London penetration depth of *d*-wave superconductors

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The temperature dependence of the London penetration depth is calculated for a model superconductor stabilized by antiferromagnetic spin fluctuations. The order parameter has *d*-wave symmetry. Results are compared with *s*- and *p*-wave calculations and anisotropic as well as strong-coupling effects are discussed.

I. INTRODUCTION

The temperature dependence of the London penetration depth $\lambda(T)$ is often used as a means of distinguishing between different coupling models in superconductivity. The two-fluid model of superconductivity gives a simple power-law behavior for the ratio $[\lambda(0)/\lambda(T)]^2$ from T_c down to $T=0$, namely $1-t^4$ with $t=T/T_c$. Other models used to describe the temperature dependence of this quantity result is significant deviations from the two-fluid-model prediction. Recently there have been many measurements of the London penetration depth in the high-T_c oxides[1-7] and to a lesser degree in the heavy-fermion superconductors.[8,9] While the situation for the high-T_c oxides is still somewhat controversial, the heavy-fermion superconductors UBe_{13} and UPt_3 display simple power-law behavior of $\lambda(T)\sim T^2$ at very low tempertures. This is in contradiction to the predictions for an isotropic order parameter (whether it is weak or strong coupling, clean or dirty limit) which displays the typical exponential T dependence for low temperatures because no quasiparticles can be excited into low-energy states. For the high-T_c oxides there seems to be a consensus that the experimentally observed temperature dependence is *s*-wave-like, but different models from two-fluid[1] to clean-limit BCS (Ref. 2) have been fitted to the experimental data with equal success. In this work we calculate the temperature dependence of the London penetration depth for a mechanism of superconductivity which is derived from the exchange of anisotropic antiferromagnetic spin fluctuations between quasiparticles. It leads generally to an order parameter with *d*-wave symmetry and has recently been proposed for the heavy fermions.[10] Studies of the pair breaking via inelastic scattering off spin fluctuations,[11] the thermodynamics,[12,13] and critical fields[14] have already been worked out within this model. It could also be related to the high-T_c oxides in the doping regime, where the long-range Néel state is qualitatively changed, but spin correlations are still observed. Unfortunately, the critical temperature of this model seems to be intrinsically lower than that of *s*-wave states. At least in the parameter range studied and within the simplifying assumptions made about the effective coupling interaction, this is the case. In Sec. II we sketch the necessary theory. Results are to be found in Sec. III, while Sec. IV is a brief conclusion.

II. THEORY

In order to derive an expression for the London penetration depth in the spin fluctuation model we first have to solve the corresponding Eliashberg equations below T_c, which have been given by Millis, Sachdev, and Varma.[11] We have extended this theory to include the effect of T-matrix scattering,[15] which seems to be necessary for the heavy fermions because the transport coefficients of this class of superconductors are not properly described by a simple Born approximation for the scattering of quasiparticles by impurities.

The corresponding equations for the order parameter $\tilde{\Delta}$ and the renormalized Matsubara frequencies $\tilde{\omega}$ are

$$\tilde{\omega}_{\mathbf{k}}(n)=\omega_n+\pi T\sum_m \langle \lambda_{\mathbf{k},\mathbf{k}'}(n-m)\Omega_{\mathbf{k}'}(m)\rangle' +\pi\Gamma^+\frac{\langle \Omega_{\mathbf{k}'}(n)\rangle'}{c^2+[\langle \Omega_{\mathbf{k}'}(n)\rangle']^2+[\langle D_{\mathbf{k}'}(n)\rangle']^2}, \quad (1)$$

$$\tilde{\Delta}_{\mathbf{k}}(n)=-\pi T\sum_m \langle \lambda_{\mathbf{k},\mathbf{k}'}(n-m)D_{\mathbf{k}'}(m)\rangle' +\pi\Gamma^+\frac{\langle D_{\mathbf{k}'}(n)\rangle'}{c^2+[\langle \Omega_{\mathbf{k}'}(n)\rangle']^2+[\langle D_{\mathbf{k}'}(n)\rangle']^2}, \quad (2)$$

with

$$D_{\mathbf{k}}(n)=\frac{\tilde{\Delta}_{\mathbf{k}}(n)}{[\tilde{\omega}_{\mathbf{k}}(n)^2+\tilde{\Delta}^2_{\mathbf{k}}(n)]^{1/2}}, \quad (3)$$

$$\Omega_{\mathbf{k}}(n)=\frac{\tilde{\omega}_{\mathbf{k}}(n)}{[\tilde{\omega}_{\mathbf{k}}(n)^2+\tilde{\Delta}^2_{\mathbf{k}}(n)]^{1/2}}, \quad (4)$$

where $\omega_n=\pi T(2n+1)$, $n=0,\pm1,\pm2,\ldots,$ and $\langle\ \rangle'$ denotes an average over the Fermi surface. If we consider only the Born approximation to the impurity scattering process then the T-matrix terms in the second line of Eqs. (1) and (2) have to be replaced by

$$\pi t^+\langle \Omega_{\mathbf{k}'}(n)\rangle' \quad (5)$$

in (1) and

$$\pi t^+\langle D_{\mathbf{k}'}(n)\rangle' \quad (6)$$

in (2), respectively. The anisotropic and energy-dependent coupling function λ is given in a separable form[11,14] as

$$\lambda(\mathbf{k},\mathbf{k}',n-m)=\frac{2}{I^2\pi}[J_0-J_1\eta_i(\mathbf{k})\eta_i(\mathbf{k}')]\times\int_0^\infty d\omega\frac{\omega A(\omega)}{(\omega_n-\omega_m)^2+\omega^2}\,, \quad (7)$$

where the functions η_i have d-wave symmetry, e.g.,

$$\eta_1(\mathbf{k})=\frac{\sqrt{15}}{2}(\hat{\mathbf{k}}_x^2-\hat{\mathbf{k}}_y^2)\,,$$
$$\eta_2(\mathbf{k})=\frac{\sqrt{5}}{2}(\hat{\mathbf{k}}_x^2+\hat{\mathbf{k}}_y^2-\hat{\mathbf{k}}_z^2)\,. \quad (8)$$

I is the coupling constant of the spin-fluctuation mediated interaction, the ratio $g=J_1/J_0$ describes the degree of repulsion of the interaction, which is repulsive for all wave vectors $\mathbf{q}$, but leads to an attraction in the d-wave gap channel. $g<1$ is the case that seems to be relevant to the heavy fermions. Formally $A(\omega)$ is the electron spin-fluctuation spectral density, which we will model with a δ function. In Eqs. (5) and (6) we have the impurity terms in the Born approximation, valid for weak scattering with

$$t^+=n_I N(0)|v(k_F)|^2\,, \quad (9)$$

where n_I is the impurity concentration, $N(0)$ is the density of states at the Fermi surface, and v is the scattering potential evaluated at the Fermi momentum k_F. In the T-matrix approximation we have

$$\Gamma^+=\frac{n_I}{N(0)\pi^2}\,,\quad c=\cot\delta_0\,. \quad (10)$$

Both the Born- and T-matrix terms were derived for pure s-wave scattering. In the T-matrix case δ_0 is the scattering phase shift. For very large c ($\delta_0\rightarrow 0$) we recover the Born approximation formula for the scattering, while for $c=0$ we are in the so-called unitarity limit ($\delta_0=\pi/2$), which is proposed to be realized in the heavy-fermion systems because of the underlying Kondo lattice.[16]

It is well known that anisotropic order parameters with zeros of the gap at the Fermi surface lead to collective mode contributions in the electromagnetic response. A simple way out of this problem without having to characterize the specific collective modes is to use hydrodynamic theory,[8] which takes into account the phase variation of the order parameter. The supercurrent in $\mathbf{q}$ space induced by a vector potential $\mathbf{A}$ is then given by

$$j_\mu^s=-\frac{e^2}{mc}K_{\mu\nu}A_\nu=-\frac{e^2}{mc}\left[\overleftrightarrow{n}^s-\frac{(\hat{\mathbf{q}}\cdot\overleftrightarrow{n}^s):(\overleftrightarrow{n}^s\cdot\hat{\mathbf{q}})}{\hat{\mathbf{q}}\cdot\overleftrightarrow{n}^s\cdot\hat{\mathbf{q}}}\right]_{\mu\nu}A_\nu\,, \quad (11)$$

where $\overleftrightarrow{n}^s$ is the superfluid tensor, which replaces the superfluid density n^s for isotropic order parameters. In (11) e is the electron charge and c is the velocity of light. The electromagnetic response $K_{\mu\nu}$ is an odd function of the wave vector. As shown by Hirshfeld, Woelfle, and Einzel,[17] we do not have to consider vertex corrections for the d-wave order parameters which are even functions of $\mathbf{k}$ as long as we consider only isotropic s-wave scattering as above. In our case the superfluidity tensor for an isotropic Fermi surface is given by

$$n_{\mu\nu}^s=\frac{3n}{4T}\sum_n\int_0^\pi\sin\theta\,d\theta\int_0^{2\pi}d\phi\,\hat{\mathbf{p}}_\mu\hat{\mathbf{p}}_\nu\frac{\tilde{\Delta}_{\hat{\mathbf{p}}}^2(n)}{[\tilde{\omega}_{\hat{\mathbf{p}}}^2(n)+\tilde{\Delta}_{\hat{\mathbf{p}}}^2(n)]^{3/2}}\,, \quad (12)$$

and we can calculate the two eigenvalues of the penetration depth corresponding to the vector potential $\mathbf{A}$ being parallel ($\lambda_\parallel$) and normal ($\lambda_\perp$) to the principal axis of gap symmetry $\mathbf{l}$:

$$\lambda_\parallel(T)=\left[\frac{mc^2}{4\pi e^2 n}\right]^{1/2}\left[\frac{3}{2T}\sum_{n=1}^\infty\int_0^\pi\sin\theta\cos^2\theta\,d\theta\int_0^{2\pi}d\phi\frac{\tilde{\Delta}_{\hat{\mathbf{p}}}^2(n)}{[\tilde{\omega}_{\hat{\mathbf{p}}}^2(n)+\tilde{\Delta}_{\hat{\mathbf{p}}}^2(n)]^{3/2}}\right]^{-1/2}\,, \quad (13)$$

$$\lambda_\perp(T)=\left[\frac{mc^2}{4\pi e^2 n}\right]^{1/2}\left[\frac{3}{2T}\sum_{n=1}^\infty\int_0^\pi\sin\theta\sin^2\theta\,d\theta\int_0^{2\pi}\sin^2\phi\,d\phi\frac{\tilde{\Delta}_{\hat{\mathbf{p}}}^2(n)}{[\tilde{\omega}_{\hat{\mathbf{p}}}^2(n)+\tilde{\Delta}_{\hat{\mathbf{p}}}^2(n)]^{3/2}}\right]^{-1/2}\,. \quad (14)$$

For the gap functions η_1 and η_2, which belong to the two-dimensional (2D) representations of the tetragonal and cubic crystal systems, these eigenvalues turn out to be sufficient to describe the electromagnetic response because nondiagonal terms in $n_{\mu\nu}^s$ vanish or are extremely small.

III. RESULTS

In Fig. 1 we present the ratio $[\lambda(T=0)/\lambda(t=T/T_c)]^2$ for the η_2 gap with both orientations $\mathbf{A}\|\mathbf{l}$ and $A\perp l$ and compare with the isotropic s-wave case. The letters s and w denote the strong- or weak-coupling case, characterized by the ratio T_c/ω_E (specified in the figure) with T_c the critical temperature and ω_E the characteristic excitation energy of the the frequency-dependent part of the interaction parameter $\lambda_{\mathbf{k},\mathbf{k}'}(n-m)$. It is easily seen that both directions give a linear behavior $\lambda(T)=\alpha T$ at low temperatures with the same prefactor α. Strong coupling does not change this power-law behavior, but tends to decrease $\lambda(t)$ with respect to $\lambda(0)$ at intermediate temperatures in a similar way as in the s-wave case. Choices of

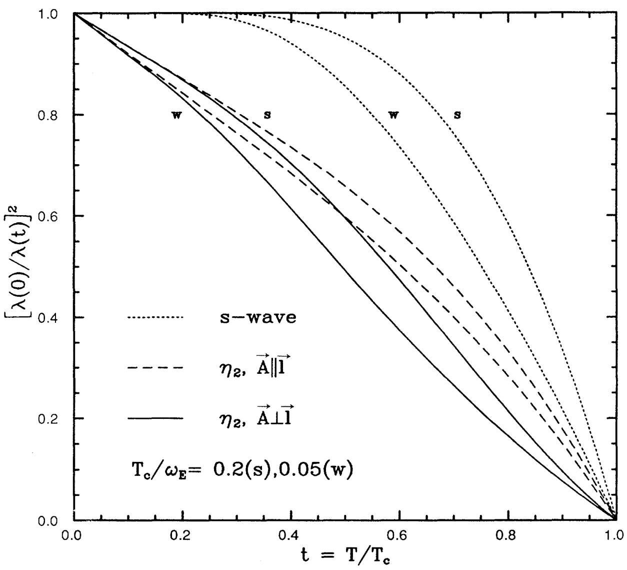

FIG. 1. Normalized penetration depth vs reduced temperature $t = T/T_c$. Strong (s) and weak coupling (w) results for s-wave and d-wave order parameter η_2.

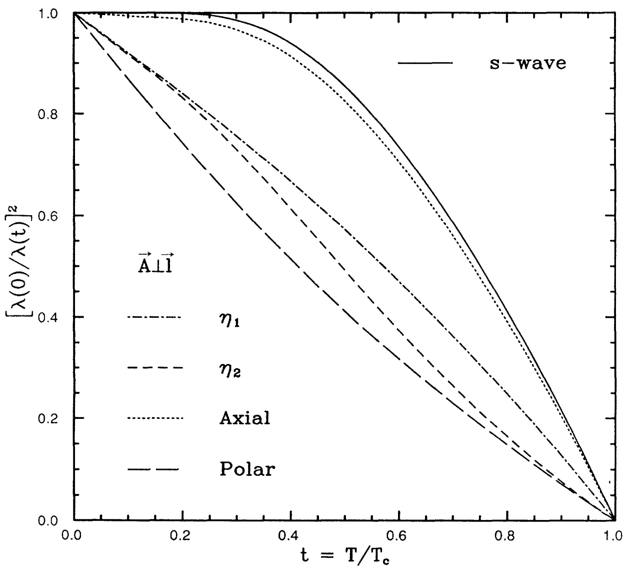

FIG. 3. Comparison of the normalized penetration depth for the d-wave case with p- and s-wave results, **A**⊥l.

the ratio $g = J_1/J_0$ different from the ones used here do not change these normalized curves at all.

Figure 2 shows the results for the η_1 case. We used the same interaction parameters and see $\lambda(t) \sim t$ behavior at low temperatures, now with different constants of proportionality for the two main directions. The linearity in T is expected for gap functions with lines of nodes. In the case of the p-wave polar state this linearity is favored by the orientation effects on the l axis caused by the magnetic field and superflow,[8] while here both directions show the same power law proportional to the temperature.

In Figs. 3 and 4 we show the weak-coupling results of Figs. 1 and 2 and compare them with the results for the p-wave case, namely, the polar and axial state results used by Einzel *et al.*[8] to analyze the $\lambda(T) \sim T^2$ behavior of UBe_{13} at low temperatures. The axial **A**$\|l$ case seems to be the proper state to analyze the experiment because it was favored by orientation effects and was the only one

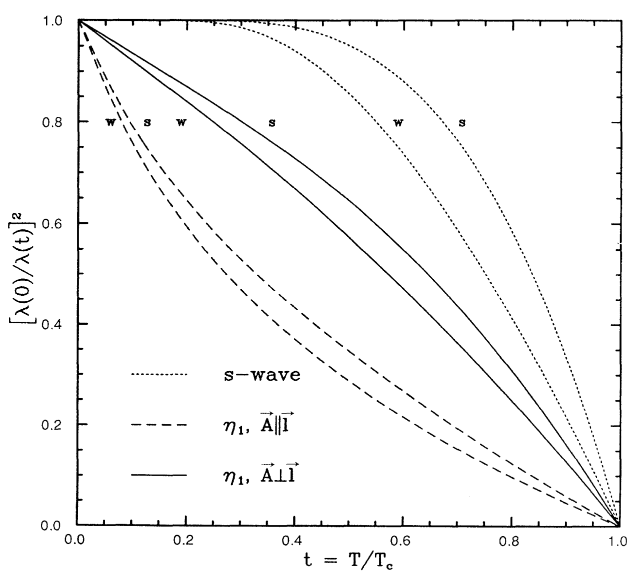

FIG. 2. Same as Fig. 1 but for d-wave order parameter η_1.

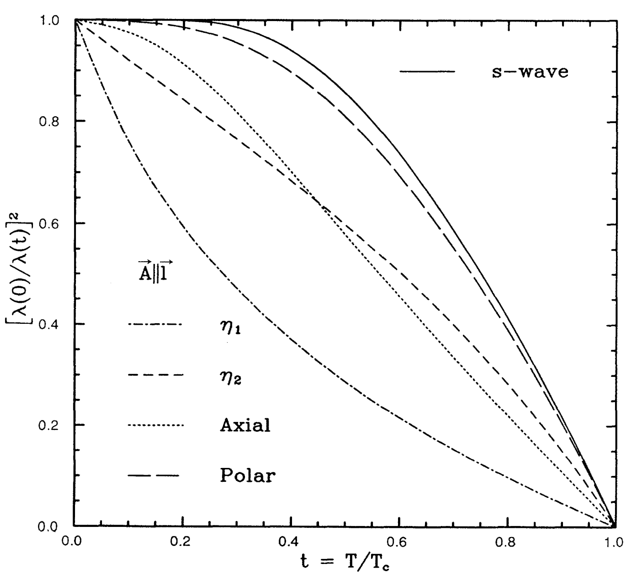

FIG. 4. Same as Fig. 3 but for **A**$\|l$.

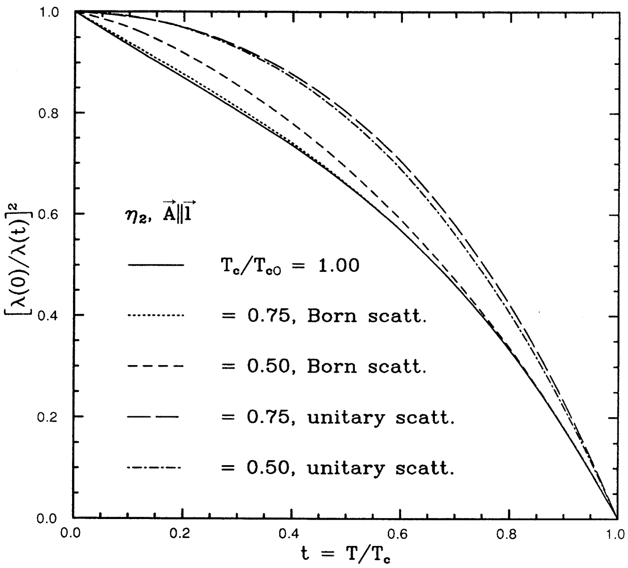

FIG. 5. Normalized penetration depth for the η_2, $\mathbf{A}\|\mathbf{l}$, case in the clean limit, for Born scattering and for T-matrix scattering in the unitary limit. Note that there is a depression of the clean limit critical temperature T_{c0} when impurities are introduced.

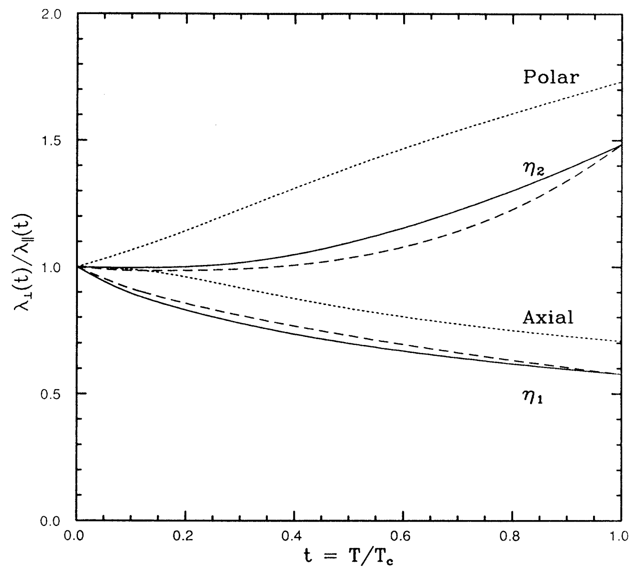

FIG. 6. Anisotropy of the penetration depth for the axial and polar, η_1 and η_2, cases. The dashed lines in the d-wave case are strong-coupling results and the solid lines represent the weak-coupling limit.

with T^2 dependence. Meanwhile, it has turned out that T^2 behavior is introduced immediately by T-matrix impurity scattering and also by weak scattering in the polar case. Figure 5 shows the η_2, $\mathbf{A}\|\mathbf{l}$, case for the clean case and for different scattering mechanisms. It shows quite clearly that unitary scattering, which introduces a finite density of states at zero energy leads to T^2 behavior near $T=0$. It is interesting to note that the unitary scattering curves differ only slightly from BCS–clean limit curves in the temperature range considered experimentally by some groups[2] in their work on high-T_c oxides. It seems clearly crucial to extend measurements to very low temperatures where the most profound differences between isotropic and anisotropic order parameters show up. Muon-spin-rotation measurements on Y-Ba-Cu-O (Ref. 1) do this and totally exclude any of the clean-limit anisotropic models considered, whether it is p-wave or d-wave, because deviations from experimental data exist over the whole temperature range. For the heavy fermions, the d-wave order parameters we considered also do not give a T^2 dependence, except when the effect of impurity scattering is included.

Finally, we show in Fig. 6 the anisotropy ratio $\lambda_\perp/\lambda_\|$ as a function of the reduced temperature t in the clean limit. This ratio goes to 1 for $t\rightarrow 0$ and monotonously approaches some finite value at $T=T_c$. Strong coupling, as indicated by the dashed line, does not change the value of this ratio at T_c, but changes the temperature dependence slightly. The temperature dependences for the polar and axial case are also shown for comparison. These anisotropy ratios are not very large and are probably dominated by other anisotropies as, e.g., different effective masses derived from nonspherical Fermi surfaces.

IV. CONCLUSIONS

To summarize, we have calculated the London penetration depth for d-wave superconductors stabilized by antiferromagnetic spin fluctuations. A $\lambda(T)\sim T$ power law is found at low temperatures, but impurity scattering—as in the case of p-wave order parameters—changes this into a T^2 dependence. We have shown that strong-coupling effects do not change these power laws, but the temperature dependence at higher temperatures is changed in a similar fashion as in the s-wave case. The ratio $g=J_1/J_0$ has no influence at all on the ratio $\lambda(0)/\lambda(T)$. The anisotropy of λ for two different angles between the vector potential $\mathbf{A}$ and the gap symmetry axis $\mathbf{l}$ is of the same order as for model p-wave superconductors with polar or axial symmetry and is not very large.

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