# High-temperature resistivity minima in Co-rich amorphous ferromagnets

A. Das and A. K. Majumdar
*Department of Physics, Indian Institute of Technology, Kanpur 208 016, Uttar Pradesh, India*
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High-resolution electrical-resistivity data have been obtained for the cobalt-rich amorphous ferromagnets $Fe_5Co_{50}Ni_{17-x}Cr_xB_{16}Si_{12}$ ($x=0$, 5, 10, and 15) and $Fe_5Co_{50}Mn_{17}B_{16}Si_{12}$ between 8 and 300 K. The resistance minimum, which is $\simeq 15$ K for $x=0$, shifts to temperatures $\simeq 200$ K and above on substitution by Cr or Mn. In a sample with $x=0$, in the region above $T_{\min}$, there is clear evidence of a magnetic term $\propto T^2$ in addition to the structural term. A quantitative analysis, based on electron-electron interaction and localization effects, is reported here for ferromagnetic systems exhibiting resistance *minima at high temperatures.* We find for $T < T_{\min}$ ($x > 0$), three distinct regions where the conductivity varies as $\sqrt{T}$, $T$, and $\sqrt{T}$ at low, intermediate, and high temperatures, respectively, in agreement with the predictions of existing theories. The magnetic scattering seems to contribute very little below $T_{\min}$.

## I. INTRODUCTION

The interpretation of transport-property measurements has always been difficult and controversial. The difficulty arises from the numerous assumptions that the theories make in explaining the observations. The resistivity behaviors of amorphous ferromagnets are explained by the diffraction model, which does not take into consideration the magnetic state of the material. The analyses by Kaul *et al.*[1] for $Fe_{80}B_{20-x}C_x$ ferromagnetic metallic glasses have shown that there exists a contribution to the electrical resistivity over and above the structural term which they attribute to the magnetic scattering. Controversy shrouds the origin of resistance minima, which are characteristic features of metallic glasses. The argument focuses on the theories[2] based on structural (two-level states) and magnetic scattering (Kondo mechanism). Traditionally the region below $T_{\min}$ was fitted to a $-\ln T$ behavior, the functional form of which was predicted by both the above theories. The presence of two-level states and the applicability of Kondo phenomena in concentrated ferromagnets are both debatable. Rapp *et al.*[3] have reanalyzed their data, previously fitted to a $-\ln T$ behavior, and have concluded that a $-\sqrt{T}$ fits better below $T_{\min}$. Expermental studies by Howson *et al.*[4] and Cochrane *et al.*[5] confirm the predictions of the theories based upon quantum corrections to the Boltzmann conductivity valid for disordered materials. Measurements and an interpretation based on these theories have only been made on nonmagnetic simple metallic glasses and ferromagnetic ones with low-temperature minima.

In this paper we analyze our data for a series of *ferromagnetic* metallic glasses, where by continuous Cr or Mn substitution the $T_{\min}$ could be shifted to higher temperatures. We have made an attempt here to explain the results below $T_{\min}$ along lines similar to those adopted in the case of nonmagnetic materials but including magnetic effects. The region above $T_{\min}$ is explained by the diffraction model coupled with magnetic scattering, similar to the analysis of Kaul *et al.*[1] For clarity the regions $T < T_{\min}$ and $T > T_{\min}$ are discussed separately.

## II. THEORY

Generalizing the Ziman theory of liquid metals, and taking into account the various scattering processes, the temperature dependence of the resistance is obtained in the form[6]

$$r(T)=R(T)/R(300\ \text{K})=a+bT^2 \quad \text{for } T<\Theta_D \tag{1}$$

and

$$r(T)=R(T)/R(300\ \text{K})=c+dT \quad \text{for } T>\Theta_D\ , \tag{2}$$

where $\Theta_D$ is the Debye temperature. In deriving the above expressions a dynamical structure factor and a Debye phonon spectrum were assumed. Expanding the structure factor into various phonon processes and retaining only the terms up to one phonon process, and on subsequent simplification, it is shown that $r \propto (T/\Theta_D)^2 I(x)$ where $x=\Theta_D/T$ and $I(x)$ is the Debye integral. At high $T$, $I(x)\rightarrow x$, and thus $r(T)\propto T$ and at low temperatures, $I(x)\rightarrow$ const and, therefore, $r(T)\propto T^2$.

In the case of amorphous ferromagnetic materials Richter *et al.*[7] have shown that for $T \ll T_c$, the total magnetic resistivity at low $T$ varies as

$$\frac{\rho_{\text{mag}}(T)}{\rho_{\text{mag}}(0)}=a+bT^{3/2}+cT^2\ . \tag{3}$$

In the above equation the $T^{3/2}$ term, which vanishes for crystalline ferromagnets, results from the partial cancellation of two competing terms. (a) A ($-T^{3/2}$) term originates from purely elastic scattering where electrons are scattered by randomly distributed temperature-dependent local moments and (b) A $T^{3/2}$ term arises from the incoherent momentum nonconserving electron-magnon scattering process. The positive contribution overcompensates the negative one. The $T^2$ term is due to the

 

coherent spin-wave scattering. According to the estimate of Richter *et al.*, based on the data of Mook *et al.*[7] on $Co_4P$, the $T^{3/2}$ term dominates the low-temperature behavior, and its coefficient $b$ is 2 orders of magnitude larger than that of the $T^2$ term.

It has been suggested that, for materials with high degrees of disorder characterized by high residual resistivity, theories based on Boltzmann conductivity relations do not rightly describe the observations.[8] Following the works of Abrahams *et al.*,[9] and Altshuler and Aronov,[10] we understand that in these materials corrections to the normal Boltzmann conductivity apply which are widely known as (a) interaction and (b) localization effects. The interaction effect considers the modification of the effective Coulomb interaction in the presence of a structural disorder. The characteristic temperature dependence of the conductivity $\sigma(T)$, calculated using the Kubo formula and valid in the limits of weak scattering for three-dimensional materials, is given by[8]

$$\Delta\sigma=\sigma(T)-\sigma(0)=\frac{1.3}{\sqrt{2}}\frac{e^2}{4\pi^2\hbar}\left(\frac{4}{3}-\frac{3}{2}\tilde{F}_\sigma\right)(k_BT/\hbar D)^{1/2}\ , \tag{4}$$

where $D$ is the diffusion constant and $\tilde{F}_\sigma$ is the screening factor for the Coulomb interaction. For strong scattering alloys in the same temperature regime, scaling theory of metal-insulator transition proposed by McMillan[11] yields

$$\sigma(T)=\sigma(0)[1+(T/\Delta)^{1/2}]\ , \tag{5}$$

where $\Delta$ denotes a correlation gap in the density of states at the Fermi level. The above $\sqrt{T}$ dependence is observed experimentally at sufficiently low temperatures.

The physics of interaction and localization effects is very well described by Dugdale.[12] The concept of localization is based upon the fact that, when the mean free path of conduction electrons is of the order of interatomic distance, the electrons do not travel in classical trajectories; rather they travel by diffusing from site to site. This results in a mutual interference between waves scattered from nearby ions and leads to a phase coherence between the scattered partial waves. As a result there exists a finite probability for an electron to return to its origin, i.e., a tendency to localize. Altshuler and Aronov[10] illustrate the role of multiple elastic scattering which enhances the probability for an electron to return to the origin and hence get localized. There are processes which destroy the phase coherence and reduce the additional resistivity. Two such processes are inelastic scattering and the effect of magnetic field. At $T\neq0$, $H=0$, inelastic scattering due to electron-phonon interaction makes the major contribution in enhancing the conductivity with temperature. By using a scaling approach to localization (Abrahams *et al.*[9]) and on further simplification by Howson,[13] the correction to conductivity is obtained as

$$\Delta\sigma(T)=\left[\frac{e^2}{\pi^2\hbar}\right]\frac{1}{L_i(T)}\ , \tag{6}$$

where $L_i$ is the inelastic diffusion length. $L_i^2(T)=\frac{1}{2}\,l_i(T)l_e$, where $l_e$ and $l_i$ are the elastic and inelastic mean free paths. The tempeature dependence of $\Delta\sigma$ arises from $l_i(T)$ which is of the form $l_i(T)\propto T^{-2}$ for $T<\Theta_D$, the $T^{-2}$ dependence arising from the electron-electron and electron-phonon scattering. $l_i(T)\propto T^{-1}$ for $T>\Theta_D$ from the usual electron-phonon scattering. These lead to

$$\Delta\sigma\propto T \quad \text{for } T<\Theta_D \tag{7}$$

and

$$\Delta\sigma\propto\sqrt{T} \quad \text{for } T>\Theta_D\ . \tag{8}$$

## III. EXPERIMENTAL

Electrical resistance of the melt-spun samples was measured by the standard four-probe method. A combination of a temperature controller (Lake Shore DRC-82C), a digital multimeter (Datron 1071), and a closed-cycle helium refrigerator (Cryosystems and CTI) was used to measure the resistance of the samples (typical dimension $20\times1\times0.027$ mm$^3$) between 8 and 300 K. The current and voltage contacts with the sample were made with a nonsuperconducting Zn-Cd solder. Absolute accuracy in the measurements of the resistivity was limited by the error (within 10%) in determining the geometry of the samples. The relative accuracy however was better than 1 part in $10^5$. The temperature stability during each measurement was $\pm0.1$ K.

## IV. RESULTS AND DISCUSSION

The samples studied are (a) $Fe_5Co_{50}Ni_{17-x}Cr_xB_{16}Si_{12}$ ($x=0$, 5, 10, and 15 designated as $A1$, $A2$, $A3$, and $A4$, respectively) and (b) $Fe_5Co_{50}Mn_{17}B_{16}Si_{12}$ ($A5$). The ferromagnetic transition temperatures, obtained from $\chi_{ac}$ measurements, are between 180 and 400 K. $T_c$ decreases with the increase of Cr due to its antiferromagnetic interaction. All the samples except the one containing Mn are ferromagnetic down to 18 K. The Mn containing sample has reentrant behavior below 30 K. All the samples indicate a resistance minimum. The corresponding $T_{min}$, which is $\approx15$ K for $x=0$, shifts to $\approx200$ K on addition of Cr or Mn. Figures 1 and 2 show the normalized resistance $r$ [$=R(T)/R(300\text{ K})$] versus temperature $T$. There is no indication of a second minimum at the lower temperatures as observed in some of the earlier works (Olivier *et al.*,[14] Rao *et al.*[15]). The resistivity minima are fairly broad and the percentage change in resistance ($\Delta R/R$) between $T_{min}$ and 8 K are $\approx1\%$ for Cr containing samples and 4% for the Mn containing sample. There is no obvious discontinuity in the resistivity plot near their respective $T_c$'s which almost always lie in the region around the broad minima. The room temperature resistivity of these samples increases with the addition of Cr or Mn. The values of the temperature coefficient of resistance $\alpha$ [$=1/R(dR/dT)$] at 300 K are rather small. Table I summarizes the values of $T_c$, $T_{min}$, $\rho(300\text{ K})$, $\alpha(300\text{ K})$, and $\Delta R/R$.

In the discussion that follows the choice of the temper-

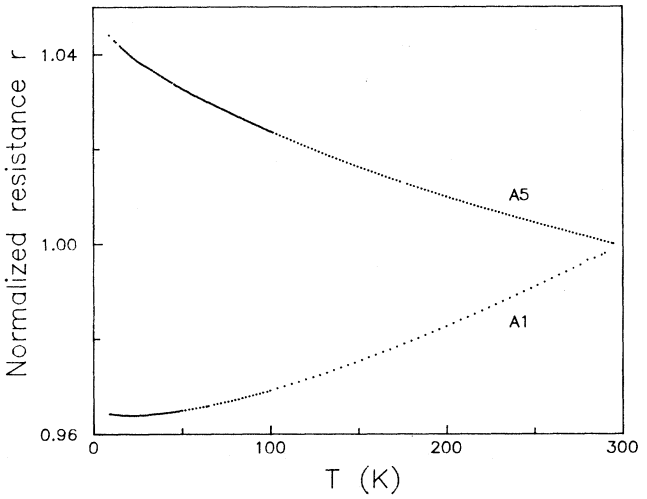


FIG. 1. Temperature dependence of the normalized resistance $r=R(T)/R(300\ \mathrm{K})$ for samples $A1$ ($Fe_5Co_{50}Ni_{17}B_{16}Si_{12}$) and $A5$ ($Fe_5Co_{50}Mn_{17}B_{16}Si_{12}$).

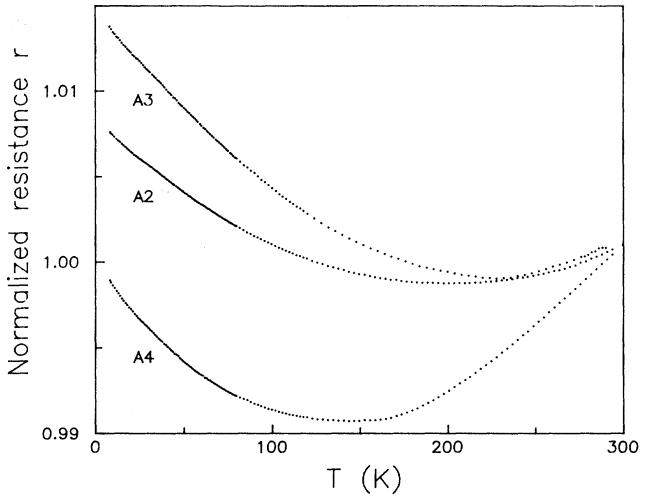


FIG. 2. Same as in Fig. 1 except for samples $A2$, $A3$, and $A4$ ($Fe_5Co_{50}Ni_{17-x}Cr_xB_{16}Si_{12}$, $x=5$, 10, and 15).

ature regions and the equations to be fitted have been influenced by the $\chi^2$ values obtained from a nonlinear least-squares fitting program.

### A. $T > T_{min}$

The data of sample $A1$ ($x=0$) between 30 and 80 K could be fitted well to Eq. (1). The value of $\chi^2=2.5\times10^{-10}$ (Table II) is very close to the experimental resolution in $r(T)$. The fit with a $T^{3/2}$ term in addition to the $T^2$ term gave an unphysical negative coefficient for the $T^{3/2}$ term. This observation is at variance with the results of Kettler and Rosenberg[16] and Singhal and Majumdar[17] who find that their results are better described by a positive $T^{3/2}$ magnetic term over and above the structural $T^2$ term of Eq. (1). However, this result is in agreement with that of Kaul *et al.*[1] who observe that the low-temperature resistivity in amorphous magnetic materials could be described by a $T^2$ term alone coming from both the magnetic and structural contributions. The coefficient of the $T^2$ term ($5.8\times10^{-7}\ K^{-2}$) of sample $A1$ (Table II) agrees well with that reported by Kaul *et al.*[1] on $Fe_{80}B_{20}$ ($\simeq 5.5\times10^{-7}\ K^{-2}$) showing similar behavior of Co- and Fe-rich samples. Thummes *et al.*[18] obtain a much larger coefficient of the $T^2$ ($\simeq 30\times10^{-7}\ K^{-2}$) term in a series of Ni-rich samples. They argue that on alloying with Ni there is a significant contribution to the $d$-band conductivity. This is supported by the evidence that the density of states [$N(E_F)$] that they obtain [4.5 (atom eV)$^{-1}$] is $\simeq$10 times higher than that of the $s$ band. Our analysis on Cr ($x\neq0$) substituted samples shows that $N(E_F)$ in these alloys are roughly 1 (atom eV)$^{-1}$. We assume the same value of $N(E_F)$ for alloy $A1$ ($x=0$). The smaller value of $N(E_F)$ for our alloys compared with those of Thummes *et al.*[18] justifies the relatively smaller coefficient of the $T^2$ term.

In the temperature region 200 K $< T <$ 300 K the data are fitted to an expression

$$r(T)=p+qT+sT^2\ . \tag{9}$$

The $\chi^2$ value improves by about a factor of 20 (Table II) when fitted to this expression as against Eq. (2). The additional $T^2$ term is ascribed solely to the magnetic scattering at high temperatures whereas it is indistinguishable from the structural $T^2$ term at low temperatures. It is difficult to choose between a $T^{3/2}$ and a $T^2$ term to be added to the $T$ term in Eq. (9) at high temperatures, purely on the basis of $\chi^2$ values (Table II) which

TABLE I. Composition, Curie temperature ($T_c$), temperature at resistivity minimum ($T_{min}$), room temperature resistivity ($\rho$), temperature coefficient of resistance ($\alpha$), and percentage change in resistance between $T_{min}$ and 8 K, of $Fe_5Co_{50}Ni_{17-x}Cr_xB_{16}Si_{12}$ and $Fe_5Co_{50}Mn_{17}B_{16}Si_{12}$.

| Sample | Composition | $T_c$ (K) | $T_{min}$ (K) | $\rho$(300 K) ($\mu\Omega$ cm) | $\alpha=1/R(dR/dT)$ ($10^{-5}\ K^{-1}$) at 300 K | $\Delta R/R$ (%) |
|---|---|---|---|---|---|---|
| $A1$ | $x=0$ | 395 | 20 | 150 | 1.7 | |
| $A2$ | $x=5$ | 265 | 200 | 162 | 4.4 | 0.90 |
| $A3$ | $x=10$ | 222 | 228 | 162 | 4.8 | 1.50 |
| $A4$ | $x=15$ | 180 | 144 | 220 | 9.5 | 0.83 |
| $A5$ | $Mn_{17}$ | 300 | >300 | 254 | −10.2 | 4.40 |

TABLE II. Fit of sample $A1$ in the region $T > T_{min}$: Equations, temperature range, coefficients, and $\chi^2$ values.

| Fit of $r$ to | Range of $T$ (K) | $b$ [a] | $c$ ($K^{-2}$ or $K^{-3/2}$, | $\chi^2$ [b] |
|---|---|---|---|---|
| $a+bT^2$ | 30–80 | $5.8\times10^{-7}$ | | $2.5\times10^{-10}$ |
| $a+bT$ | 200–300 | $1.7\times10^{-4}$ | | $73.0\times10^{-10}$ |
| $a+bT+cT^2$ | 200–300 | $1.1\times10^{-4}$ | $1.3\times10^{-7}$ | $4.2\times10^{-10}$ |
| $a+bT+cT^{3/2}$ | 200–300 | $4.0\times10^{-5}$ | $5.5\times10^{-6}$ | $4.0\times10^{-10}$ |

[a]Units of $b$ are $K^{-1}$, $K^{-3/2}$, $K^{-2}$ for $T$, $T^{3/2}$, and $T^2$ terms, respectively.
[b]$\chi^2=1/N\sum_i^N[Y_i(\text{fit})-Y_i(\text{data})]^2/[Y(\text{mean})]^2$.

show equally good fit for both. However, since the $T^{3/2}$ was discarded in the low-temperature fit, we believe it is the $T^2$ term which describes the magnetic contribution. The necessity of the $T^2$ term, we find extends to $T\simeq0.75$ $T_c$ and the coefficient of this term ($s=1.3\times10^{-7}$ $K^{-2}$) is even higher than those reported with analyses done only up to 0.5 $T_c$.[1,17] At temperatures even beyond $T_c$, the $T^2$ term ($s=0.77\times10^{-7}$ $K^{-2}$), in addition to the linear term in $T$ ($q=5.1\times10^{-5}$ $K^{-1}$), is distinctly observed in the case of sample $A4$ ($T_c=180$ K) in the region 240 K $< T <$ 300 K. The existence of a magnetic term at $T > T_c$, which is increasing even beyond $T_c$ and saturating at $T\simeq2$ $T_c$, has been shown and discussed by Kettler and Rosenberg[16] for a FeNi system. The value of $\Theta_D$ for sample $A1$ is obtained from the coefficients of the high- and low-temperature fits to Eqs. (9) and (1), respectively. The relation for $\Theta_D$, taking into account the additional $T^2$ term in the high-temperature fit, is[1]

$$\Theta_D=\frac{\pi^2}{6}\left[\frac{q}{b-s}\right].$$

From this we obtain a realistic value for $\Theta_D=402$ K.

### B. $T < T_{min}$

We have used $\sigma(T)$ in the discussion which follows, as opposed to $\Delta\sigma(T)=\sigma(T)-\sigma(0)$ used elsewhere.[19] The value of $\sigma(0)$ [the extrapolated value of $\sigma(T)$ to 0 K] in our case does not remain the same when extrapolated from different regions of temperature. This result is different from that of Howson and Greig[19] who obtained $\sigma(0)$ to be the same, within experimental error, when extrapolated from different regions of temperature. Thus $\sigma(0)$ in our case is a temperature region-dependent parameter which however does not in any way influence our analysis and conclusions. As a matter of fact we find that the choice of $\sigma(0)$ hardly affects the coefficients of $T$ and $\sqrt{T}$ terms of Eqs. (7) and (8), respectively.

In the temperature region between 8 K (the lowest attainable here) and 20 K, for all the samples ($A2$–$A5$) the data could be fitted to $\sigma(T)=a+b\sqrt{T}$ as given by Eqs. (4) and (5). The coefficients and the $\chi^2$ values are summarized in Table III. The coefficient of $\sqrt{T}$ [(400–1000)$(\Omega\text{m})^{-1}$ $K^{-1/2}$] is in good agreement (in spite of the narrow range of temperature used here) with the near universal value of 600 $(\Omega\text{m})^{-1}$ $K^{-1/2}$ obtained by Cochrane and Strom-Olsen[5] and Rapp *et al.*,[3] based on analysis of a large class of nonmagnetic as well as magnetic materials. Similar values have also been quoted by Thummes *et al.*[18] and Olivier *et al.*[14] on ferromagnetic materials. The diffusion constant $D$, obtained from the slope of $\sigma(T)$ plot and Eq. (4), keeping $\tilde{F}_\sigma=0$, is in the range $(0.7\text{–}4.0)\times10^{-5}$ $m^2/s$ for all the four samples (Table III). Within themselves they however do not show any concentration dependence. The values are similar to those obtained by Olivier *et al.*[14] ($\simeq3\times10^{-5}$ $m^2/s$) and Thummes *et al.*[18] ($\simeq1.3\times10^{-5}$ $m^2/s$). Consequently, the value of the density of states, calculated using the Einstein relation $D=[\rho(0)e^2N(E_F)]^{-1}$, is $N(E_F)\simeq1$ (atom eV)$^{-1}$ for Cr containing samples and $\simeq3.2$ (atom eV)$^{-1}$ for the sample with Mn (Table III).

In explaining the nonuniversality of the Mooij correlation Tsuei[20] has suggested that electron localization plays a significant role in determining the sign and magnitude of TCR (temperature coefficient of resistivity) at temperatures even as high as room temperature. Howson and Greig[4] have analyzed the data of a few nonmagnetic samples (Cu-Ti, Cu-Hf, Cu-Zr), which show resistance minima at high temperatures, between 30 and 300 K and have observed the temperature dependence as given in Eqs. (7) and (8). In our case the samples $A2$, $A3$, and $A4$ have the disadvantage that this analysis could not be carried over a large range of temperature because of resistivity

TABLE III. Fit to $\sigma(T)=\sigma(0)+a\sqrt{T}$ below $T_{min}$(8–20 K): Samples, coefficients, density of states $N(E_F)$, diffusion constant ($D$), and $\chi^2$ values.

| Sample | $\sigma(0)$ $10^5(\Omega\text{m})^{-1}$ | $a$ $10^2(\Omega\text{m})^{-1}$ $K^{-1/2}$ | $N(E_F)$ (atom eV)$^{-1}$ | $D$ $10^{-5}$ $m^2\,s^{-1}$ | $\chi^2$ |
|---|---|---|---|---|---|
| $A2$ | 6.119 87 | 4.30 | 0.95 | 4.0 | $1.7\times10^{-10}$ |
| $A3$ | 6.079 21 | 5.54 | 1.50 | 2.4 | $4.6\times10^{-10}$ |
| $A4$ | 4.540 92 | 4.86 | 0.89 | 3.2 | $2.0\times10^{-10}$ |
| $A5$ | 3.755 18 | 10.09 | 3.20 | 0.7 | $3.3\times10^{-10}$ |

minima occurring around 200 K only. However, sample $A5$ is free from such restrictions; it has a $T_{\min} > 300$ K and also a $T_c = 300$ K. Therefore more emphasis has been given on the analysis of data of sample $A5$.

The region between 55 and 90 K of $A5$ shows a linear temperature dependence as given by Eq. (7). In the region between 100 and 300 K a $\sqrt{T}$ behavior, as given by Eq. (8), is found. The coefficients and the $\chi^2$ values of the fit are summarized in Table IV. The $\chi^2$ values obtained are comparable to the experimental accuracy indicating excellent fit. Figure 3 shows the plot of $\ln\Delta\sigma$ against $\ln T$. In ln-ln *plots* the use of $\Delta\sigma$ is essential for finding the exponent of $T$. The three regions corresponding to $\sqrt{T}$ at low temperature, $T$ in the intermediate region and again $\sqrt{T}$ at high temperature are amply evident from the figure. In calculating $\Delta\sigma[\sigma(T)-\sigma(0)]$, $\sigma(0)$ is obtained from the respective regions of fit (Table IV). The same procedure is followed for samples $A2$–$A4$ and the resulting plots, similar to that of sample $A5$, albeit in a contracted temperature scale, are shown for $A2$ and $A3$ in Fig. 4, and the best-fitted coefficients and the $\chi^2$ values are included in Table IV. In both Figs. 3 and 4 the data points in the region intermediate between two regions of fit are not included. Similar plots are reported for the nonmagnetic amorphous systems in $\sqrt{T}$ and $T$ regions at low and intermediate temperatures. Here we are able to clearly demonstrate all the *three* regions on a single figure hitherto only theoretically predicted. It is interesting to note here that the electron-phonon scattering mechanism had led Meisel and Cote[6] to formulate the diffraction model and explain the familiar temperature dependence above $T_{\min}$, $\rho \propto T^2$ at $T < \Theta_D$ and $\rho \propto T$ at $T > \Theta_D$. In this model the electron-phonon scattering increases the resistivity, or in other words, decreases the conductivity as a function of temperature, as it should be for normal metallic systems. In the localization model the *same* inelastic electron-phonon scattering is used to explain the reduction of resistivity from its elastic collision-assisted value at $T=0$. Assuming the elastic-mean-free-path length $l_e \simeq 3$ Å (Tsuei),[20] we obtain from Eq. (6) and the slope of the $\Delta\sigma(T)$ curve for $A5$, the value of the inelastic-mean-free-path length to be $9\times10^{-4}$ m T$^{-2}$ in the range 55–90 K and $2.5\times10^{-6}$ m T$^{-1}$ in the range 100–300 K. These values are similar to those of

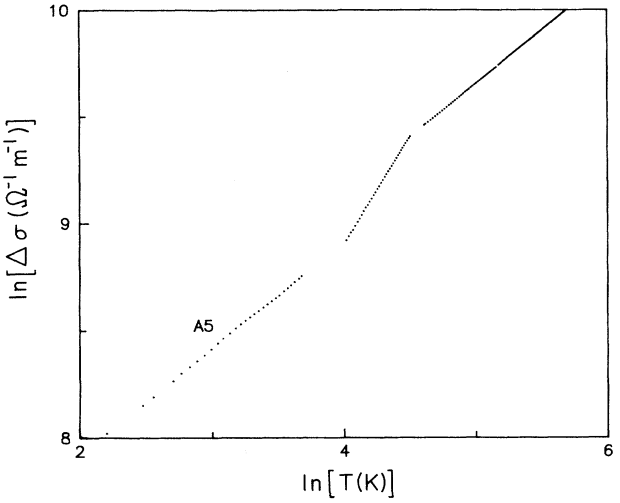


FIG. 3. Plot of $\ln\Delta\sigma$ vs $\ln T$ for sample $A5$ ($Fe_5Co_{50}Mn_{17}B_{16}Si_{12}$) showing three distinct regions of $\sqrt{T}$, $T$, $\sqrt{T}$ dependences in agreement with the predictions of interaction and localization theories. The points in the overlap regions are omitted.

$7.7\times10^{-4}$ m T$^{-2}$ for $T < \Theta_D/3$ and $1.8\times10^{-6}$ m T$^{-1}$ for $T > \Theta_D/3$ reported in the case of nonmagnetic $Cu_{50}Ti_{50}$ (Howson).[13]

The present samples being ferrromagnetic we have made an attempt to find if contributions to resistivity arising from spin disorder, which gives an inelastic term, could be detected. Our method of analysis is as follows. In the region between 55 and 90 K localization theory predicts that

$$r(T)-r(0)=-aT\ , \tag{10}$$

where $r(T)=R(T)/R(300\text{ K})$. In the presence of magnetic scattering the mean free path would be modified as

$$l_{\text{eff}}^{-1}(T)=l_i^{-1}(T)+l_m^{-1}(T)\ , \tag{11}$$

where $l_m^{-1}(T)$ is the mean free path attributed to magnetic scattering. Replacing $l_i$ by $l_{\text{eff}}$ in Eq. (6). and noting that $l_m^{-1}(T) \propto T^{3/2}$ [Eq. (3)] we get for $T < T_c$

TABLE IV. Fit in the intermediate- and high-temperature regions below $T_{\min}$ : Samples, equations, temperature range, coefficients, and $\chi^2$ values.

| Sample | Fit of $\sigma$ to | Range of $T$ (K) | $a$ $10^5(\Omega\text{m})^{-1}$ | $b$ $(\Omega\text{m K})^{-1}$ or $(\Omega\text{m})^{-1}\text{K}^{-1/2}$ | $\chi^2$ |
|---|---|---|---|---|---|
| $A2$ | $a+bT$ | 25–50 | 6.129 61 | 48.5 | $0.6\times10^{-10}$ |
| | $a+b\sqrt{T}$ | 55–100 | 6.108 02 | 648.2 | $2.7\times10^{-10}$ |
| $A3$ | $a+bT$ | 25–50 | 6.090 75 | 66.3 | $3.3\times10^{-10}$ |
| | $a+b\sqrt{T}$ | 55–110 | 6.053 32 | 989.4 | $2.6\times10^{-10}$ |
| $A4$ | $a+bT$ | 20–40 | 4.553 33 | 47.8 | $4.7\times10^{-10}$ |
| | $a+b\sqrt{T}$ | 45–70 | 4.539 24 | 528.0 | $5.5\times10^{-10}$ |
| $A5$ | $a+bT$ | 55–90 | 3.794 57 | 66.9 | $3.2\times10^{-10}$ |
| | $a+b\sqrt{T}$ | 100–300 | 3.733 04 | 1276.0 | $7.5\times10^{-10}$ |

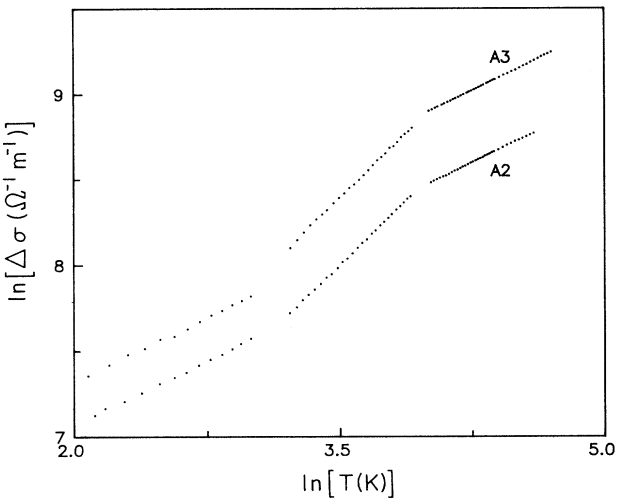


FIG. 4. Same as in Fig. 3 except for samples $A2$ and $A3$ ($Fe_5Co_{50}Ni_{17-x}Cr_xB_{16}Si_{12}$, $x=5$ and 10).

$$[r(T)-r(0)]^2=aT^2+bT^{3/2} \quad \text{for } T<\Theta_D \tag{12}$$

and

$$[r(T)-r(0)]^2=aT+bT^{3/2} \quad \text{for } T>\Theta_D \ . \tag{13}$$

The above two relations are on the assumption that a $T^{3/2}$ will be dominating the magnetic scattering term as predicted by Richter *et al.*[7] However, we observe that in $A1$ the inclusion of the $T^{3/2}$ term did not give any meaningful result. A $T^2$ term was essential in addition to the $T$ term at higher temperatures. The $T^2$ term is identified as the contribution from magnetic scattering which is indistinguishable from the $T^2$ term due to structural contribution in the low-temperature region. If this is so then $l_m^{-1}(T)\propto T^2$, which gives

$$[r(T)-r(0)]^2=aT^2 \quad \text{for } T<\Theta_D \tag{14}$$

and

$$[r(T)-r(0)]^2=aT+bT^2 \quad \text{for } T>\Theta_D \ . \tag{15}$$

On fitting the data to Eqs. (12)–(15) we find that the $T^{3/2}$ *terms* in Eqs. (12) and (13) are smaller by a factor of 1000 than the $T^2$ and $T$ terms. In Eq. (14) the magnetic term is indistinguishable from the structural term. For $T>\Theta_D$ [Eq. (15)], the magnetic $T^2$ term is smaller again by a factor of 1000 in comparison to the $T$ term. The values of $\chi^2$ also have not improved on addition of the magnetic term, be it $T^{3/2}$ or $T^2$. Thus we conclude that the magnetic term, if present, is very small and does not influence the above analysis in terms of the localization theory.

## V. CONCLUSIONS

We have studied the electrical resistivity of a cobalt-rich ($Fe_5Co_{50}Ni_{17-x}Cr_xB_{16}Si_{12}$, $x=0$, 5, 10, and 15 and $Fe_5Co_{50}Mn_{17}B_{16}Si_{12}$) amorphous system which is ferromagnetic and at the same time shows resistivity minima at relatively high temperatures. We find that with substitution of Ni by Cr or Mn $T_c$ decreases while $T_{\min}$ and $\rho$ increase. The temperature dependence of resistivity for sample $A1$ ($x=0$) above $T_{\min}$ and $T<T_c$, has been explained in the framework of the diffraction model, taking into account the magnetic state of the material. The magnetic term varies as $T^2$ and not as $T^{3/2}$ as predicted theoretically.[7] The magnetic $T^2$ term could be distinguished from the structural term only at high temperatures. This result is in agreement with that of Kaul *et al.*[1] An evidence of the magnetic $T^2$ term is found at temperatures *even* beyond $T_c$, with the coefficient comparable to that at $T<T_c$.

In the region below $T_{\min}$ ($x>0$), we find that the conductivity varies as $\sqrt{T}$, $T$, and $\sqrt{T}$ at low-, intermediate-, and high-temperature regions, respectively, arising from electron-electron interaction and localization effects. We show here for the *first time,* all the three distinct regions predicted theoretically. The coefficient of the $\sqrt{T}$ term at low temperatures [(400–1000) $(\Omega m)^{-1}\,K^{-1/2}$] is in agreement with the universal value of 600 $(\Omega m)^{-1}\,K^{-1/2}$ obtained by Cochrane and Strom-Olsen[5] for nonmagnetic systems with strong electron-electron interaction. Also the diffusion constant of electrons and the density of states at the Fermi level of these materials are calculated from the above coefficients.

The magnetic spin-disorder resistivity is found to be smaller than the electron-phonon term by a factor of about 1000. Hence we conclude that, on the basis of our analysis below $T_{\min}$ in terms of localization theory, the resistance minima observed are of nonmagnetic origin.

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