Magneto-optics of multilayers with arbitrary magnetization directions

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Equations are derived for determining magneto-optic coefficients in multilayer systems with arbitrary directions of their magnetizations. The equations are cast in a matrix form that is suitable for numerical simulations. This establishes a framework for calculating the Kerr and Faraday effects for a multilayer system in much the same way as has been applied previously to the bulk. Numerical Kerr results are presented for the following systems: bulk Fe, an overlayer of 50 Å of Fe on Au, and for an Fe/Au superlattice for different directions of the magnetization and different angles of incidence.

I. INTRODUCTION

There has recently been much interest in the magneto-optics of thin magnetic overlayers[1–6] and superlattices,[7–13] While magneto-optics is more than a century old,[14] it is the development of laser-based information-storage systems[15–17] that has created demand for high-performance materials. Dielectric coatings on magnetic substrates have been successfully used to enhance the magneto-optical signal.[18,19] This has led to the theoretical analysis of layered structures.[20–24] Conventionally, both theory and experiment deal with three distinct configurations for the direction of the magnetization **M** with respect to the plane of incidence (normal unit vector $\hat{\mathbf{x}}$) and plane of separation between layers (normal unit vector $\hat{\mathbf{z}}$): (1) polar—$\hat{\mathbf{x}}\perp\mathbf{M}\parallel\hat{\mathbf{z}}$, (2) longitudinal—$\hat{\mathbf{x}}\perp\mathbf{M}\perp\hat{\mathbf{z}}$, and (3) transversal—$\hat{\mathbf{x}}\parallel\mathbf{M}\perp\hat{\mathbf{z}}$. From the point of view of theory the choice of the above configurations leads to considerable simplifications enabling one to derive explicit expressions for the magneto-optic coefficients.[20–30] Also, the three configurations are convenient to work with experimentally. However, there seems to be no compelling reason why one should exclusively work with these configurations.[31] It is of interest to extend magneto-optics to encompass arbitrary directions of the magnetization in the different layers of the system.

In this paper we derive expressions for calculating magneto-optic coefficients for any general configuration of the magnetization in a multilayer system. We do so by using our recently developed universal approach to magneto-optics.[29] This approach is based on the medium boundary A and propagation $\bar{D}$ matrices. The central features of this approach are outlined in what follows. Assume that the xy plane (see Fig. 1) represents the separation plane between two media 1 and 2. We denote by F and P the column vectors

$$F=\begin{bmatrix}E_x\\E_y\\H_x\\H_y\end{bmatrix},\quad P=\begin{bmatrix}E_s^{(i)}\\E_p^{(i)}\\E_s^{(r)}\\E_p^{(r)}\end{bmatrix}, \tag{1}$$

where F is built from the x and y components the electric **E** and magnetic **H** fields, and in P, $E_s^{(i)}$ is the perpendicular and $E_p^{(i)}$ the parallel to the plane of incidence (in Fig. 1 it is the yz plane) components of the incident wave $\mathbf{E}^{(i)}$ (similar notations are used for the reflected wave $\mathbf{E}^{(r)}$) The medium boundary matrix A is then defined by the expression[29]

$$F=AP\ . \tag{2}$$

For a nonmagnetic medium the calculation of A is very simple. One just uses the connection between **H** and **E** from Maxwell's equations, $\mathbf{H}=\mathbf{N}\times\mathbf{E}$ (N is the refractive index, and **N** is in the direction of the wave propagation), and the geometry in Fig. 1. The result for A becomes [Eq. (32) of Ref. 29]

$$A=\begin{bmatrix}1&0&1&0\\0&\alpha_z&0&-\alpha_z\\0&-N&0&-N\\\alpha_zN&0&-\alpha_zN&0\end{bmatrix}, \tag{3}$$

where $\alpha_z=\cos\theta$ (θ is the angle between **N** and the z axis). The calculation of A for a magnetic medium with a general direction of magnetization **M** will be described in Sec. II. (In Ref. 29 this was done separately for the polar and longitudinal configurations. In a later manuscript[30] the medium boundary matrix A was derived for a general **M** in the plane of incidence.) Having the matrix A, it becomes very simple to write down the boundary matching conditions for a two-media problem in Fig. 1:

$$A_1P_1=A_2P_2\ . \tag{4}$$

This is a set of four linear equations with the unknowns $E_{1s}^{(r)},E_{1p}^{(r)},E_{2s}^{(i)},E_{2p}^{(i)}$ ($E_{2s}^{(r)}=E_{2p}^{(r)}=0$, see Fig. 1). The medium boundary matrix A solves, therefore, the problem for a single boundary. When there is more than one boundary (as in a multilayer system) we also need to know the wave propagation inside the medium. This is given by the medium propagation matrix $\bar{D}$ which is defined in the following way.[29,30] Equation (4) can also be interpreted as determining $P_2(0)$ (the wave components at $z=0$ in medium 2, see Fig. 1), when $P_1(0)$ is known. The ques-

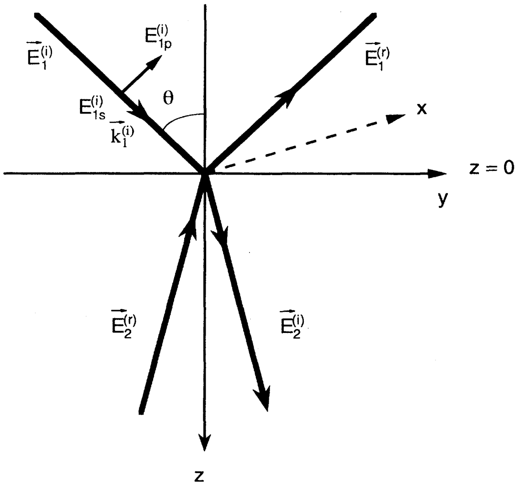

FIG. 1. Two media separated by the xy plane. Light is incident from medium 1. The plane of incidence is the yz plane. E_s and E_p are the electric-field components relative to the plane of incidence: s stands for perpendicular and p for parallel. **k** is the wave vector of the wave.

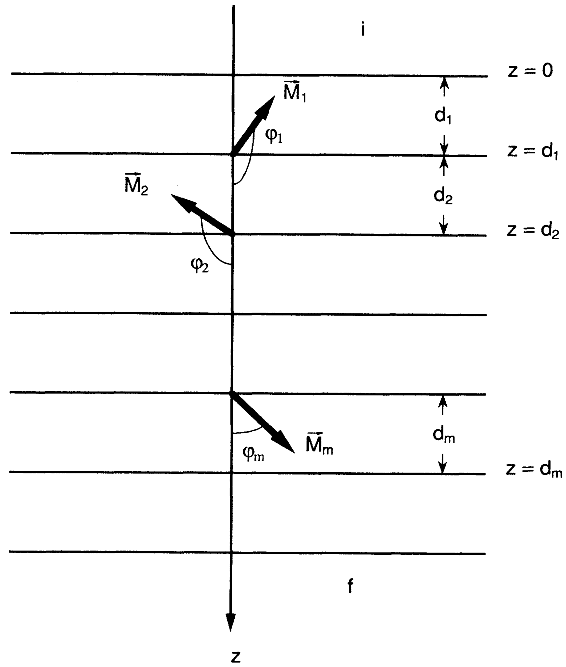

FIG. 2. Multilayer system. M is the magnetization, φ is the angle of M with the z direction, and d is the thickness of the layer. i and f label the initial and final media. m is the running index of the layers.

tion is then, what is $P_2(z)$ at any level z in medium 2? This question is answered by the propagation matrix $\bar{D}_2$. By definition one has[29,30]

$$P_2(0)=\bar{D}_2(z)P_2(z) \ . \tag{5}$$

Having the medium propagation matrix $\bar{D}$ one can solve the scattering problem (reflection transmission) for any multilayer system as shown in Fig. 2. The light starts out in the initial medium i, goes through the multilayer systems, and ends up in the substrate or final medium f. From Eqs. (4) and (5) one finds for the multilayer system the following formula[29] [Eqs. (43)–(46) in Ref. 29]:

$$A_iP_i=\prod_{m=1}^{l}(A_m\bar{D}_m A_m^{-1})A_fP_f \ , \tag{6}$$

where l is the number of layers in the system.

For being able to apply formula (6) to a system with an arbitrary configuration of magnetization **M** in the layers (Fig. 2), one has to find the medium boundary A and propagation $\bar{D}$ matrices. In our previous publications A and $\bar{D}$ were found for polar and longitudinal configurations in Ref. 29 and for a general direction of **M** in the plane of incidence in Ref. 30. In this work the A and $\bar{D}$ matrices are calculated for an arbitrary configuration of the directions of **M**.

In Sec. II we outline the calculation of the medium boundary matrix A. Section III describes the derivation of the medium propagation matrix. Section IV presents our numerical simulations. Section V gives a summary and conclusions.

II. MEDIUM BOUNDARY MATRIX A

We find in this section A for an arbitrary direction of the magnetization **M** with respect to the plane of incidence yz and the plane of separation xy (see Fig. 3). We shall specify the **M** direction by means of the polar angles φ and γ in the xyz system (Fig. 3). Thus, for a polar configuration $\varphi=0$, while for a longitudinal one $\varphi=\pi/2$, $\gamma=\pi/2$, and for a transversal configuration $\varphi=\pi/2$, $\gamma=0$. In general,

$$\begin{aligned} M_x&=M\sin\varphi\cos\gamma \ , \\ M_y&=M\sin\varphi\sin\gamma \ , \\ M_z&=M\cos\varphi \ . \end{aligned} \tag{7}$$

By assuming that in the polar configuration the dielectric tensor is[29]

$$\varepsilon=N^2\begin{bmatrix} 1 & iQ & 0 \\ -iQ & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{8}$$

we find for the general direction of **M** the following expression for ε (by coordinate transformation):

$$\varepsilon=N^2\begin{bmatrix} 1 & i\cos\varphi Q & -i\sin\gamma\sin\varphi Q \\ -i\cos\varphi Q & 1 & i\cos\gamma\sin\varphi Q \\ i\sin\gamma\sin\varphi Q & -i\cos\gamma\sin\varphi Q & 1 \end{bmatrix} , \tag{9}$$

where Q is the magneto-optic constant.

For calculating the medium boundary matrix A, the connection has to be found between the vectors F and P

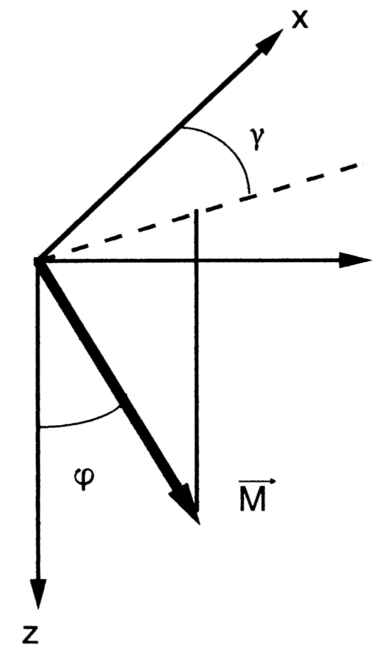

FIG. 3. Spherical coordinates for the magnetization in the *xyz* system.

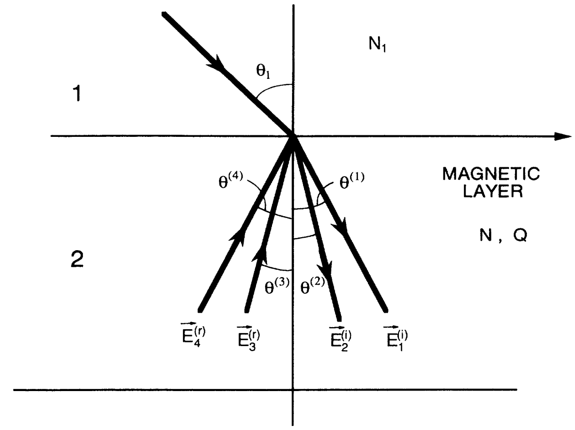

FIG. 4. The four eigenmodes of the electromagnetic wave in a magnetic medium. According to our notations, waves going from medium 1 to medium 2 are denoted by $\mathbf{E}^{(i)}$, while those going from 2 to 1 by $\mathbf{E}^{(r)}$. $\theta^{(j)}$, $j=1,2,3,4$ are the angles between the propagation directions of the four waves and the z direction. N is the refractive index of the medium and Q is its magneto-optic constant.

[Eqs. (1) and (2)]. In a magnetic medium, the components of the **E** vector in an eigenmode are related by the equation [Eq. (82.12) in Ref. 32]

$$\frac{D_y'}{D_x} = \mp i \ , \tag{10}$$

where y' is in the direction of the **D**-wave propagation vector **k**, and the $\mp$ signs are for the two circularly polarized waves. The refraction indices for these two waves are

$$n = N(1 \pm \tfrac{1}{2} gQ) \ , \tag{11}$$

where $g=\cos(\mathbf{k},\mathbf{M})$ is the cosine of the angle between the propagation vector **k** and the magnetization. As is well known[20–24] and as was described in detail in Refs. 29 and 30, there are four waves propagating in a magnetic layer: two are going into the medium $E_1^{(i)}, E_2^{(i)}$, and two coming out of the medium (see Fig. 4). Correspondingly, the following notations are used: $n^{(1,2)}$ are the refractive indices for $E_1^{(i)}, E_2^{(i)}$; $n^{(3,4)}$ are for $E_3^{(r)}, E_4^{(r)}$. Also, g_i will be used for the angle cosine of the incident wave $\mathbf{E}^{(i)}$ in Eq. (11), while g_r is for the reflected wave $\mathbf{E}^{(r)}$. In Fig. 4 we show the direction angles $\theta^{(j)}$, $j=1,2,3,4$, for the four waves. Snell's law for the two media in Fig. 4 will assume the form

$$N_1 \sin\theta_1 = N\sin\theta = n^{(1,2)}\sin\theta^{(1,2)} = n^{(3,4)}\sin\theta^{(3,4)} \ , \tag{12}$$

where N_1 is the refractive index of the incidence medium (medium 1 in Fig. 4), θ_1 is the angle of incidence, N is the refractive index of the magnetic layer, and θ is an auxiliary angle that is used throughout the paper. From Eq. (12) we have the following expressions for the sine and cosine directions of the four waves:

$$\begin{aligned}
\sin\theta^{(1,2)} &\equiv \alpha_y^{(1,2)} = \alpha_y(1 \mp \tfrac{1}{2} g_i Q) \ , \\
\sin\theta^{(3,4)} &\equiv \alpha_y^{(3,4)} = \alpha_y(1 \mp \tfrac{1}{2} g_r Q) \ , \\
\cos\theta^{(1,2)} &\equiv \alpha_z^{(1,2)} = \alpha_z(1 \pm \tfrac{1}{2} g_i Q) \ , \\
\cos\theta^{(3,4)} &\equiv \alpha_z^{(3,4)} = -\alpha_z \left[1 \pm \frac{1}{2}\frac{\alpha_y^2}{\alpha_z^2} g_r Q \right] \ ,
\end{aligned} \tag{13}$$

where $\alpha_y = \sin\theta$ and $\alpha_z = \cos\theta$. It is now straightforward to find the relations between the components of the electric-field vector **E** in the magnetic medium. For this we use Eq. (10), the dielectric tensor (9), and the expressions (13) for the angles of the four waves. With the same algebra as in Refs. 29 and 30, one finds

$$\begin{aligned}
E_y^{(1,2)} &= (\mp i\alpha_z^{(1,2)} + i\alpha_y^2 \cos\varphi Q - i\alpha_y\alpha_z \sin\gamma \sin\varphi Q \pm \alpha_y \cos\gamma \sin\varphi Q) E_x^{(1,2)} \ , \\
E_z^{(1,2)} &= (\pm i\alpha_y^{(1,2)} - i\alpha_z^2 \sin\gamma \sin\varphi Q + i\alpha_y\alpha_z \cos\varphi Q \pm \alpha_z \cos\gamma \sin\varphi Q) E_x^{(1,2)} \ , \\
E_y^{(3,4)} &= (\mp i\alpha_z^{(3,4)} + i\alpha_y^2 \cos\varphi Q + i\alpha_y\alpha_z \sin\gamma \sin\varphi Q \pm \alpha_y \cos\gamma \sin\varphi Q) E_x^{(3,4)} \ , \\
E_z^{(3,4)} &= (\pm i\alpha_y^{(3,4)} - i\alpha_z^2 \sin\gamma \sin\varphi Q - i\alpha_y\alpha_z \cos Q \mp \alpha_z \cos\gamma \sin\varphi Q) E_x^{(3,4)} \ .
\end{aligned} \tag{14}$$

From here we can find the tangential components E_x, E_y, H_x, and H_y expressed via $E_x^{(j)}$, $j=1,2,3,4$. Eventually, for calculating the medium boundary matrix A, one needs to know the tangential components as functions of $E_s^{(i)}$, $E_p^{(i)}$, $E_s^{(r)}$, $E_p^{(r)}$ [Eq. (2)]. One can show that[29]

$$\begin{aligned} E_x^{(1,2)} &= \tfrac{1}{2}(E_s^{(i)} \pm iE_p^{(i)}) , \\ E_x^{(3,4)} &= \tfrac{1}{2}(E_s^{(r)} \pm iE_p^{(r)}) , \end{aligned} \tag{15}$$

where use was made of the equations (see Fig. 1)

$$\begin{aligned} E_p^{(i)} &= E_y\alpha_z - E_z\alpha_y , \\ E_p^{(r)} &= -E_y\alpha_z - E_z\alpha_y . \end{aligned} \tag{16}$$

This information leads us to the following expression for the medium boundary matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ \frac{i}{2}\frac{\alpha_y}{\alpha_z}Q(\alpha_y g_i - 2\sin\varphi\cos\gamma) & \alpha_z + i\alpha_y \sin\varphi\cos\gamma Q & -\frac{i}{2}\frac{\alpha_y}{\alpha_z}Q(\alpha_y g_r - 2\sin\varphi\cos\gamma) & -\alpha_z + i\alpha_y \sin\varphi\cos\gamma Q \\ \frac{i}{2}Ng_iQ & -N & \frac{i}{2}Ng_rQ & -N \\ N\alpha_z & \frac{iN}{2\alpha_z}g_iQ & -N\alpha_z & -\frac{iN}{2\alpha_z}g_rQ \end{bmatrix} , \tag{17}$$

where

$$\begin{aligned} g_i &= \cos\varphi\alpha_z + \alpha_y \sin\varphi\sin\gamma , \\ g_r &= -\cos\varphi\alpha_z + \alpha_y \sin\varphi\sin\gamma . \end{aligned} \tag{18}$$

This is the A matrix for an arbitrary direction of the magnetization **M** [Eq. (7)]. When **M** is the plane of incidence, A assumes the form in Ref. 29 and 30.

III. MEDIUM PROPAGATION MATRIX $\bar{D}$

For a multilayer system one has to know how the phase of the wave changes when the wave propagates through the medium. This change is accounted for by the medium propagation matrix.[29,30] The latter is derived in the following way. The four components $E_x^{(j)}$, $j=1,2,3,4$ vary in the medium according to the equation[22,29]

$$E_x^{(j)}(0) = E_x^{(j)}(z)\exp\left[-i\frac{2\pi}{\lambda}n^{(j)}\alpha_z^{(j)}z\right] , \tag{19}$$

where $z=0$ is the boundary between two media (Fig. 1), and z is the depth into the material. For finding the medium propagation matrix $\bar{D}$ we need to write Eq. (19) for the P vectors [see Eq. (5)]. This we obtain by using Eq. (15), and for propagation through a layer of thickness d, we find[29,30]

$$\bar{D} = \begin{bmatrix} U & U\delta_i & 0 & 0 \\ -U\delta_i & U & 0 & 0 \\ 0 & 0 & U^{-1} & -U^{-1}\delta_r \\ 0 & 0 & U^{-1}\delta_r & U^{-1} \end{bmatrix} , \tag{20}$$

where [g_i and g_r are given in Eq. (18)]

$$\begin{aligned} U &= \exp\left[-i\frac{2\pi}{\lambda}Nd\alpha_z\right] , \\ \delta_i &= \frac{\pi}{\lambda}Nd\frac{Q}{\alpha_z}g_i , \\ \delta_r &= \frac{\pi}{\lambda}Nd\frac{Q}{\alpha_z}g_r . \end{aligned} \tag{21}$$

The matrix $\bar{D}$ in Eq. (20) is given to first order in Q. When **M** is in the plane of incidence, $\bar{D}$ in Eq. (20) goes over into the medium propagation matrix in Refs. 29 and 30 (the g_r in the present paper has an opposite sign to g_r in Ref. 30).

IV. NUMERICAL SIMULATIONS AND DISCUSSION

In this section we apply Eq. (6) with the expressions for the medium boundary matrix A [Eq. (17)] and the medium propagation matrix $\bar{D}$ [Eq. (20)] for calculating Kerr rotations and ellipticities in the following systems: (1) bulk Fe, (2) 50 Å of Fe on a Au substrate, and (3) a 50 period Fe/Au superlattice on Au with the modulation period 10 Å Fe+10Å Au. The calculations are carried out for the He-Ne laser light $\lambda=6328$ Å ($h\nu=1.96$ eV). For this wavelength the refractive indices ($N=N_R+iN_I$) for Fe and Au are[33]

$$\begin{aligned} &\text{Fe:}\ \ N_R = 2.87,\ \ N_I = 3.36 , \\ &\text{Au:}\ \ N_R = 0.12,\ \ N_I = 3.29 . \end{aligned} \tag{22}$$

For Fe we also have to know the magneto-optic constant $Q=Q_R+iQ_I$. Q(Fe) can be found from the data for the off-diagonal matrix element ϵ_{xy} of the dielectric tensor[34]

$$Q_R = 0.376, \quad Q_I = 0.0066 . \tag{23}$$

In calculating the magneto-optic coefficients it is more convenient to rewrite Eq. (6) in the following form:[29]

$$P_i = M P_f , \tag{24}$$

where

$$M = A_i^{-1} \prod_m A_m \bar{D}_m A_m^{-1} A_f \equiv \begin{bmatrix} G & H \\ I & J \end{bmatrix} . \tag{25}$$

In Eq. (25) G, H, I, and J are 2×2 matrices. One can then show that[29]

$$\begin{bmatrix} t_{ss} & t_{sp} \\ t_{ps} & t_{pp} \end{bmatrix} = G^{-1} , \tag{26}$$

$$\begin{bmatrix} r_{ss} & r_{sp} \\ r_{ps} & r_{pp} \end{bmatrix} = I G^{-1} , \tag{27}$$

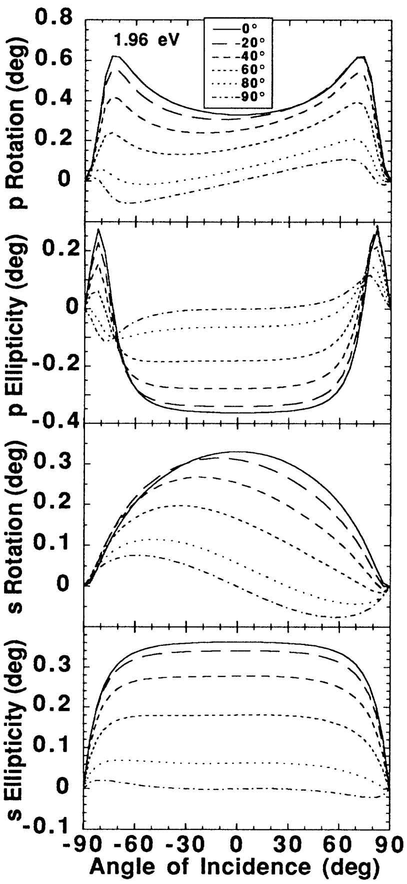

FIG. 5. Kerr rotations and ellipticities for bulk Fe as a function of angle of incidence. The photon energy is $h\nu = 1.96$ eV. The curves in each panel are for different directions of the magnetization from the z axis (normal to the boundary). 0° is for the polar configuration, 90° is for the longitudinal one. The other angles are for the magnetization directions between the polar and longitudinal configurations. s and p denote polarization of the light (perpendicular and parallel to the plane of incidence, respectively).

where t are the transmission and r the reflection magneto-optic coefficients. Our computer program is based on the subdivision of the M matrix [Eq. (25)] into 2×2 matrices and the use of Eqs. (26) and (27).

For calculating the Kerr rotation ϕ' and ellipticity ϕ'', the following formulas are used[35] (for s and p polarizations):

$$\phi_s' + i\phi_s'' = \frac{r_{ps}}{r_{ss}} , \tag{28}$$

$$-\phi_p' + i\phi_p'' = \frac{r_{sp}}{r_{pp}} . \tag{29}$$

Our computational results of Kerr rotations and ellipticities are presented in Figs. 5–7. The curves in these figures are arranged in the same way as in the classic paper for bulk Fe by Metzger, Pluvinage, and Torguet.[35] The figures give rotations and Kerr ellipticities as functions of the angle of incidence. Each panel contains six curves corresponding to different directions of the magnetization inside the plane of incidence. The degrees from 0° to 90° in each figure are for the angles between the magnetization and the z axis (the axis perpendicular

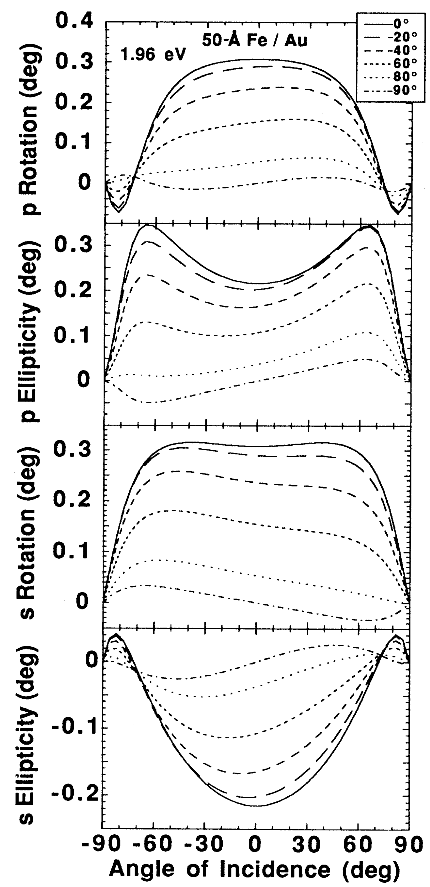

FIG. 6. Kerr rotations and ellipticities for an overlayer of 50 Å of Fe on a Au substrate. All other notations are as in Fig. 5.

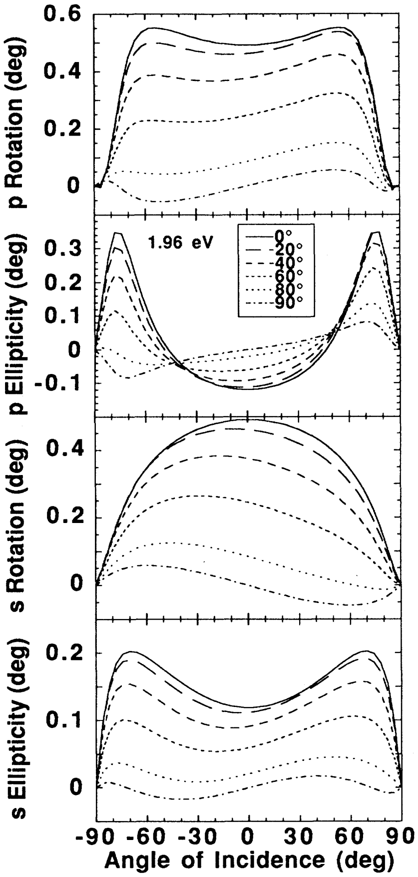

FIG. 7. Kerr rotations and ellipticities for a 50[10-Å Fe/10-ÅAu]/Au superlattice. It contains 50 periods of a 10-Å Fe plus 10-Å Au bilayer on a Au substrate. All other notations are as in Fig. 5.

to the boundary, as in Figs. 1 and 2). 0° is for the polar Kerr effect and, correspondingly, 90° is for the longitudinal Kerr effect. All the results are for He-Ne laser light.

The curves in Fig. 5 for bulk Fe are in full qualitative agreement with the published results in Ref. 35 (the latter is for Na yellow light). When comparing the overlayer and superlattice results (Figs. 6 and 7) with those for bulk (Fig. 5), the following picture emerges: the superlattice and bulk results are qualitatively quite similar; the Kerr effect for the overlayer differs quite strongly from them. For example, the p rotation in bulk (Fig. 5) and in the superlattice (Fig. 7) is close to maximum for incidence angles around 70°, while this same rotation for the overlayer (Fig. 6) is close to zero. A general trend for all the curves in Figs. 5–7 is the decrease of the Kerr signal with the increase of the angle between the magnetization and the z axis. The biggest signal is for the polar configuration and as a rule it is an order of magnitude larger than the longitudinal one. Altogether, Figs. 6 and 7 contain many new detailed results of Kerr signals from overlayers and superlattices with general magnetization directions, and it is anticipated that these computational data will encourage experimental interest in these systems.

V. SUMMARY AND CONCLUSIONS

A matrix method is developed for calculating the magneto-optic coefficients from multilayers with arbitrary directions of the magnetizations in individual layers. The method is based on two types of matrices—the medium boundary A and medium propagation $\bar{D}$ which are explicitly given in the paper [Eqs. (17) and (20), respectively]. This enables one to calculate the Kerr and Faraday effects for any multilayer system of magnetic and nonmagnetic films with completely general magnetization directions. We have a computer program for numerical simulations of the Kerr effect from such general multilayer systems. The results in Figs. 5–7 were generated by this program. In these figures the magnetization is in the plane of incidence in accordance with the bulk results in Ref. 35. However, our method and the computer program are also applicable to arbitrary magnetization directions.

In conclusion, we would like to make a remark about our 4×4 matrix method and the computational algorithm,[29,30] as compared to the 2×2 matrix scheme that is used in Ref. 31. While our matrix method itself uses 4×4 matrices [Eq. (6)], the final computational algorithm is simplified by the use of 2×2 matrices [Eqs. (26) and (27)]. It looks, therefore, that our computational algorithm is quite similar to the one used in Ref. 31.

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