RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 43, NUMBER 7 1 MARCH 1991

Flux creep and current relaxation in high-T_c superconductors

M. V. Feigel'man and V. B. Geshkenbein
Landau Institute for Theoretical Physics, Moscow, U.S.S.R.

V. M. Vinokur*
Institute for Solid State Physics, Chernogolovka, Moscow Region, U.S.S.R.
(Received 25 September 1990)

The long-time asymptotics of the current (magnetization) relaxation in high-temperature superconductors is studied within the framework of collective creep theory. The inverse normalized relaxation rate S^{-1} depends linearly on $T\ln(t/\tau_0)$. The attempt time τ_0 is shown to be much longer than a "microscopic" time scale and is strongly size and j_c dependent.

It is generally agreed that thermal fluctuations of vortex lines (VL) are of extraordinary importance in the formation of the mixed state in high-T_c superconductors (HTS) (see, for review, Refs. 1 and 2). The most indicative properties are (i) very fast fall of the critical current j_c with temperature,[3] and (ii) giant flux creep.[4] One can distinguish two principal kinds of thermal fluctuations of VL: (i) phononlike fluctuations of VL near the equilibrium positions within metastable valleys generated by random pinning potential, and (ii) thermally activated jumps of the segments of VL between different valleys. It was shown in our previous papers[5-7] that phononlike fluctuations smear the pinning potential, providing the decrease in the critical current with temperature increase. On the other hand, thermally activated jumps are known to be responsible for the creep phenomenon.

One of the most important problems is the nature of pinning in HTS. Usually the data concerning pinning force and barriers are extracted from measurements of magnetic hysteresis loops and of the rate of magnetic relaxation. Due to the presence of giant flux creep the width of the magnetic hysteresis loop measures the actual persistent current $j(t)$ which can be considerably lower than the true critical current j_c [defined as the value of current corresponding to substantial deviations from high-current linear $V(j)$ behavior]. Another consequence of giant flux creep is that the logarithmical relaxation rate $S=|d\ln M(t)/d\ln t|$ [where $M(t)$ is the sample magnetization] generally cannot be directly related with some well-defined pinning barrier energy U_0 as it is given by the Anderson formula $S=T/U_0$.[8] The point is that the whole distribution of pinning barriers is generated by random potential, so that the actual value of the pinning barrier (corresponding to a given stage of relaxation process) depends on the actual value of persistent current.

In this paper we consider magnetization-current relaxation due to flux creep. We shall use basically the theory of collective flux creep developed in Ref. 5 and demonstrate how the pinning characteristics can be inferred from the current (magnetization) relaxation measurements.

In HTS materials the values of persistent currents $j(t)$ are considerably lower than j_c. It was shown[5] that at $j \ll j_c$ the activation barriers $U(j)$ between different metastable states should grow with current decrease according to a power law:

$$U(j) \simeq U_0(j_c/j)^{\alpha}, \qquad (1)$$

where U_0 is the characteristic energy scale and the exponent α depends on the dimensionality of the problem and on the particular regime of flux creep. In the three-dimensional (3D) case $\alpha=\frac{1}{7}$ in the weak-field, low-temperature region where creep is dominated by the motion of the individual flux lines; at higher field collective creep due to small ($R_b \ll \lambda_L$) bundles takes place and $\alpha=\frac{3}{2}$, wheres at still higher fields the bundle size R_b is much larger than the London penetration depth λ_L, and $\alpha=\frac{7}{9}$. For 2D collective creep $\alpha=\frac{9}{8}$.[9] The above-mentioned results are valid for the case of hopping distances u much shorter than the lattice constant a_0. The opposite limit $u \gtrsim a_0$ was considered by Nattermann,[10] who has ground $\alpha=\frac{1}{2}$. The $U(j)$ behavior (1) gives rise to essentially nonlinear current-voltage characteristics, $V \propto \exp(-A/j^{\alpha})$, and so to the zero linear resistance $\rho_{\text{lin}}=(dV/dj)|_{j=0}=0$, which is the main feature of the vortex-glass state.[11]

If the current relaxation is due to some thermal activation process, then the relevant activation barrier U_a is related with the persistent current $j(t)$ by the relation[12]

$$U_a(j(t))=T\ln(t/\tau_0), \qquad (2)$$

where τ_0 is some attempt time which will be determined below. Combining Eqs. (1) and (2), one obtains[5] the asymptotic form of the current relaxation law:

$$j(t) \simeq j_c\left[\frac{U_0}{T}\ln\left(\frac{t}{\tau_0}\right)\right]^{-1/\alpha}. \qquad (3)$$

At initial stage of relaxation process, when $(j_c-j) \ll j_c$, the Anderson formula

$$j(t)=j_c\left(1-\frac{T}{U_c}\ln\frac{t}{\tau_0}\right) \qquad (4)$$

should be valid, where U_c is a characteristic energy barrier for the case of $j \approx j_c$. Then one can write down the interpolation formula for the whole process of current relaxation:[7]

$$j(t)=j_c\left[1+\alpha\frac{T}{U_c}\ln\left(\frac{t}{\tau_0}\right)\right]^{-1/\alpha}. \tag{5}$$

The comparison between Eqs. (5) and (3) shows that $U_c=\alpha U_0$. The result (5) can also be obtained from the relation (2) and Eq. (1) modified to the form $U(j)=U_0[(j_c/j)^\alpha-1]$ [which is necessary to have $U(j_c)=0$]. In the case of single-vortex creep $\alpha\ll 1$ and Eq. (5) can be approximated [at $(T/U_c)\ln(t/\tau_0)\ll\alpha^{-1}$] by the power law[7]

$$j(t)=j_c\exp\left[-\frac{T}{U_c}\ln\left(\frac{t}{\tau_0}\right)\right]=\left(\frac{t}{\tau_0}\right)^{-T/U_c}. \tag{6}$$

The result (5) leads to the following formula for the normalized creep rate S:

$$S=\frac{T}{U_c+\alpha T\ln(t/\tau_0)}. \tag{7}$$

That means that if one defines the apparent pinning barrier as $U_a=T/S$, then U_a is constant at low temperatures but grows linearly with T at higher temperatures, which is just the result obtained in other works.[13-15] Note also, that in some work (e.g., Ref. 13) the value of U_a was deduced from the experimental data using the formula

$$U_a=T\left[\ln\left(\frac{t}{\tau_0}\right)-\frac{j(t)}{dj(t)/d\ln(t)}\right], \tag{8}$$

which is the consequence of the Anderson formula (4) and, therefore, should be used at $T\ln(t/\tau_0)\ll U_c$ only. As it is seen from Eqs. (7) and (8), the use of Eq. (8) leads to an overestimation of the temperature-dependent term in U_a.

A number of experimental data on the creep rate S were analyzed recently by Malozemoff and Fisher[16] and was shown to obey (at least qualitatively) the behavior following from Eq. (7). The same result was recently obtained in a very clear way by Konczykowski.[17] However, it was shown in Ref. 17 that the results are meaningful if one would use unusually long attempt time $\tau_0\geq 10^{-4}$ sec. Therefore, we encounter the problem of a calculation of the attempt time τ_0 which was usually considered to be "microscopic" ($\sim 10^{-12}$ sec).

The attempt time τ_0 can be found in the framework of the Anderson model, as it was done by Beasley, Labusch, and Webb[18] (see also Fisher and Nattermann[19]). To do this, let us write down the equation describing flux penetration into the sample of the slab geometry (the slab thickness is equal to d):

$$\frac{\partial B}{\partial t}=\nabla(vB). \tag{9}$$

We shall consider the case of full penetration only $[d\ll\Delta B_{\text{ext}}(c/4\pi j_c)]$. Differentiating Eq. (9) with respect to the coordinate x across the slab and using the usual expression for the VL velocity due to thermally activated processes, $v\simeq\omega_m u\exp[-U(j)/T]$, one obtains

$$\frac{\partial j}{\partial t}=\frac{c\omega_m}{4\pi}\frac{\partial^2}{\partial x^2}\{uB\exp[-U(j)/T]\}, \tag{10}$$

where ω_m is some microscopic (i.e., size independent) frequency, and u is the length over which the flux bundle is hopping. We shall look for the solution of Eq. (10) in the form $j(t,x)=j(t)+\delta j(t,x)$, where $|\delta j(t,x)|\ll j(t)$ (cf. Refs. 18 and 19). This kind of solution can indeed be justified in the cases when B dependence of $U(j)$ and j_c is irrelevant, i.e., if (i) the case of single-vortex creep is considered, or (ii) a response to a relatively weak magnetic field step is considered, so that $\delta B(x)$ variation across the sample is much smaller than the background field in the sample and the effect of $\delta B(x)$ on j_c and $U(j)$ can be neglected.

Assuming at least one of these conditions to be fulfilled, we can rewrite Eq. (10) in the form

$$\frac{\partial j}{\partial t}=\frac{c\omega_m}{4\pi}\exp\left(-\frac{U(j)}{T}\right)\frac{\partial^2}{\partial x^2}\times\left[uB\exp\left(-\frac{\partial U}{\partial j}\frac{\delta j(x,t)}{T}\right)\right], \tag{11}$$

and find the solution for $\delta j(x,t)$:

$$\delta j(x,t)=-\frac{T}{(\partial U/\partial j)}\ln\frac{1+x^2/d^2}{B(x)/B_{\text{ext}}}. \tag{12}$$

This means, indeed, that $|\delta j(x,t)|\sim[T/U(j)]j(t)\ll j(t)$. Then the current relaxation $j(t)$ is governed by the following equation:

$$\frac{\partial j}{\partial t}=-\frac{2Buc\omega_m}{\pi d^2}\exp[-U(j)/T]. \tag{13}$$

Integration of Eq. (13) leads to Eq. (2) with the inverse attempt time

$$\tau_0^{-1}\simeq\frac{u\omega_m cB}{\pi d^2 T}\left|\frac{\partial U(j)}{\partial j}\right|. \tag{14}$$

Here B is the value of the magnetic induction at the slab boundary. Thus we have found that the attempt time τ_0 grows with the size of the sample d. The reason for that dependence is simple: The sample magnetization is proportional to the sample volume, whereas the rate of its time variation is proportional to the sample surface area (new vortices should penetrate into the sample from the surface). The value of ω_m^{-1} can be estimated as $\mathcal{A}u/v_c$, where $v_c\approx cj_c\rho_{\text{flow}}B^{-1}$ is the value of FL lattice velocity at $j\geq j_c$ (so that ω_m^{-1} is the traveling time of the flux bundle inside one metastable state); ρ_{flow} is the flux-flow resistivity at $j\approx 2j_c$ in the mixed state with the induction B. The numerical factor $\mathcal{A}$ is at present unknown, but one can speculate that $\mathcal{A}$ grows with the volume of flux bundle (cf. Ref. 20, where such an effect is shown to exist for the case of thermally activated hopping of an elastic line in periodic potential). Also, using Eq. (1) to obtain $|dU(j)/dj|$, one arrives at

$$\tau_0^{-1}\simeq\mathcal{A}\frac{c^2}{d^2}\rho_{\text{flow}}\frac{U_c}{T}. \tag{15}$$

Note that τ_0 appears to be U_c dependent. For the case of collective pinning,[6,7] where U_c grows with j_c decrease, Eq. (15) means that j_c decrease leads to the increase of τ_0^{-1}, which is just the opposite of the behavior of the microscopic frequency ω_m that is proportional to j_c. The same effect can be enlarged by the factor $\mathcal{A}$, that also grows with the j_c decrease. The result (15) was derived for the case of the flux penetration into the sample (or, more generally, for the case of finite external magnetic field B). Rather frequently the experiments are performed in another way: initially presented magnetic field B_0 is switched off and relaxation of remanent magnetization $M_{\rm rem}(t)$ is measured. In this case at the sample boundary $B=0$ and Eq. (14) cannot be applied. However, it seems reasonable to estimate the effective value $B_{\rm eff}$ which should enter Eq. (14) as

$$B_{\rm eff} \approx B(x \approx u) \approx \frac{u}{d} B_0 . \tag{16}$$

The last estimate in Eq. (16) holds if the critical current j_c is weakly dependent on B, so the B profile can be considered as a linear one. In any case the parameters $\rho_{\rm flow}$ and U_c entering Eq. (15) should be taken at the magnetic field value $B_{\rm eff} \ll B_0$, which means that $\tau_{0(\rm REM)}^{-1} \ll \tau_0^{-1}$. However, due to substantial uncertainty in the estimation of $B_{\rm eff}$ for the case of usual remanence measurements with $B_{\rm ext}=0$, it seems to be more useful to perform relaxation experiments at finite value of the external field.

To conclude, we have shown that (i) inverse normalized logarithmic relaxation rate S^{-1} grows linearly with $T\ln(t/\tau_0)$, and (ii) the characteristic attempt time τ_0 is much longer than it was usually assumed and depends strongly on the sample size and on the value of j_c [cf. Eq. (15)].

We are grateful to M. Konczykowski and A. P. Malozemoff for numerous useful discussions.

---

*Present address: Materials Science Division, Argonne National Laboratory, Argonne, IL 60439.

[1] A. P. Malozemoff *et al.*, in *Strong Correlations and Superconductivity,* edited by H. Fukuyama, S. Maekava, and A. P. Malozemoff, Springer Series in Solid State Sciences Vol. 89 (Springer-Verlag, Heidelberg, 1989), p. 349.

[2] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).

[3] M. V. Kartsovnik *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 595 (1988) [JETP Lett. **47**, 691 (1988)].

[4] Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988).

[5] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).

[6] M. V. Feigel'man and V. M. Vinokur, Phys. Rev. B **41**, 8986 (1990).

[7] V. B. Geshkenbein, A. I. Larkin, M. V. Feigel'man, and V. M. Vinokur, Physica C **162-164**, 239 (1989).

[8] P. W. Anderson, Phys. Rev. Lett. **9**, 309 (1962).

[9] M. V. Feigel'man, V. B. Geshkenbein, and A. I. Larkin, Physica C **167**, 177 (1990); V. M. Vinokur, P. H. Kes, and A. E. Koshelev, Physica C **168**, 29 (1990).

[10] T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990).

[11] R. C. Koch *et al.*, Phys. Rev. Lett. **63**, 1511 (1989).

[12] V. B. Geshkenbein and A. I. Larkin, Zh. Eksp. Teor. Fiz. **95**, 1108 (1989) [Sov. Phys. JETP **68**, 639 (1989)].

[13] G. M. Stollman, B. Dam, J. H. P. M. Emmen, and J. Pankert, Physica C **162-164**, 1191 (1989).

[14] Y. Xu, M. Suenaga, A. R. Moodenbaugh, and D. G. Welch, Phys. Rev. B **40**, 10882 (1989).

[15] I. A. Campbell and L. Frutcher, Phys. Rev. Lett. **64**, 1561 (1990).

[16] A. P. Malozemoff and M. P. A. Fisher (unpublished).

[17] M. Konczykowski (private communication).

[18] M. R. Beasley, R. Labusch, and W. W. Webb, Phys. Rev. **181**, 682 (1969).

[19] K. H. Fischer and T. Nattermann (unpublished).

[20] M. Buttiker and R. Landauer, Phys. Rev. A **23**, 1397 (1981).