to longitudinal optical-phonon absorption beside the usual CR peak was predicted.

The effective mass of 2D electrons has also attracted theoretical interest. Polaronic effects in $GaAs/Ga_{1-x}Al_xAs$ heterojunctions were calculated by Das Sarma and Mason[25] using static screening. Lei[26] has calculated the effective-mass renormalization due to optical phonons including the dynamical screening. Other calculations based on different methods have also been reported.[22,27]

The motivation of the present work derives from the recent experiments of Hopkins *et al.*[10] and Brummell *et al.*[9] on the CRLW in $GaAs/Ga_{1-x}Al_xAs$ heterojunc-

tions. They have measured[10] the temperature dependence of the CRLW over a wide range, and concluded that for $T<50$ K, the dominant scattering mechanism at high fields is the electron-impurity scattering. They have emphasized the importance of calculating CRLW self-consistently.[10] Wu and Peeters[16] in their memory-function calculation attempted to describe the observed cyclotron-resonance linewidths by considering individual scattering mechanisms. They obtained agreeable results at high temperatures, but at low temperatures their CRLW differed from the experimental data. Here we concentrate on the low-temperature region where the electron-impurity and electron–acoustic-phonon effects are most important. Also measured in these experiments[9] was the reduction in the cyclotron mass as a function of temperature due to electron-phonon interactions. As a by-product of our calculations we provide an order of magnitude result for the effective mass due to the electron–acoustic-phonon interaction.

Recently, we have studied[28] the contribution of electron-phonon scattering to the frequency-dependent Landau-level broadening and the density of states of a 2D electron gas in a strong magnetic field. We have found that electron-phonon and electron-disorder interactions influence each other at low temperatures through the density of states, and it is necessary to treat their contributions to the Landau-level broadening self-consistently.

Our goal in this paper is to present calculations of the temperature dependence of the CRLW in the quasi-2D system $GaAs/Ga_{1-x}Al_xAs$ single quantum well. We do not investigate the theory of cyclotron-resonance but relate the cyclotron-resonance linewidth to the Landau-level broadening in a simple approximation to compare with the experiments. Hopkins *et al.*[10] have stressed the necessity of calculating the CRLW self-consistently including the density of states, and it is our aim here to attempt a calculation towards this. We also calculate the cyclotron mass renormalization due to electron–acoustic-phonon interaction and discuss qualitatively the interpretation of the experimental data.

The organization of this paper is as follows. We present the details of our calculation in Sec. II. Our results and comparison with experiment will be discussed in Sec. III. We give a brief summary of our findings and make concluding remarks in Sec. IV.

II. THEORY

A. Hamiltonian

The Hamiltonian of the quasi-two-dimensional system of electrons under a strong magnetic field in the $GaAs/Ga_{1-x}Al_xAs$ single-quantum-well structure we consider is written as

$$H=\sum_{\lambda}\omega_{\lambda}a_{\lambda}^{\dagger}a_{\lambda}+\sum_{\mathbf{Q}}\omega_{\mathbf{Q}}(b_{\mathbf{Q}}^{\dagger}b_{\mathbf{Q}}+\tfrac{1}{2})+\sum_{\mathbf{Q}}\sum_{\lambda\lambda'}M_{\lambda\lambda'}^{\mathrm{ap}}(\mathbf{Q})a_{\lambda}^{\dagger}a_{\lambda'}(b_{\mathbf{Q}}^{\dagger}+b_{-\mathbf{Q}})+\sum_{\mathbf{Q}}\omega_{L}(c_{Q}^{\dagger}c_{Q}+\tfrac{1}{2})+\sum_{\mathbf{Q}}\sum_{\lambda\lambda'}M_{\lambda\lambda'}^{\mathrm{op}}(\mathbf{Q})a_{\lambda}^{\dagger}a_{\lambda'}(c_{\mathbf{Q}}^{\dagger}+c_{-\mathbf{Q}})+H_{e-d}\ , \tag{1}$$

in which λ stands for the Landau level n and subband quantum number p. Here $a_{\lambda}^{(\dagger)}$ is the electron annihilation (creation) operators in the nth Landau level and pth subband, $b_{\mathbf{Q}}^{(\dagger)}$ is the acoustic-phonon annihilation (creation) operators, and $c_{\mathbf{Q}}^{(\dagger)}$ is the optical-phonon annihilation (creation) operators with wave vector $\mathbf{Q}$. The energy of an electron in the quantum level λ is denoted by $\omega_{\lambda}=(n+\frac{1}{2})\omega_c+p^2E_0$, where $\omega_c=eB/m_bc$ is the cyclotron frequency with m_b the band mass and $E_0=\hbar^2\pi^2/2m_bL$ is the subband energy with L the square-well width. The phonon energies are $\omega_{\mathbf{Q}}$ and ω_L for the acoustic and optical phonons, respectively. The third and fifth terms in the Hamiltonian describe the coupling of different types of phonons to the electrons and their matrix elements can be expressed in the Landau basis as

$$|M_{\lambda\lambda'}^{\mathrm{ap(op)}}|^2=\sum_{\mathbf{Q}}|V_{\mathbf{Q}}^{\mathrm{ap(op)}}|^2|J_{nn'}(Q_{\perp})|^2|F_{pp'}(Q_z)|^2\ . \tag{2}$$

In Eq. (2), the electron-phonon interaction is given by

$$|V_{\mathbf{Q}}|^2=\begin{cases}\dfrac{\hbar D^2}{2dv_s}Q & \text{for acoustic phonons,}\\[2ex] \dfrac{2\pi\alpha}{Q^2}\left(\dfrac{\omega_L^3}{m_b}\right)^{1/2} & \text{for optical phonons ,}\end{cases} \tag{3}$$

in which D is the deformation potential constant, d the mass density, v_s the speed of sound in GaAs, and α is the polar-optical-phonon coupling constant. The form factors for the motion of electrons perpendicular and parallel to the applied magnetic field are

$$J_{nn'}(Q_{\perp})=(-1)^{(n'-n)}\sqrt{n'!/n!}x^{(n'-n)/2}L_n^{n'-n}(x^2)\times e^{-x^2/2},\quad x=Q_{\perp}l/\sqrt{2}; \tag{4}$$

$$|F_{pp'}(Q_z)|^2=\left|\int dz\,\phi_p(z)e^{iQ_zz}\phi_{p'}^{*}(z)\right|^2, \tag{5}$$

respectively, where $\phi_p(z)$ is the electron wave function in the z direction, with the subband quantum number p; $L_n^m(x)$ is the associated Laguerre polynomial; and $l=(\hbar/m_b\omega_c)^{1/2}$ is the cyclotron radius.

The last term in Eq. (1) contains the electron-disorder interaction and here we consider short-range impurities and treat their effect in a phenomenological way within the self-consistent Born approximation.[29] We defer the discussion of the model for the electron-impurity interaction to the following sections.

B. Cyclotron resonance and Landau-level broadening

In the cyclotron resonance experiments, the magneto-absorption spectrum is observed. The absorption coefficient due to the cyclotron transition from the nth Landau level to the n'th is given by[30]

$$\alpha(\omega)\sim\frac{\Gamma_{nn'}}{(\omega-\omega_{n'}+\omega_n-\Delta_{nn'})^2+\Gamma_{nn'}^2}\ . \tag{6}$$

The cyclotron peaks will appear in the absorption spectrum when the energy difference between the two Landau

levels is equal to the energy of the incoming electromagnetic radiation. Here $\Gamma_{n'n}$ and $\Delta_{n'n}$ describe the broadening and line shift, respectively, of the cyclotron resonance during the transition, and they can be calculated in various approximations.[13,16,31,32] We use the following expression for the cyclotron-resonance linewidth:

$$\Gamma_{CR}=\Gamma_{nn'}=\tfrac{1}{2}[\Gamma_n(\omega_n)+\Gamma_{n+1}(\omega_{n+1})] , \tag{7}$$

where we have assumed that the Landau levels are decoupled. It was found[13,20] that the above expression for Γ_{CR} holds to a very good approximation at high magnetic fields (quantum limit). We note the difference between the approach taken here to calculate Γ_{CR} and that of Wu and Peeters.[16] Our expression [Eq. (7)] involves the broadenings due to two adjacent Landau levels and can be obtained from the absorption spectrum under certain approximations. In contrast, Wu and Peeters[16] take $\Gamma_{CR}=-\mathrm{Im}\Sigma(\omega_c)$ where $\Sigma(\omega)$ is the memory function. That both methods give similar results will be shown in the subsequent sections.

Since the Wu and Peeters work has agreement with the experiments of Hopkins *et al.*[10] at high temperatures, but shows some discrepancy at low temperatures, our aim is to calculate the Landau-level broadening at low temperatures due to electron-phonon and electron-disorder interactions to obtain the cyclotron-resonance linewidth Γ_{CR}.

The electron self-energy due to electron–acoustic-phonon interaction in second-order perturbation theory can be written as[28]

$$\Sigma_n^{e\text{-ap}}(\omega)=\sum_{n'p'}\int\frac{d^3Q}{(2\pi)^3}|V_Q^{ap}|^2|J_{nn'}(Q_\perp)|^2|F_{pp'}(Q_z)|^2\int d\omega'\rho_{n'}(\omega')\left[\frac{n_B(\omega_Q)+n_F(\omega')}{\omega-\omega'+\omega_Q-i\eta}+\frac{n_B(\omega_Q)+1-n_F(\omega')}{\omega-\omega'-\omega_Q-i\eta}\right] . \tag{8}$$

In the above expression, n_B and n_F are the Bose and Fermi distribution functions, respectively, and $\rho_n(\omega)$ is the spectral function [related to the density of states $D(\omega)$] of the nth Landau level. Since we are not necessarily concerned with the high-temperature regime only, we especially keep the Fermi function n_F. It was found[19] that the location of the Fermi level ε_F within the Landau level brings about prominent features in the Landau-level broadening. The real and imaginary parts of $\Sigma_n^{e\text{-ap}}$ define the shift in energy $\Delta_n^{e\text{-ap}}$ and the Landau-level broadening $\Gamma_n^{e\text{-ap}}$. The Landau-level broadening due to electron–acoustic-phonon interaction reads

$$\Gamma_n^{e\text{-ap}}(\omega)=\pi\int\frac{d^3Q}{(2\pi)^3}|V_Q^{ap}|^2|J_{nn}(Q_\perp)|^2|F_{pp'}(Q_z)|^2 \times\{\rho_n(\omega+\omega_Q)[n_B(\omega_Q)+n_F(\omega+\omega_Q)]+\rho_n(\omega-\omega_Q)[n_B(\omega_Q)+1-n_F(\omega-\omega_Q)]\} . \tag{9}$$

The shift in the energy $\Delta_n^{e\text{-ap}}$ is simply the real part of the electron self-energy and for the acoustic-phonon interaction it can be most easily evaluated by using the Kramers-Krönig relation

$$\Delta_n^{e\text{-ap}}(\omega)=\frac{1}{\pi}\mathrm{P}\int d\omega'\frac{\Gamma_n^{e\text{-ap}}(\omega')}{\omega'-\omega} , \tag{10}$$

in which P denotes the principal part of the integral.

For the self-energy due to electron-impurity scattering we use the self-consistent Born approximation (SCBA) introduced by Ando *et al.*[29]

$$\Sigma_n^{e\text{-}d}(\omega)=\frac{\gamma^2}{4}G_n(\omega) , \tag{11}$$

where γ is the phenomenological parameter describing the effects of the impurity scattering and $G_n(\omega)$ is the full Green's function

$$G_n(\omega)=\frac{1}{\omega-\omega_n-\Sigma_n^{e\text{-ap}}(\omega)-\Sigma_n^{e\text{-}d}(\omega)} , \tag{12}$$

in which the self-energies due to electron–acoustic-phonon and electron-disorder interactions are included in an additive form. Strictly speaking, the SCBA form for the self-energy is more appropriate for short-range impurities, and a first-principles calculation of γ is possible.[29] Here we do not specify the exact nature or the form of disorder, but use γ as a parameter.

We write the real and imaginary parts of the electron self-energy due to electron-disorder interaction as

$$\Delta_n^{e\text{-}d}(\omega)=\frac{\gamma^2}{4}\frac{\omega-\omega_n-\Delta_n(\omega)}{[\omega-\omega_n-\Delta_n(\omega)]^2+\Gamma_n^2(\omega)} , \tag{13}$$

$$\Gamma_n^{e\text{-}d}(\omega)=\frac{\gamma^2}{4}\frac{\Gamma_n(\omega)}{[\omega-\omega_n-\Delta_n(\omega)]^2+\Gamma_n^2(\omega)} , \tag{14}$$

respectively, where the total Landau-level broadening $\Gamma_n(\omega)$ and the energy shift $\Delta_n(\omega)$ are

$$\Gamma_n(\omega)=\Gamma_n^{e\text{-ap}}(\omega)+\Gamma_n^{e\text{-}d}(\omega) , \tag{15}$$

$$\Delta_n(\omega)=\Delta_n^{e\text{-ap}}(\omega)+\Delta_n^{e\text{-}d}(\omega) . \tag{16}$$

Finally, the density of states ρ_n that enters the calculation of $\Gamma_n^{e\text{-ap}}$ is expressed as

$$\rho_n(\omega)=\frac{1}{\pi}\frac{\Gamma_n(\omega)}{[\omega-\omega_n-\Delta_n(\omega)]^2+\Gamma_n^2(\omega)} , \tag{17}$$

where Γ_n and Δ_n are given in Eqs. (15) and (16).

Since the self-energies depend on the density of states, and the density of states depends on the electron self-energy, the above set of equations needs to be solved self-consistently. Such a self-consistent treatment of the Landau-level broadening and the density of states for a quasi-two-dimensional system under magnetic field, to our knowledge, has not been attempted previously. The importance of the self-consistent determination of the Landau-level broadening and the density of states for the purpose of comparison with cyclotron-resonance experiments was emphasized by Hopkins *et al.*[10] We have solved the system of equations that determine $\Gamma_n^{e\text{-ap}}$, $\Gamma_n^{e\text{-}d}$, and $\rho_n(\omega)$ as functions of frequency, iteratively until consistent. The full-frequency-dependent properties of the electron self-energy due to electron-phonon interactions will be discussed elsewhere.[28]

The contribution of the optical phonons to the CRLW was not included in the self-consistent loop. Instead, at high temperatures we take a simplified approach. The Landau-level broadening $\Gamma^{e\text{-op}}(\omega_c/2)$ due to the electron–optical-phonon interaction is evaluated in the elastic scattering approximation[11,13] by solving

$$\Gamma_n^{e\text{-op}}=\left\{\int\frac{d^3Q}{(2\pi)^3}|V_Q^{\text{op}}|^2|J_{0n}(Q_\perp)|^2|F_{11}(Q_z)|^2\times[2n_B(\omega_L)+1]\right\}^{1/2}, \tag{18}$$

where n is determined by the resonant condition $\omega-\omega_n+\omega_L=0$.

III. RESULTS AND DISCUSSION

In this study we chose $\phi_p(z)=(2/L)^{1/2}\sin(2\pi pz/L)$, wave functions appropriate for an infinite square well, with width $L\approx 100$ Å and concentrate on the lowest subband $p=1$. The two lowest Landau levels ($n=0$ and 1) were considered for the evaluation of the cyclotron-resonance linewidth Γ_{CR}. We shall drop reference to the indices n and p in the following discussion. We have only partially investigated the effect of the square-well width L on the CRLW. Our results remain insensitive to L in the range 50 Å $<L<$ 500 Å to the degree of accuracy sought here.

We used material parameters appropriate for the $GaAs/Ga_{1-x}Al_xAs$ single-quantum-well structures. The electron–acoustic-phonon interaction was considered in the deformation-potential model, and we took $D=13.5$ meV for the deformation potential, $d=5.3$ g/cm^3 for the mass density of the system, and $v_s=5.37\times10^5$ cm/s for the speed of sound in GaAs. The optical phonons are treated as dispersionless with the energy $\omega_L=36.2$ meV, and the electron–optical-phonon coupling constant α was taken to be 0.07. Throughout we used $m_b=0.067m_e$ for the band mass of electrons, where m_e is the bare electron mass.

In Fig. 1, the temperature dependence of the CRLW due to electron–acoustic-phonon and electron–optical-phonon interactions is shown to indicate their relative importance. The broadening due to electron–acoustic-phonon interaction results from a self-consistent calculation, whereas the broadening due to electron–optical-phonon interaction is calculated in the elastic-scattering approximation using Eq. (18). We note that the contribution of electron–acoustic-phonon interaction (solid line) to the CRLW is not negligible even at $T\sim100$ K. The electron–optical-phonon interaction becomes appreciable for $T>50$ K, and is the dominant scattering mechanism at high temperatures. The temperature dependence of the CRLW due to electron–acoustic-phonon coupling is not as strong as that due to electron–optical-phonon coupling. Similar conclusions were reached by Wu and Peeters.[16] Our calculations indicate that the broadening due to electron-disorder interaction $\Gamma^{e\text{-}d}$ is essentially temperature dependent,[28] and decreases slightly as the temperature is increased. The actual value of the electron-disorder interaction parameter γ we used is 0.5 meV. We have found that although the frequency-dependent line shape of $\Gamma^{e\text{-}d}$ depends on the choice of γ, the magnitude of $\Gamma^{e\text{-ap}}$ is nearly independent of it.

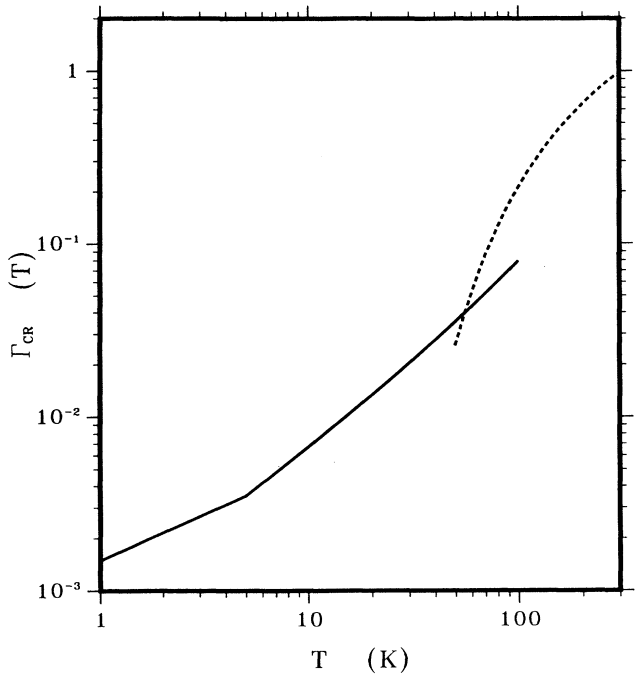

FIG. 1. Calculated cyclotron resonance linewidth Γ_{CR} due to electron–acoustic-phonon (solid line) and electron–optical-phonon (dashed line) against temperature at $B=6.2$ T and $n_s=10^{11}$ cm^{-2}.

To make realistic contact with experiments, we have done our calculations at the electron density of $n_s=10^{11}$ cm^{-2} and magnetic field $B=6.2$ T. Defining the filling factor $\nu=2\pi n_s l^2$, we deduce that for these parameters the Fermi energy $\varepsilon_F\approx3.6$ meV, so that it lies below the center of the first Landau level. Although at high temperatures the exact location of ε_F becomes immaterial for the Landau-level broadening, we have shown that it gives rise to noticeable effects at low temperatures (e.g., when $k_BT\sim\Gamma$).[28]

We compare the results of our calculation of the

cyclotron-resonance linewidth with the experiments of Hopkins *et al.*[10] We show in Fig. 2 the calculated CRLW in the present approximation (solid line), the treatment of Wu and Peeters[16] (dashed line), and the experimental data[10] (solid squares). The cyclotron-resonance linewidth Γ_{CR} in Fig. 2 is calculated by adding contributions from electron–acoustic-phonon, electron–optical-phonon, and electron-impurity couplings. For the broadening due to electron-impurity scattering we have taken $\Gamma^{e-d} \simeq 0.05$ T. Ando and Murayama[21] give $\Gamma^{e-d} \simeq 0.01$ T in their self-consistent calculation of electron-impurity interactions including screening, so the value we use here to fit our theoretical curve to the experimental data is comparable to theirs. The small value of Γ^{e-d} used to fit our curve to the experiment may indicate the high quality of the samples in which the impurity effects are secondary to the electron–acoustic-phonon scattering at low temperatures. We have not included the piezoelectric-phonon contribution to the CRLW since it is considered rather small.[16]

The overall agreement we obtain with the data of Hopkins *et al.*[10] is reasonable, especially if the entire range of temperature is regarded. The electron–optical-phonon interactions contribute substantially to Γ_{CR} for $T > 50$ K. Since this contribution was not calculated self-consistently, but using the elastic-scattering approximation, we obtain slightly larger values of Γ_{CR} at high temperatures. Strictly speaking the elastic-scattering approximation will not be valid in the temperature region $T < 300$ K and the elastic nature of this scattering mechanism will be important. However, we mainly focus our attention to the low-temperature region where the electron–acoustic-phonon and the electron-disorder scatterings are dominant. Also not included in the calculation of $\Gamma^{e\text{-op}}$ were the nonparabolicity effects and the higher subbands, which may become important at the temperatures considered. Wu and Peeters[16] have also calculated Γ_{CR}, and compared their results with the data of Hopkins *et al.*[10] They have calculated the CRLW due to the individual scattering mechanisms through the memory-function formalism, and introducing a broadening parameter to fit their calculation to the experiment, they have obtained reasonable agreement. As can be seen in Fig. 2 (dashed line), the calculations of Wu and Peeters[16] display a discrepancy at low temperatures, and this was attributed to the electron-impurity scattering and self-consistent screening by them.

It is difficult to give an analytic expression for the temperature dependence of the CRLW for the whole range of temperatures considered. The temperature dependence of the Landau-level broadening in our numerical calculations comes from two factors. First, the Bose and the Fermi distribution functions determine the number of phonons at a given temperature. Second, the density of states or the spectral function $\rho_n(\omega)$ depends on temperature when evaluated self-consistently [see Eqs. (8)–(17)]. The $\rho_n(\omega)$ scales qualitatively with temperature such that its peak value decreases as T is increased, and it acquires more weight in the wings away from the peak position keeping it normalized.[28]

In Fig. 3, we show the magnetic field dependence of the

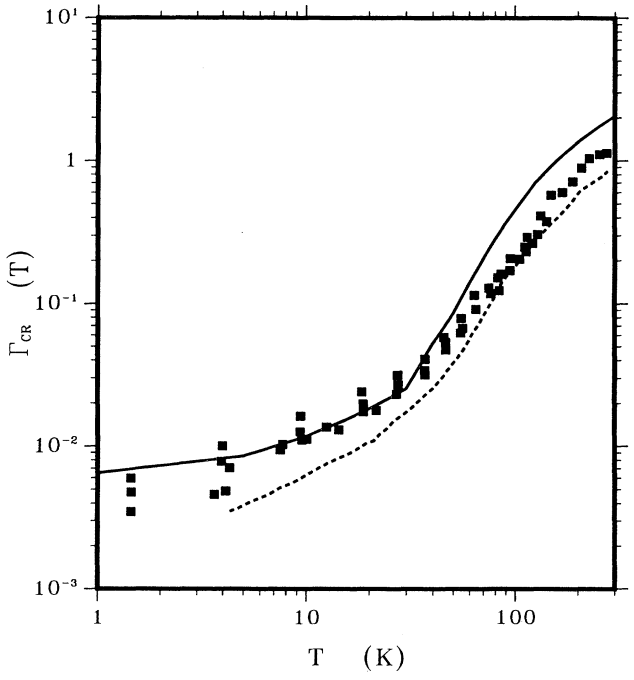

FIG. 2. Comparison of our calculation with the experimental data (solid squares, Ref. 10). The solid line represents our fit including the electron-disorder contribution. The dashed line is the calculation of Ref. 16.

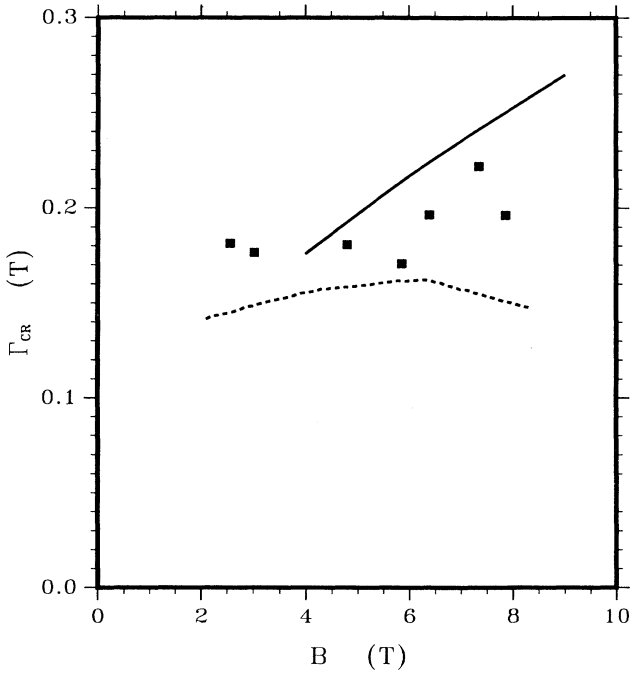

FIG. 3. Magnetic field dependence of the cyclotron-resonance linewidth Γ_{CR} at $T = 90$ K and $n_s = 1.4 \times 10^{11}$ cm^{-2}. Our calculation is indicated by the solid line, solid squares refer to the experimental data of Ref. 10, and the dashed line is the memory-function calculation of Ref. 16.

CRLW at $T=90$ K. The experimental results of Hopkins *et al.*[10] is indicated by solid squares, and the dashed curve is the calculation of Wu and Peeters.[16] It is interesting to note that the curve due to Ref. 16 gives a roughly constant Γ_{CR} as a function of magnetic field, whereas the data (solid squares) seems to have oscillations. The discrepancy between Wu and Peeter's calculation and the data was attributed to possible screening effects. Our calculation exhibits a slight magnetic-field dependence of the CRLW so that Γ_{CR} increases with increasing B, in contrast to the findings of Wu and Peeters.[16] We do not see any anomalous peak or oscillations.

The magnetic field dependence of the CRLW in our model comes in part from the dependence of the form of the electron-phonon scattering potential on the cyclotron radius, i.e., the dependence of the form factor $J_{nn'}(Q_\perp)$ on the magnetic radius l. It also enters through the self-consistent determination of the density of states $\rho_n(\omega)$, rendering an explicit expression difficult. Qualitatively, the peak height of the density of states $\rho_n(\omega)$ increases, making it a sharper function, and the weight in the wings decreases as the magnetic field is increased. To obtain our calculated Γ_{CR} in Fig. 3 we took the electron density $n_s=1.4\times10^{11}$ cm^{-2} fixed. As the magnetic field changes from $B=2$ to 10 T the position of the Fermi level moves within the Landau level. Since the temperature is high enough ($k_BT\sim7.7$ meV), this does not effect the value of Landau-level broadening.

The cyclotron mass was calculated by various authors[22,25–27] in different approximations. Most of these calculations were concerned with the enhancement of the effective mass due to electron–optical-phonon coupling. We define the mass renormalization (after Wu *et al.*[22]) due to the electron–acoustic-phonon interaction by

$$\frac{\Delta m}{m_b}=\left.\frac{\partial\Delta^{e\text{-ap}}}{\partial\omega}\right|_{\omega_c/2}, \tag{19}$$

where $\Delta^{e\text{-ap}}$ is the real part of the electron self-energy. We plot the mass renormalization $\Delta m/m_b$ against temperature in Fig. 4. The real part of the electron self-energy due to electron–acoustic-phonon interaction $\Delta^{e\text{-ap}}$ is obtained through the self-consistent calculation at $B=6.2$ T and $n_s=1.4\times10^{11}$ cm^{-2}. At the physical parameters used to calculate $\Delta^{e\text{-ap}}$, the Fermi energy $\varepsilon_F\approx5.02$ meV, which is close to $\omega_c/2\approx5.38$ meV, i.e., ε_F lies near the center of the Landau level. Therefore the quasiparticle effective mass and the cyclotron mass evaluated using Eq. (19) at $\omega=\omega_c/2\approx\varepsilon_F$ coincide. We have not attempted to compare our result of Fig. 4 directly with the experiments of Brummell *et al.*[9] because of the approximate nature of our calculation and the nonparabolicity effects present in the data. Instead, without going into a much detailed analysis of the calculation of effective mass, we mention here some of the qualitative results. The calculation of Wu *et al.*[22] of the same quantity $\Delta m/m_b$, for polar optical phonons were somewhat below the experimental data of Brummell *et al.*[9] The magnitude of $\Delta m/m_b$ in the temperature range $10<T<75$ K in our calculation is close to that obtained in Ref. 22. This suggests the possibility of the electron–acoustic-phonon interaction being responsible for part of the observed decrease in the cyclotron mass at low temperatures. It was noted[33] that the mass renormalization due to electron-purity scattering will be small compared to that due to electron-phonon scattering. Indeed, in our self-consistent calculation we find the quantity $\partial\Delta^{e\text{-}d}/\partial\omega|_{\omega_c/2}$ to be quite small.

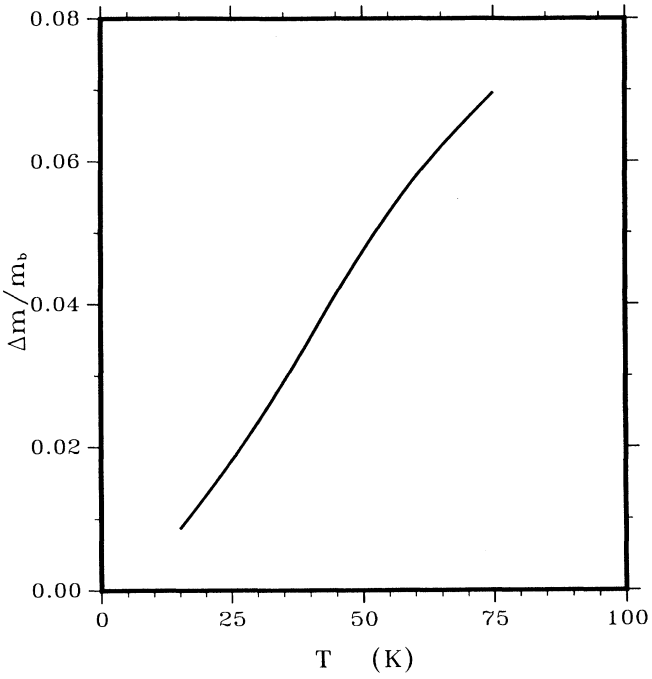

FIG. 4. Effective-mass renormalization $\Delta m/m_b$ in GaAs/$Ga_{1-x}Al_xAs$ due to the electron–acoustic-phonon interaction as a function of temperature. The calculation for $\Delta m/m_b$ is done at $B=6.2$ T and $n_s=1.4\times10^{11}$ cm^{-2}.

In the present calculation, we assumed that the applied magnetic fields are strong enough so that the Landau levels are completely decoupled and can be evaluated independently. The CRLW was calculated using Eq. (7) where only the transition from $n=0$ to 1 levels was considered. At a higher level of approximation, one may include the effects of coupling of Landau levels by calculating the broadening from the following expression of CRLW:

$$\Gamma_{\lambda\lambda'}(\omega)=\sum_{\mathbf{Q}}\sum_{\lambda'',\pm}N_{\mathbf{Q}}^{\pm}|V_{\mathbf{Q}}|^2[|J_{n''n}(Q_\perp)|^2|F_{p''p}(Q_z)|^2\rho\lambda''\lambda(\omega\pm\omega_{\mathbf{Q}})+|J_{n'n''}(Q_\perp)|^2|F_{p'p''}(Q_z)|^2\rho_{\lambda'\lambda''}(-\omega\pm\omega_{\mathbf{Q}})], \tag{20}$$

where $\rho_{\lambda\lambda'}(\omega)$ is given by

$$\rho_{\lambda\lambda'}(\omega)=\frac{1}{\pi}\frac{\Gamma_{\lambda\lambda'}}{(\omega+\omega_\lambda-\omega_{\lambda'}+\Delta_{\lambda\lambda'})^2+\Gamma^2_{\lambda\lambda'}} . \quad (21)$$

In the above equations, λ denotes the Landau level n and the subband quantum number p, $N_Q^{\pm}=n_B(\omega_Q)+\frac{1}{2}\pm\frac{1}{2}$, and $\Delta_{\lambda\lambda'}$ is the real part of the self-energy which can be obtained from $\Gamma_{\lambda\lambda'}$ using the Kramers-Krönig relation. Clearly, the Landau-level broadening depends on the density of states $\rho_{\lambda\lambda'}$, which itself depends on $\Gamma_{\lambda\lambda'}$. Hence the set of equations has to be solved self-consistently, which would be a very involved task.

The experiments of Hopkins *et al.*[10] were performed on heterostructures. The finite thickness of the electron layer in the $GaAs/Ga_{1-x}Al_xAs$ system was incorporated by the use of the infinite-square-well wave function. Wu and Peeters[16] in their memory-function approach have used the Fang-Howard variational wave function[29] in the lowest subband. Both calculations give similar results for the cyclotron-resonance linewidths, indicating that the choice of the electronic wave function in the z direction is not essential. In fact, in our investigation[28] of the electron-phonon contribution to the frequency-dependent Landau-level broadening we have found that the square-well and the Fang-Howard wave functions gave very similar results provided the appropriate variational parameter is chosen.

IV. CONCLUSION

In conclusion, we have evaluated the cyclotron-resonance linewidth Γ_{CR} as a function of temperature in $GaAs/Ga_{1-x}Al_xAs$ single-quantum-well structures. At temperatures up to ~ 100 K, we have performed self-consistent numerical calculations of the Landau-level broadening due to electron–acoustic-phonon and electron-disorder interactions. At higher temperatures we have used the elastic-scattering approximation to obtain the contribution of optical phonons to the CRLW. The results in these two regimes were combined when comparing with the experimental data, which cover temperatures up to 300 K. A comparison of our results with the relevant experiments and other theoretical accounts gave good agreement. The main result of our calculation is that the electron–acoustic-phonon interaction contributes significantly to the Landau-level broadening and hence to the cyclotron-resonance linewidth, even at temperatures where the electron–optical-phonon scattering is the dominant mechanism. We have illustrated the importance of calculating the Landau-level broadening self-consistently.

In the present work, we have calculated the Landau-level broadening including contributions to it from electron–acoustic-phonon scattering and electron-impurity scattering self-consistently. The effect of the electron–optical-phonon scattering was not treated self-consistently with the other scattering mechanisms. It would be interesting to treat the electron–optical-phonon interaction on equal footing with the others. Even perhaps the inclusion of the screening effects through electron-electron interaction would be worthwhile to obtain better, parameter-free agreement with the experimental data. Up to now, most of the calculations that include screening were limited to the frequency-independent case using $\epsilon(q)$. Our self-consistent method of calculating the Landau-level broadening could be extended to include a frequency-dependent dielectric function $\epsilon(q,\omega)$, opening the possibility of studying the effects of dynamic screening. A full self-consistent theory of the cyclotron-resonance linewidth to cover both the low- and high-temperature regions is very much needed.

We have also calculated the correction to the cyclotron mass due to the electron–acoustic-phonon interactions. Our preliminary results indicate that the experimentally observed anomalous mass increase with temperature may partly be explained in terms of acoustic phonons.

The self-consistent method with which we calculate the Landau-level broadening in this paper is quite general. We have applied it to calculate the cyclotron-resonance linewidth in the $GaAs/Ga_{1-x}Al_xAs$ single-quantum-well structure, but the calculation could easily be extended to multiple quantum wells, and even different kind of semiconductor materials.

ACKNOWLEDGMENTS

One of the authors (M.S.) acknowledges the Natural Science and Engineering Research Council of Canada and Centre for Chemical Physics at the University of Western Ontario for financial support.

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