PHYSICAL REVIEW B VOLUME 43, NUMBER 7 1 MARCH 1991

Coherence properties of holes subject to a fluctuating spin chirality

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(Received 6 November 1990; revised manuscript received 14 January 1991)

The coherence properties of holes coupled to short-ranged chiral spin fluctuations with a characteristic chiral spin-fluctuation time $\tau_{\rm ch}=\omega_{\rm ch}^{-1}$ are investigated in two dimensions. At temperatures $kT \ll 4\pi^2\langle\phi^2\rangle^{-1}\hbar\omega_{\rm ch}$ hole quasiparticles exist and propagate with a renormalized mass $m^*/m = 1+\langle\phi^2\rangle\hbar/16\pi m a_0^2\omega_{\rm ch}$. $\langle\phi^2\rangle$ is the amplitude of the local fictitious flux fluctuation and a_0 is a lattice cutoff. At temperatures $kT \gg 4\pi^2\langle\phi^2\rangle^{-1}\hbar\omega_{\rm ch}$ an effective-mass approximation is invalid and we find that the hole diffuses according to a *logarithmic* diffusion law in the quasistatic chiral field. The unusual diffusion law is a consequence of the long-ranged nature of the gauge field. The result shows that the holes do not form a coherent quantum fluid in the quasistatic regime.

The motion of charge carriers in doped Mott insulating states in two dimensions has been studied in detail during recent years in the belief that the normal and superconducting states of high-temperature superconductors are states of this kind.[1] In the t-J model the hole motion is frustrated by short-range antiferromagnetic spin correlations (and vice versa) such that the *coherent* bandwidth for holes propagating in the correlated spin background is limited by a few times J, the antiferromagnetic exchange constant.[2] This can be a substantial reduction of the coherent bandwidth below the bare hopping scale t. Experiments on high-temperature superconductors are, of course, performed at temperatures well below the antiferromagnetic energy scale so it is important to understand the quasiparticle or other behavior of holes in the temperature regime $T \ll J$. Recently, several authors[3] have argued that the linear temperature coefficient of resistance (and possibly the unusual Hall effect) seen universally in the cuprates derives from hole scattering off fluctuations of the local spin chirality. (We refer to holes throughout this paper although the same arguments apply for the double-occupancy case also.) Fluctuations of the local spin chirality[4] represent, from the point of view of the propagating hole, a fluctuating fictitious flux of order of a flux quantum per plaquette. If this is the dominant scattering mechanism of charge from spin at temperatures much less than J, then it follows that chiral fluctuations play an important role in setting the intrinsic coherence temperature of the hole system. This can be very low experimentally; the material Bi-Sr-Cu-O for instance[5] shows linear temperature coefficient of resistance down to about 10 K; thus, it is interesting to know if chiral fluctuations can account for such low coherence temperatures. The essential result of this paper will be that coherence is absent when the holes experience *short-ranged, quasistatic* chiral fluctuations.

The coupling of holes to spin chirality is a consequence of the backflow of spin accompanying the motion of holes. The Hamiltonian for holes (created by $h_i^\dagger$) moving in a *static* spin background is[6]

$$H = -t\sum_{\langle i,j\rangle} z_i^* z_j h_i^\dagger h_j \,,$$

where $z_i^* = [\cos(\theta_i/2),\ \sin(\theta_i/2)\exp i\phi_i]$ represents an up-spin state with respect to a local quantization axis $\mathbf{n}_i = (\sin\theta_i\cos\phi_i,\ \sin\theta_i\sin\phi_i,\ \cos\theta_i)$. The form of z_i^* follows by rotating the spin frame through angle θ_i about the axis $\mathbf{a}_i = (\sin\phi_i, -\cos\phi_i, 0)$. The required spin-$\frac{1}{2}$ rotation operator is $R = \exp i(\sigma\cdot\mathbf{a}/2)$ where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. In addition, it can be shown that the resulting overlap of spinor states z_i and z_j can be expressed as[7]

$$z_i^* z_j = e^{(i/2)\Omega_{n_i,n_j,z}}\left|\frac{1+\mathbf{n}_i\cdot\mathbf{n}_j}{2}\right|^{1/2}$$

where $\Omega_{n_i,n_j,z}$ is the solid angle subtended by the spin axes at i and j, and the z axis. The solid angle $\Omega_{n_i,n_j,z}$ is positive or negative depending on the handedness of n_i, n_j, z. (Another common measure of the handedness or chirality is $S_{\mathbf{n}_i}\cdot S_{\mathbf{n}_j}\times \mathbf{z}$.) Now when a hole is taken around an arbitrary closed path C in the spin background it acquires a phase equal to the total solid angle subtended by the spins along the particle path. This is a consequence of the overlap of displaced spinor wave functions: $\langle\psi_i|\psi_f\rangle = \prod_C z_i^* z_j$. Spin chirality, as measured by the solid angle, generates Aharonov-Bohm phases or effective magnetic fields normal to the plane.

Spin-singlet background states generally exhibit appreciable short-ranged chiral fluctuations. They are present, for instance, in a quantum antiferromagnetic state where the fluctuation frequency scale is of order of the Heisenberg exchange constant. The precise structure of chiral spin correlations present in the relevant doped Mott insulating states is still not known for certain. One possibility is that long-range chiral or staggered chiral order develops with order parameter $\langle \mathbf{S}_i\cdot\mathbf{S}_{i+\hat{x}}\times\mathbf{S}_{i+\hat{x}+\hat{y}}\rangle$.[4,8] In the work of Nagaosa and Lee[3] the chiral fluctuations correspond to the transverse current fluctuations of a pseudo-Fermi sea[9]

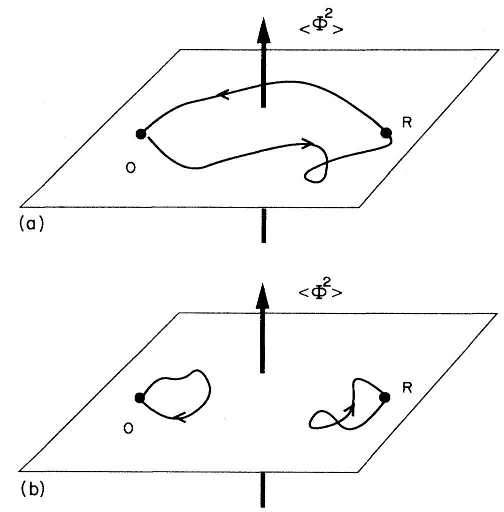

FIG. 1. Gauge-invariant coherence temperature. (a) Two-particle exchange $Z_{12}(R,\beta)$. (b) The "classical" contribution Z_0^2 dominates in the incoherent regime. At the coherence temperature $Z_{12}(R)\sim Z_0^2$ for $R\sim$mean particle separation.

(chiral symmetry is unbroken). In a recent paper[10] Ioffe, Kalmeyer, and Wiegmann argue that holes see a quasistatic flux distribution in this model even at low temperature.

Motivated by their work, we investigate a situation where the spatial range of correlation is negligible and the chirality decays in real time with definite decay time $\tau_{\rm ch}=\omega_{\rm ch}^{-1}$. (A local, diffusive mode.) The fictitious flux fluctuations (expressed in the continuum) satisfy

$$\langle\phi(\mathbf{r},t)\phi(0,0)\rangle=a_0^2\langle\phi^2\rangle\delta(\mathbf{r})\exp-\omega_{\rm ch}t\,. \qquad (1)$$

$\langle\phi^2\rangle$ is a dimensionless parameter expected, for typical spin backgrounds, to be of the order of ϕ_0^2. (The flux quantum $\phi_0=2\pi$ in our units.) As we shall see the model (1) enables us to study both quasistatic and dynamic limits.

A hole traveling around a loop in a fluctuating background flux such as expression (1) returns with a random Aharonov-Bohm phase. As pointed out by Nagaosa and Lee this scattering process has the effect of suppressing the contribution of closed paths which enclose a large "area" relative to paths which almost retrace themselves. In the lattice case the limit where the loop contribution is completely absent is familiar as the retraced path approximation. As shown by Brinkman and Rice,[11] the retraced path restriction on a lattice radically alters the physics of holes. For instance, it modifies the behavior of the density of states near the renormalized band edges (in more than one dimension; in one dimension all loops are retraced). This effect goes beyond an effective-mass picture which just gives a renormalized density of states. Similarly, we shall verify below that chirality fluctuations can lead to a breakdown of the effective-mass approximation.

The single hole density matrix or the propagator is not a gauge invariant object in a fluctuating chiral background. Despite this the coherence temperature of the hole system remains well defined as the temperature below which the free energy becomes sensitive to hole statistics, i.e., Fermi versus Bose statistics. Ordinarily the coherence temperature is the temperature at which the thermal length λ_T becomes comparable with the interparticle spacing. This picture assumes an effective-mass approximation. More generally, we may define the coherence temperature as the temperature below which ring exchange processes make a substantial contribution to the system free energy. The multiparticle partition function Z involves a sum over particle permutations or ring exchanges when expressed in terms of an imaginary-time path integral; in the incoherent phase the action associated with ring exchange is high and these processes are suppressed. The elementary ring exchange shown in Fig. 1(a) involves a *closed* particle path, and so, makes a gauge-invariant contribution to the partition function.

The single-particle partition function expressed as an imaginary-time path integral is ($\hbar=1$):

$$Z=\int_{\mathbf{r}(0)=\mathbf{r}(\beta)}D\mathbf{r}(\tau)e^{-S_0}\left\langle\exp\frac{1}{2\pi}\oint d\tau\mathbf{A}\cdot\dot{\mathbf{r}}\right\rangle_A\,, \qquad (2)$$

where the averaging is over gauge-field configurations $\mathbf{A}(\nabla\times\mathbf{A}=a_0^{-2}\phi\hat{\mathbf{z}})$ and S_0 is the free-particle action: $S_0=a_0^{-2}W^{-1}\int_0^\beta\dot{\mathbf{r}}^2(\tau)d\tau$, where $\beta=1/kT$, $W=\hbar^2/2ma_0^2$, and a_0 is a lattice cutoff. m is the partially renormalized particle mass. Expanding the exponential, averaging and reexponentiating in the usual way, the effective action becomes

$$S[\mathbf{r}(\tau)]=S_0+\frac{1}{8\pi^2}\int_0^\beta\int_0^\beta d\tau d\tau'\dot{r}_\alpha(\tau)\dot{r}_\beta(\tau')\times D_A^{\alpha\beta}[\mathbf{r}(\tau)-\mathbf{r}(\tau'),\tau-\tau']\,, \qquad (3)$$

where $D_A^{\alpha\beta}(\mathbf{r},\tau)$ is the time-ordered A-field propagator in imaginary time. The "gauge-field" fluctuations corresponding to Eq. (1) may be expressed in the Feynman gauge, where only diagonal components of the gauge-field propagator are nonzero:[12]

$$\langle A_q^\alpha(t)A_{-q}^\beta(0)\rangle=a_0^{-2}\langle\phi^2\rangle\frac{1}{q^2}e^{-\omega_{\rm ch}t}\delta^{\alpha\beta},\ t>0\,. \qquad (4)$$

From (4) we find for the imaginary-time propagator: $D_A^{\alpha\beta}(\mathbf{r},\tau)=D_A^{\alpha\beta}(\mathbf{r},0)\cos\omega_{\rm ch}|\tau|$, $0<\tau<\beta$.

We first consider the high-temperature limit $\beta\omega_{\rm ch}\rightarrow 0$. In this limit the explicit τ dependence of $D_A(\mathbf{r},\tau)$ can be neglected; i.e., the hole moves in a quasistatic random chiral field with spatial correlation $D_A(\mathbf{r},0)$. In two dimensions the local amplitude of gauge-field fluctuation $D_A(\mathbf{r},0)$ is logarithmically divergent from Eq. (4). This divergence should not appear in gauge-invariant properties which see only finite flux $\langle\phi^2\rangle$.

Using expression (4) in the effective action (3) we obtain

$$S=S_0+\frac{\langle\phi^2\rangle}{8\pi^2a_0^2}\int_0^{2\pi a_0^{-1}}\frac{dq}{q}\int_0^\beta\int_0^\beta d\tau d\tau'\dot{r}^\alpha(\tau)J_0(q|\mathbf{r}(\tau)-\mathbf{r}(\tau')|)\dot{r}_\alpha(\tau')\,. \qquad (5)$$

J_0 is a Bessel function. Consider the evaluation of the effective action (5) for a closed loop of "dimension" L. [L can be taken as the greatest separation between points on the loop $\mathbf{r}(\tau)$.] The q integration is then cut off below $q_L \sim 2\pi L^{-1}$. To see this, note that for $q \ll L^{-1}$ the argument of the Bessel function is small at all times and $J_0 \simeq 1 - \frac{1}{4}q^2|\mathbf{r}(\tau)-\mathbf{r}(\tau')|^2$. For closed paths the first term in this expansion gives zero contribution, cutting off the small q singularity. Notice that for open paths the logarithmic divergence of the action survives, a consequence of absence of gauge invariance for open paths. Adding zero to (5) in the form

$$0 = \frac{\langle\phi^2\rangle}{8\pi^2 a_0^2}\int_0^\beta\int_0^\beta d\tau d\tau' \dot{r}^a(\tau) \ln\frac{L}{a_0}\dot{r}_a(\tau')$$

we have finally

$$S \simeq \int_0^\beta\int_0^\beta d\tau d\tau' \dot{r}^a(\tau)\left[\frac{1}{a_0^2 W}\delta(\tau-\tau') + \frac{\langle\phi^2\rangle}{8\pi^2 a_0^2} \times \ln\frac{|\mathbf{r}(\tau)-\mathbf{r}(\tau')|}{a_0}\right]\dot{r}_a(\tau') . \quad (6)$$

The action expression (6) is not Gaussian and the problem cannot be treated exactly. The form of the effective action however suggests a self-consistent approximation on the kernel of expression (6) (the quantity in large parenthesis). Then it is not hard to argue that (6) leads to a logarithmic, rather than linear diffusion law for long times. Replace the argument of the logarithm by its root-mean-square value; $|\mathbf{r}(\tau)-\mathbf{r}(\tau')|^2 \rightarrow f(\tau-\tau') \equiv \langle|\mathbf{r}(\tau)-\mathbf{r}(\tau')|^2\rangle$. The action is Gaussian with a kernel which is nonlocal in time. The problem reduces to finding the diffusion law $f(\tau)$ self-consistently with the action.

Suppose $f(\tau)$ is a power law τ^p. [More properly, one must remember that $f(\tau)$ has period β so that $f(\tau) = a_0^2\beta W \sin^p \pi|\tau|/\beta$, for instance, is a more appropriate trial form.] The kernel obtained by this replacement is $p\ln|\tau-\tau'|$. The resulting action is the well-known Caldeira-Leggett action of dissipative quantum mechanics.[13] At times long compared to the inelastic lifetime [$\tau \gg 1/\langle\phi^2\rangle W$ in this case], the diffusion law in the Caldeira-Leggett model is logarithmic rather than linear, in imaginary time. The assumption that $f(\tau)$ is a power law is contradicted; therefore, no power-law diffusion can be a self-consistent solution. The essential reason for the altered form of the diffusion law is that the kernel in expression (6) is long ranged in space and effectively also in time. For a short-ranged kernel a linear diffusion law would hold at sufficiently long times.

These observations suggest a logarithmic solution for $f(\tau)$, and hence a double log behavior of the self-consistent kernel. To verify this solution, we express the action in Fourier space, with arbitrary kernel $k(\tau) = \ln f(\tau)$; $\omega_n = 2\pi n\beta^{-1}$. Evaluating $f(\tau)$, the self-consistency equation reads

$$\lim_{\beta\to\infty} f(\tau) = \lim_{\beta\to\infty}\frac{a_0^2 W}{2}\beta^{-1}\sum_n \frac{1-\cos\omega_n\tau}{\omega_n^2(1+k_n)} \rightarrow \frac{a_0^2 W}{2\pi}\int_0^\infty d\omega \frac{1-\cos\omega\tau}{\omega^2[1+k(\omega)]}$$

with

$$\lim_{\beta\to\infty} k_n = \lim_{\beta\to\infty}\frac{\langle\phi^2\rangle W}{16\pi^2}\int_0^\beta ds \cos\omega_n s \ln f(s) \rightarrow \frac{\langle\phi^2\rangle W}{16\pi^2}\int_0^\infty ds \cos\omega s \ln f(s)$$

$k_n \equiv 0$ for $\omega_n > \omega_c$. So,

$$f(\tau) = \frac{a_0^2 W}{2\pi}\int_0^\infty (1-\cos\omega\tau)\left[\omega^2 + \frac{\langle\phi^2\rangle W}{16\pi^2}\omega\int_0^\infty ds \sin\omega s \frac{f'(s)}{f(s)}\right]^{-1} \simeq \frac{8a_0^2}{\langle\phi^2\rangle}\int_0^{\langle\phi^2\rangle W}\frac{1-\cos\omega\tau}{\omega}\left[\int_0^\infty ds \sin\omega s \frac{f'(s)}{f(s)}\right]^{-1} . \quad (7)$$

The later approximation is valid at long times. Inspection of expression (7) shows that

$$f(\tau) \simeq \frac{8a_0^2}{\pi\langle\phi^2\rangle}\ln(\langle\phi^2\rangle W\tau), \quad \langle\phi^2\rangle W\tau \gg 1 \quad (8)$$

is self-consistent to logarithmic accuracy.

It is interesting to compare these results with the retraced-path-restriction (RPR) lattice result for the single-particle density of states near the band edge mentioned earlier. In the RPR, the low-temperature hole partition function is $Z \sim \beta^{-3/2}e^{-\beta e_0}$; i.e., states at the band edge are depleted and there is a square root rather than step singularity in the density of states at the renormalized band edge at e_0. We compare this to the behavior in the Caldeira-Leggett model. With an upper cutoff ω_c [equivalent to a short distance cutoff on the logarithm in expression (6)] it can be shown that[14]

$$Z_{\mathrm{CL}} = N\int \prod_{n=1}^{\infty} dy_n dx_n \exp\left[-\beta/a_0^2 W \sum_n (\omega_n^2 + \langle\phi^2\rangle W/8\pi^2|\omega_n|)(x_n x_{-n} + y_n y_{-n})\right] \sim \frac{A\langle\phi^2\rangle}{a_0^2}\exp\left[-\frac{\beta\langle\phi^2\rangle W}{16\pi^3}\left(1+\ln\frac{\omega_c}{\langle\phi^2\rangle}\right)\right] .$$

The (infinite) normalization constant N is known from the free particle partition function, and A is the system area. The upward shift of the band edge $\sim\langle\phi^2\rangle W$ is in qualitative agreement with RPR; however, the form of the density of states near the renormalized band edge is ω^{-1} in Caldeira-Leggett versus $\omega^{1/2}$ in RPR. The discrepancy probably indicates that approach to the lattice RPR result from a continuum-limit calculation is subtle as one might expect. Z calculated with self-consistent kernel is a complicated task but the result should be similar to Caldeira-Leggett. In any case an effective-mass approximation is invalid and the density of states (or equivalently, the low-

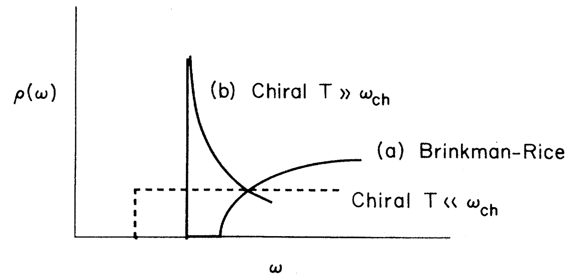

FIG. 2. Hole density of states near the band edge. (a) The Brinkman-Rice retraced path approximation. (b) The Caldeira-Leggett density of states $\sim\omega^{-1}$ approximating the effect of a quasistatic field at temperatures $T \gg \omega_{ch}$ in two dimensions. The impossibility of Bose condensation in this regime is evident. The dashed line is the renormalized hole density of states, $T \ll \omega_{ch}$.

temperature partition function) is changed from the two-dimensional step singularity, as summarized in Fig. 2.

Returning to the result (8), one sees that a logarithmic diffusion law effectively suppresses coherence effects in the multihole system.[15] Roughly speaking coherence is suppressed because the distance diffused in imaginary time $\sim\beta/2$ is less than the order of the mean particle separation. A more precise criterion is that the action cost associated with the process shown in Fig. 1(a) grows as $R^2 \ln\ln\beta$, if the particle path is constrained to pass through a distance R. (Again a similar behavior occurs in the Caldeira-Leggett model where the classical action cost grows as $R^2 \ln\beta$.[15])

The behavior is qualitatively altered at temperatures $\beta\omega_{ch} \to \infty$. The explicit time dependence of the kernel must be retained and a quasistatic approximation is invalid. $D_A^{\alpha\beta}(\mathbf{r},\tau)$ oscillates in imaginary time on a scale which is fast compared with the inverse temperature and the kernel is effectively short ranged. The self-consistent equation can be developed along the above lines. Essentially at long times $\tau \gg \omega_{ch}^{-1}$, one can approximate

$$D_A^{\alpha\beta}(\mathbf{r},\tau) \simeq a_0^2 \langle\phi^2\rangle \frac{\pi\delta(\tau)}{\omega_{ch}} \delta^{\alpha\beta}$$

to logarithmic accuracy, so that the diffusion is linear in τ (with a high-frequency motion superposed). An effective-mass approximation holds

$$W^* \simeq W \Big/ \left[1 + \frac{\langle\phi^2\rangle}{8\pi} \frac{W}{\omega_{ch}} \right]. \quad (9)$$

The form of this result is familiar, mirroring Einstein phonons which give a mass enhancement proportional to the inverse phonon frequency. Also (9) is consistent with the conclusion that the regime $T > \omega_{ch}$ is incoherent. For $\omega_{ch} \ll W$ the coherent bandwidth is $\sim\omega_{ch}$ so that the crossover to a coherent hole state is expected below $\sim\omega_{ch}$ at a temperature determined by the hole density.[16]

As yet there is no experimental determination of the frequency scale for chiral fluctuations. Shastry and Shraiman[17] have noted that local chiral spin fluctuations should show up in the B_{2g} Raman geometry, via a nominally "higher-order" effect in $t/(U-\omega_{laser})$.

J.M.W. thanks Robin Ball for helpful conversations.

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