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Dynamical Jahn-Teller effect as a possible microscopic mechanism for anyonic superconductivity

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We show that a (2+1)-dimensional Chern-Simons gauge theory where the gauge field couples to flavored fermions (i.e., fermions carrying an auxiliary quantum number) arises in a natural way from a chiral two-band model describing the dynamical Jahn-Teller effect. The theory exhibits finite-temperature superconductivity without Cooper pairing and a second-order phase transition.

I. INTRODUCTION

According to a suggestion by Laughlin, high-temperature superconductivity is due to the fractional statistics of the relevant quasiparticles, the *semions*.[1] Seminonic or, more generally, anyonic behavior is a feature of quasi-two-dimensionality, and once we have a physical system that is essential two dimensional, we have to take into account the possibility of fractional statistics from first principles of quantum theory alone.[2] An important consequence is that, because of the statistical interweaving of kinematics and dynamics, fractional statistics gases behave as gases of weakly attractive fermions and hence become superfluids or, when coupled to electromagnetism, superconductors at zero temperature.[3] In these new types of superconductors, which are conveniently described by a Chern-Simons gauge-field theory, parity and time-reversal invariance are both violated.[4]

There has been a debate about the question of whether anyonic superconductivity survives at finite temperature, i.e., the Chern-Simons "counterterm" is stable against a finite-temperature correction, but now it seems that this is indeed the case.[5,6] It has been proven that the model displays a Meissner-Ochsenfeld effect at zero and finite temperature. However, when the model is applied to realistic situations, no second-order phase transition is observed, and the Meissner-Ochsenfeld effect persists to arbitrarily high temperature.[6]

Recently, Kapusta *et al.* showed that this serious difficulty can be handled simply by introducting a two-component flavorlike quantum number for the Chern-Simons coupled fermions, i.e., the anyons, provided that the statistical magnetic field created by the two components have opposite signs.[7] As in particle physics, by *flavor* we mean a quantum number that labels an auxiliary internal degree of freedom.

Up to now the microscopic origin for the Chern-Simons term has been rather obscure. Recently Wilczek proposed a general framework to extract anyonic superconductivity from elementary electronic interactions by giving certain condensate values to bilinear Fermi operators.[8] In this paper we show that the ansatz of Kapusta *et al.* may be put on a firm physical foundation, starting from the dynamical Jahn-Teller effect for the oxygen atoms in the CuO_2 layers.[9–12]

The paper is organized as follows: In Sec. II we briefly recapitulate the analysis of Kapusta *et al.* After a description of the relevant Jahn-Teller dynamics in Sec. III, we introduce some useful notions from field theory in Sec. IV and clarify the physical picture in Sec. V according to which the Chern-Simons field is a vector excitation of the underlying microscopic Jahn-Teller interaction.

II. FINITE-TEMPERATURE PHASE TRANSITIONS OF ANYON SUPERCONDCTIVITY

At finite temperature ordinary BCS superconductivity does not exist in two dimensions, because fluctuations overcome the energy in destroying the off-diagonal long-range order. But Hohenberg's theorem[13] does not apply to anyon superconductivity due to the inherent noncommutativity of translations.[3,8] In doing explicit calculations, this was shown by Randjbar-Daemi, Salam, and Strathdee:[6] In the case of only one fermion flavor the Meissner-Ochsenfeld effect persists at all temperatures, despite the fact that at $T\neq 0$ a new type of static magnetic interaction appears. Nevertheless, at limiting temperatures only the long-range component of the interaction is important.

In order to find a phase transition in the temperature dependence of the penetration depth Kapusta *et al.* start with *two* fermions flavors.[7] Consider the Lagrangian density

$$\mathcal{L}=\tfrac{1}{2}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda+\boldsymbol{\psi}^\dagger iD_t\boldsymbol{\psi}-\frac{1}{2m}|D_k\boldsymbol{\psi}|^2 -\frac{1}{2m}f_{12}(g'\boldsymbol{\psi}^\dagger\boldsymbol{\sigma}_3\boldsymbol{\psi}+g''\boldsymbol{\psi}^\dagger 1\boldsymbol{\psi}) , \qquad (1)$$

where $D_\mu=\partial_\mu-iga_\mu$ are the covariant Chern-Simons derivative, $f_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu$ the associated field strengths, $\boldsymbol{\sigma}_3$ a Pauli matrix, and $\boldsymbol{\psi}$ a two-component spinor. One takes $g^2/2\pi=1/N$, where N labels the fractional statistics. Call the two components "+" and "−." In the standard case the usual identity $f_{12}=-g\boldsymbol{\psi}^\dagger\boldsymbol{\psi}$ leads to an average Chern-Simons magnetic field

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$$b=-g\langle\psi^\dagger\psi\rangle \quad (2)$$

in the mean-field approximation. But in case of *two* components we obtain

$$b=-g(\langle\psi_-^\dagger\psi_-\rangle-\langle\psi_+^\dagger\psi_+\rangle)\ , \quad (3)$$

where the associated gauge couplings have opposite signs. Symmetry breaking leads to

$$\langle\psi_-^\dagger\psi_-\rangle\neq\langle\psi_+^\dagger\psi_+\rangle\ , \quad (4)$$

that is the "+" fermion has a higher energy than the "−" fermion. We may then expect that $b(T)=0$ at a certain $T=T_c\neq0$.

In the Lagrangian (1) we have two magnetic coupling constants g' and g''. It is the σ_3 coupling that induces $\langle\psi_-^\dagger\psi_-\rangle\neq\langle\psi_+^\dagger\psi_+\rangle$. In mean-field approximation the fermions move independently in the fields a_0=const and b =const. The Landau orbitals have energies

$$E_{n\sigma}=\left[n+\frac{1}{2}+\frac{g''}{2g}+\frac{g'}{2g}\sigma\right]\omega_c,\quad \omega_c=\frac{g|b|}{m}\ , \quad (5)$$

and the chemical potentials are

$$\mu_\sigma=\mu+ga_0\sigma,\quad \sigma=\pm1,\quad n=0,1,2,\ldots\ . \quad (6)$$

Minimizing the thermodynamic potential

$$\Omega=-\frac{g|b|T}{2\pi}\sum_{n,\sigma}[\tfrac{1}{2}\beta E_{n\sigma}+\ln(1+e^{-\beta(E_{n\sigma}-\mu_\sigma)})]-a_0b \quad (7)$$

at fixed T and μ (with $\beta\equiv2/k_BT$) yields

$$b=-\frac{g^2|b|}{2\pi}\sum_{n,\sigma}\sigma\rho_{n\sigma}\ , \quad (8)$$

where

$$\rho_{n\sigma}=\frac{1}{e^{\beta(E_{n\sigma}-\mu_\sigma)}+1}\ . \quad (9)$$

From this we obtain $a_0=a_0(b,\mu,T)$, which may be substituted into the thermodynamical potential (7) and the zero-point-energy shift due to the Casimir effect is given by

$$\Omega_{\text{zero-pt}}=\frac{[3(g')^2+3(g'')^2-g^2]b^2}{48\pi m}\ . \quad (10)$$

Numerical integration shows that the Chern-Simons magnetic field $b(T)$ approaches zero smoothly at $T=T_c\neq0$, indicating a second-order phase transition.

As an example, Kapusta *et al.* get, with $m=10m_e$, $N=10$ ($g^2/2\pi=0.1$), $g'/g=9$, and $\mu=0.01$ eV, the critical temperature $T_c=125$ K. (The high effective mass for the fermions will be explained by us as being due to the usual high-mass values in a Jahn-Teller polaron state later on.)

Expanding the thermodynamic potential for small b one gets to order b^4

$$\Omega(b)-\Omega(0)=-\tfrac{1}{2}c_2\omega_c^2+\tfrac{1}{4}c_4\omega_c^4\ , \quad (11)$$

where c_2 and c_4 are calculable constants, $b\neq0$ implies $c_2>0$, and T_c is obtained from $c_2(T_c)=0$. This form is rather typical for a second-order phase transition. The real Meissner-Ochsenfeld effect is studied by coupling to electromagnetic vector potential A_μ. Within perturbation theory one arrives at the conclusion that the penetration depth given at $T=0$ by

$$\lambda(T=0)=\left[\frac{m}{e^2\langle\psi^\dagger\psi\rangle}\right]^{1/2} \quad (12)$$

diverges at $T\to T_c$ for $\lambda\simeq(T_c-T)^{-1/2}$. So we may conclude that we have a real superconducting phase transition at $T=T_C$.

III. THE JAHN-TELLER-TYPE HAMILTONIANS AND LAGRANGIANS

The most general physically reasonable two-band Hamiltonian describing the interaction between phonons and electrons may be written as

$$H=H_{\text{ph}}+H_e+H_{e\text{-ph}}\ , \quad (13)$$

with

$$H_{\text{ph}}=\left[P_{\text{diag}}^2+\frac{1}{2}M_{\text{diag}}\Omega_{\text{diag}}^2Q_{\text{diag}}^2\right]+\left[\frac{P_{\text{off}}^2}{2M_{\text{off}}}+\frac{1}{2}M_{\text{off}}\Omega_{\text{off}}^2Q_{\text{off}}^2\right]\ , \quad (14)$$

$$H_e=\sum_{\mathbf{k}}E_{\mathbf{k}1}c_{\mathbf{k}1}^\dagger c_{\mathbf{k}1}+\sum_{\mathbf{k}}E_{\mathbf{k}2}c_{\mathbf{k}2}^\dagger c_{\mathbf{k}2}\ , \quad (15)$$

$$H_{e\text{-ph}}=-\sum_{\mathbf{k},\mathbf{k}}(c_{\mathbf{k}'1}^\dagger,c_{\mathbf{k}'2}^\dagger)\times\begin{bmatrix}g_{\text{diag}}Q_{\text{diag}} & g_{\text{off}}Q_{\text{off}}\\ g_{\text{off}}Q_{\text{off}} & g_{\text{diag}}Q_{\text{diag}}\end{bmatrix}\begin{bmatrix}c_{\mathbf{k}1}\\ c_{\mathbf{k}2}\end{bmatrix}\ , \quad (16)$$

where Q_{diag} and Q_{off} denote phonons associated to different symmetry types. We conventionally call them diagonal and off-diagonal phonons, respectively.

The interaction Hamiltonian $H_{e\text{-ph}}$ may be diagonalized giving

$$H_{e\text{-ph}}=-\sum_{\mathbf{k},\mathbf{k}'}(c_{\mathbf{k}'\text{I}}^\dagger,c_{\mathbf{k}',\text{II}}^\dagger)\begin{bmatrix}g_{\text{diag}}Q_{\text{diag}}+g_{\text{off}}Q_{\text{off}} & 0\\ 0 & g_{\text{diag}}Q_{\text{diag}}-g_{\text{off}}Q_{\text{off}}\end{bmatrix}\begin{bmatrix}c_{\mathbf{k}\text{I}}\\ c_{\mathbf{k}\text{II}}\end{bmatrix}\ ,$$
$$=:-\sum_{\mathbf{k},\mathbf{k}'}c_{\mathbf{k}'}^\dagger(g_{\text{diag}}Q_{\text{diag}}\mathbf{1}+g_{\text{off}}Q_{\text{off}}\tau_3)\mathbf{c}_{\mathbf{k}}\ . \quad (17)$$

We write τ_3 instead of σ_3 indicating that it is a flavor-type of degree of freedom and *not* a spin degree of freedom, which here plays the essential role.

Consequently, the phonon second-quantized form of the Hamiltonian reads

$$H_{\mathrm{ph}}=\sum_{\mathbf{q}} \hbar\omega_{\mathrm{off}_{\mathbf{q}}} a^{\dagger}_{\mathrm{off}_{\mathbf{q}}} a_{\mathrm{off}_{\mathbf{q}}}+\sum_{\mathbf{q}} \hbar\omega_{\mathrm{diag}_{\mathbf{q}}} a^{\dagger}_{\mathrm{diag}_{\mathbf{q}}} a_{\mathrm{diag}_{\mathbf{q}}} , \tag{18}$$

$$H_e=\sum_{\mathbf{k}} E_{\mathbf{k}\sigma}\mathbf{c}^{\dagger}_{\mathbf{k}\sigma}\mathbf{c}_{\mathbf{k}\sigma} , \tag{19}$$

$$H_{e\text{-ph}}=-\sum_{\mathbf{k},\mathbf{k}+\mathbf{q}} \mathbf{c}^{\dagger}_{\mathbf{k}+\mathbf{q}} g_{\mathrm{diag}}(a^{\dagger}_{\mathrm{diag}_{-\mathbf{q}}}+a_{\mathrm{diag}_{\mathbf{q}}})\mathbf{1}\mathbf{c}_{\mathbf{k}}-\sum_{\mathbf{k},\mathbf{k}+\mathbf{q}} \mathbf{c}^{\dagger}_{\mathbf{k}+\mathbf{q}} g_{\mathrm{off}}(a^{\dagger}_{\mathrm{off}_{-\mathbf{q}}}+a_{\mathrm{off}_{\mathbf{q}}})\tau_3\mathbf{c}_{\mathbf{k}} . \tag{20}$$

The model may be reformulated in Lagrangian language. The Lagrangian is given by

$$\mathcal{L}=\left[\frac{1}{c^2_{\mathrm{diag}}}\partial_t\varphi_{\mathrm{diag}}\partial_t\varphi_{\mathrm{diag}}-\partial_k\varphi_{\mathrm{diag}}\partial_k\varphi_{\mathrm{diag}}\right]+\left[\frac{1}{c^2_{\mathrm{off}}}\partial_t\varphi_{\mathrm{off}}\partial_t\varphi_{\mathrm{off}}-\partial_k\varphi_{\mathrm{off}}\partial_k\varphi_{\mathrm{off}}\right]$$
$$+\boldsymbol{\psi}^{\dagger}i\partial_t\boldsymbol{\psi}-\frac{1}{2m}|\partial_k\boldsymbol{\psi}|^2-\frac{1}{2m}g_{\mathrm{diag}}\varphi_{\mathrm{diag}}(\boldsymbol{\psi}^{\dagger}\mathbf{1}\boldsymbol{\psi})-\frac{1}{2m}g_{\mathrm{off}}\varphi_{\mathrm{off}}(\boldsymbol{\psi}^{\dagger}\tau_3\boldsymbol{\psi}) , \tag{21}$$

where c_{diag} and c_{off} are the diagonal and off-diagonal sound velocities, respectively. Notice that the coupling proportional to 1 is a conventional electron-phonon coupling, whereas the term proportional to τ_3 is due to the Jahn-Teller effect splitting the energy levels.

A concrete realization of this model was given by Yu and Anderson.[9] They take as the 1 and 2 bands the bands associated with the oxygen s and p orbitals (which are the orbitals of a different parity) and neglect the diagonal coupling. Introducing the *chiral* (that is, the left-handed and right-handed) linear combinations of the fields

$$\boldsymbol{\psi}=\begin{bmatrix}\psi_L\\ \psi_R\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}\psi_s-\psi_p\\ \psi_s+\psi_p\end{bmatrix} , \tag{22}$$

and setting $\lambda:=g_{\mathrm{off}}$ we get

$$\mathcal{L}=\left[\frac{1}{c^2}\partial_t\varphi\partial_t\varphi-\partial_k\varphi\partial_k\varphi\right]+\boldsymbol{\psi}^{\dagger}i\partial_t\boldsymbol{\psi}$$
$$-\frac{1}{2m}|\partial_k\boldsymbol{\psi}|^2-\frac{1}{2m}\lambda\varphi(\boldsymbol{\psi}^{\dagger}\tau_3\boldsymbol{\psi}) . \tag{23}$$

In this Lagrangian the phonons that act between the left-handed and right-handed electrons are *propagating*. A similar interband interaction based on propagating phonons was discussed by Weger, Englman, and Halperin,[11] and the matching of the coupling signs was considered in detail in Ref. 12.

However, this is not exactly what Yu and Anderson had in mind. Yu and Anderson addressed their attention to a so-called *local phonons*, i.e., a nonpropagating vibration mode, which is intimately connected to a "solid-state instanton," i.e., to a tunneling of an atom in a dynamically generated double-well potential.[9] In the following section we will construct a correct field-theoretical analogue describing such a situation.

IV. THE LOCAL PHONON AS A NONPROPAGATIG OGIEVETSKIĬ-POLUBARINOV PHONON

In relativistic quantum-field theory "phonon modes" are often associated with massless fields of helicity zero.[14] Whereas a massive particle of spin s has $2s+1$ states of polarization, a massless particle with spin $s\neq 0$ will have only helicity states, a massless *and* a spinless particle with only *one* helicity state. More abstractly, in the massless case a system of $2s+1$ states is no longer irreducible. This mechanism plays a distinguished role, when we take the massless limit of a massive quantum field theory. For example take a look at the case of a massive spin-1 boson i.e., a massive photon. It is evident that the two maximum polarized states become the two helicity states in the massless limit, but one is forced to ask what happens to the other state. In the case of a photon it simply disappears. Physical states are chosen in such a way that the longitudinal photon cancels against the scalar *but* negative-norm "timelike" photon component. This is the essence of the well-known Gupta-Bleuler formalism.

Some time ago, Ogievetskiĭ and Polubarinov[15] showed that one can construct tensor field theories in which exactly the opposite happens, namely, the vanishing of the helicity-(± 1) states.

Opposed to the massive Maxwell-Dirac Lagrangian,

$$\mathcal{L}=-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+\frac{m^2}{2}A^2_\mu$$
$$-\bar{\psi}(\gamma_\mu\partial^\mu+M)\psi+ie\bar{\psi}\gamma_\mu\partial^\mu\psi A_\mu , \tag{24}$$

the most simple massive Ogievetskiĭ-Polubarinov particle quantum electrodynamics is described by

$$\mathcal{L}=-\tfrac{1}{12}(\partial_\mu A_{\nu\lambda}+\partial_\lambda A_{\mu\nu}+\partial_\nu A_{\lambda\mu})^2+\frac{m^2}{4}A^2_{\mu\nu}$$
$$-\bar{\psi}(\gamma_\mu\partial^\mu+M)\psi-\tfrac{1}{2}g\,\varepsilon_{\mu\nu\lambda\rho}\bar{\psi}\gamma^\nu\psi\partial^\mu A^{\lambda\rho} . \tag{25}$$

The massless limit $m\rightarrow 0$ is taken at the end of the computation (infrared regularization). Ogievetskiĭ and Polubarinov proved that this Lagrangian can be represented as a four-fermion theory with an intermediate boson. This is exactly what we are looking for. Moreover, it can be shown that the Ogievetskiĭ-Polubarinov phonon is *nonpropagating* or, using different words, it is neither emitted nor absorbed: It is a *dummy field*.

The possibility of constructing a gauge field from fundamental fermionic interactions has been known for a long time starting with Bjorken's classical paper,[16] which, by the way, was motivated by the BCS theory. The difficulties of this approach associated with the nonrenormalizability of four-fermion interactions and the inherent breaking of Lorentz invariance are, of course, of no relevance in the case of condensed-matter systems.

Now we are able to introduce flavored electrons and write down the correct relativistic analogue of Eq. (23):

$$\mathcal{L}=-\tfrac{1}{12}(\partial_\mu A_{\nu\lambda}+\partial_\lambda A_{\mu\nu}+\partial_\nu A_{\lambda\mu})^2+\frac{m^2}{4}A_{\mu\nu}^2 -\bar{\psi}(\gamma_\mu\partial^\mu+M)\psi-\tfrac{1}{2}g\,\varepsilon_{\mu\nu\lambda\rho}\bar{\psi}\gamma^\nu\tau_3\psi\partial^\mu A^{\lambda\rho}\ . \quad (26)$$

Finally, this Lagrangian has to be reduced down to a nonrelativistic $(2+1)$-dimensional situation. Canceling the third row *and* column in the antisymmetric tensor field $A_{\mu\nu}$, we get the dual of a $(2+1)$-vector potential a_μ, such that the interaction term must have the form "field strength times a current diagonal in flavor." Since in $(2+1)$ dimensions a Chern-Simons terms *a priori* present, a minimal choice for the Lagrangian is given by

$$\mathcal{L}=\tfrac{1}{2}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda+\psi^\dagger iD_t\psi-\frac{1}{2m}|D_k\psi|^2 -\frac{1}{2m}f_{12}(\psi^\dagger\tau_3\psi)\ , \quad (27)$$

where $f_{12}=\partial_1 a_2-\partial_2 a_1$ and $D_\mu=\partial_\mu-iga_\mu$. We recognize this as a possible "local-phonon" version of Eq. (23). It coincides with the Lagrangian of Kapusta *et al.*, who suggested that the internal degree of freedom may be identified with the spin, though they did not preclude other interpretations. From this we get a statistical magnetic field

$$b=-g(\langle\psi_R^\dagger\psi_R\rangle-\langle\psi_L^\dagger\psi_L\rangle)\ , \quad (28)$$

such that a breaking of the chiral invariance at low temperatures gives b a finite value. In a study of the finite temperature Meissner-Ochsenfeld effect Kapusta *et al.* arrived at a set of four coupled integro-differential equations, indicating that superconductivity terminates at $T=T_c$. With a mean-field approximation and certain values for the coupling constants and effective mass, Kapusta *et al.* arrived at a $T_c=125$ K. In our physically motivated approach the high effective mass has its origin in the Jahn-Teller polaron state.[10]

V. A POSSIBLE PHYSICAL PICTURE

As Wilczek has pointed out,[8] to obtain a fictitious magnetic field we have to depart from an interaction Hamiltonian

$$H_{\text{int}}=\sum_{j,k,l,m}\Gamma_{jklm}c_j^\dagger c_k c_l^\dagger c_m \quad (29)$$

and search for a mechanism that produces correlations of the form

$$\langle c_j^\dagger c_k\rangle=\Lambda_{jk}\ , \quad (30)$$

where the product

$$\prod_{\text{loop}}\Lambda_{jk} \quad (31)$$

must be a complex number.[8] Ignoring fluctuations, we obtain an effective Hamiltonian, which describes the hopping of an electron is an external magnetic field:

$$H_{\text{eff}}=\sum_{j,k,l,m}\Gamma_{jklm}\Lambda_{jk}c_l^\dagger c_m\ . \quad (32)$$

Note that a necessary condition for the formation of a fictitious magnetic flux is the spontaneous breaking of parity and time-reversal invariance. In our framework this is naturally achieved by the double-well potential characterizing the dynamical Jahn-Teller effect, which forces the pairs of electron field operators to develop a nonvanishing vacuum expectation value and, consequently, generates a parity- and time-reversal-noninvariant mass term for the ψ field. Thus we may say that a *fictitious* or *statistical magnetism* or, so to speak, anyonic statistics is created from nonlinear acoustics and quantum mechanics *in harmony* with the reduction down to two space dimensions.

The breaking of chiral invariance corresponds in relativistic quantum-field theory to the spontaneous breakdown of γ_5 invariance in the Gross-Neveu model.[17] More explicitly, our theory is a certain nonrelativistic limit of a two-flavor chiral four-fermion model. In the relativistic formulation, chirality is an external quantum number (as is the spin), which becomes a flavorlike quantum number in the nonrelativistic limit. Thus it may be identified with an $N=2$ *ad hoc* flavor of the relativistic analogue. Similarly, spin degrees of freedom are treated as SU(2) flavors or colors nonrelativistically (see, e.g., Ref. 18).

Hence, our model is a certain limit of the $N=2$ chiral Thirring or Gross-Neveu model. A relation between Chern-Simons theories and the N-flavor Thirring models has already been analyzed comprehensively.[19] Mavromatos *et al.* found a low-energy equivalence between the number of flavors and the statistics parameter of the Chern-Simons theory by the correspondence

$$\int D\bar{\psi}D\psi\exp\left[i\sum_{i=1}^{N}\left[\int\bar{\psi}^i[i\gamma^\mu(\partial_\mu+iA_\mu)\pm m]\psi^i+\text{const}\times\int(\bar{\psi}^i\gamma^\mu\psi^i)^2\right]\right] \simeq\exp\left[\pm\frac{i}{2}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda-i\times\text{const}\times\int F_{\mu\nu}^2(A)\right]\ , \quad (33)$$

but they could not give any physical explanation for the interesting case $N=2$, which appears naturally in view of the Jahn-Teller effect. In our case the four-fermion interaction on the left-hand side of Eq. (33) is an effective description of the dynamic Jahn-Teller effect in the manner of Ogievetskiĭ and Polubarinov.

Let us close with a few remarks concerning the physics of the Chern-Simons term. In their pioneering work Deser, Jackiw, and Templeton stated that the physical origin of the Chern-Simons term occurring in $(2+1)$-dimensional systems can be traced back to the so-called θ term of $(3+1)$-dimensional physics.[20] This θ term describes an instantonic or, perhaps more directly, a "tunneling impurities" background on which the full (say, three-dimensional) system lives. That a "hidden long-range force" is present in quantum field theories with instantons has been known for some time, and its connection to topological gauge fields, i.e., tensor gauge fields, is almost straightforward.[21]

VI. CONCLUSION

Let us stress the essential point: The electrons are interacting *via* the dynamical Jahn-Teller effect. The interaction—mediated through a dynamically created double-well potential—generates, field theoretically speaking, a parity- and time-reversal-noninvariant mass term for the electrons. An electron, embedded in its surrounding lattice distortion and interacting with the vibration modes, may be regarded as an anyon placed under the influence of an effective magnetic flux. Now, since we have two electron chiralities, we will naturally encounter two anyon flavors just given *semionics* and a phase transition at acceptable temperatures if we insert a high effective mass for the quasiparticles that is expected from the Jahn-Teller polaron picture. The situation is somehow reminiscent of quantum chromodynamics at high temperatures where chiral symmetry is restored. The restoration of chiral symmetry also happens for the Jahn-Teller interaction at a critical temperature, thus causing a real phase transition of second order.

Perhaps it may be possible to construct a more general physical situation than the one fixed by the double-well potential giving more flavors and thus another type of long-range order. In the framework considered here it remains to be shown that because of the nonpropagating nature of the nonlinear phonons, we essentially have a pairing in position space, and thus a possible dynamical basis for an *s*-channel resonance mechanism in the manner of Friedberg and Lee.[22]

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