Energy-transport phenomena in single superconducting grains

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Superheated, superconducting grains have been proposed as detectors for dark matter and low-energy neutrinos. We present measurements with irradiation by monoenergetic α particles on single In and Sn grains at ^{3}He temperatures. A local-heating model which explains the structure of our data is developed. The difference between local heating and global heating (seen previously for Al and Zn) is explained by the different rates of quasiparticle relaxation in these materials. There are indications that the phonons have not reached thermal equilibrium with the quasiparticles when the grain flips.

I. INTRODUCTION

Energy-transport phenomena in superconducting materials under the extreme nonequilibrium conditions created by irradiation with single low-energy particles have been under study in recent years because of possible applications in a detector for low-energy neutrinos and hypothetical dark-matter particles. The proposed detection reaction, the elastic scattering of such a particle on a nucleus in the detector, yields a tiny nuclear recoil energy as the only measurable result of the scattering. This energy is a few tens of eV for neutrinos and up to 1 keV for dark-matter candidates. Superconductors are interesting in this context because these recoil energies, although small compared to the sensitivity of conventional particle detectors, are several orders of magnitude larger than the energy necessary to create excitations in a superconductor.

One of the concepts for a detector using superconductors that has been extensively studied is based on micrometer-sized grains made of a type-I superconducting material.[1] The grains are brought into a superheated, superconducting state by applying an external magnetic field H_a which is between the critical field H_c and the superheating field H_{sh} of the grains (see Fig. 1). In this state a small energy deposition can heat a grain sufficiently to cause a sudden phase transition of the entire grain into the normal conducting state (referred to here as a "flip"). This flip is accompanied by the breakdown of the Meissner effect: the magnetic field that was expelled from the grain in its superconducting state penetrates the grain when it becomes normal conducting. This flux change can be detected by a superconducting quantum interference device (SQUID) or other sensitive electronics. More detailed descriptions of this detector concept [the "superheated, superconducting grain (SSG) detector"] and the basic properties of superconducting grains can be found in Refs. 2 and 3.

An important parameter of an SSG detector is its sensitivity, i.e., the amount of energy deposit required to flip a grain. In this context, we discuss the heating mechanism in a grain after a particle interaction. Measurements presented in this paper show that it is possible to flip an In or Sn grain by heating just a fraction of the grain above the superheating phase boundary. This phenomenon is called *local heating.* Al and Zn,[4–6] on the other hand, exhibit what is called *global heating*: the entire grain must be heated above the superheating phase boundary to induce a flip. It is clear that the heating behavior of the grains has a large impact on the possible sensitivity and the shape of the energy threshold of an SSG detector.

A related question is what fraction of the energy deposited by an ionizing particle goes into phonons. This could be important for the identification of ionizing background events:[7] a low-energy recoil nucleus does not ionize efficiently[8] and will mostly excite phonons first. Since it is the condensation of electrons to Cooper pairs that is responsible for the superconductivity, some energy must be transferred into the electron system in order to destroy the superconductivity and cause a flip. An ionizing particle, on the other hand, will first interact with the

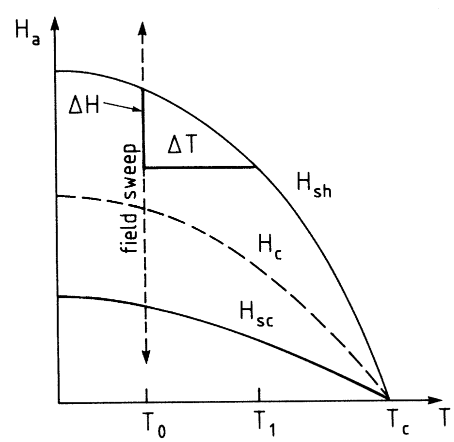

FIG. 1. Phase diagram of a type-I superconductor. H_{sh} is the superheating field, H_c the critical field, and H_{sc} the supercooling field. A temperature rise of $\Delta T = T_1 - T_0$ corresponds to a decrease of the superheating field of ΔH.

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electrons in the grain. In this case it might be possible to induce a flip by heating just the electrons in the grain. However, because of the coupling between electrons and phonons, some of the deposited energy will also go into the phonon system. Since the phonon heat capacity of In is much higher than for other grain materials investigated so far, it is a good material for studying the heating of phonons in the flip process and its effect on the sensitivity of the grain.

In this paper we report on new measurements of single In and Sn grains which were motivated by the above questions. After a short description of the experimental procedure, we discuss our data under three aspects. First, we present a local-heating model which is capable of explaining the structure of our In and Sn data. Second, we give a possible explanation of why In and Sn grains show a local-heating behavior, as opposed to Al and Zn which exhibit global heating. And third, we present a further analysis of our data that shows that the phonon system is heated, at least to a large part, when the grains flip.

II. EXPERIMENTAL SETUP AND RESULTS

In order to investigate the sensitivity and the heating mechanism of In and Sn grains, we performed irradiation experiments on single grains in the following way. At a fixed operating temperature T_0 well below the critical temperature T_c, an external magnetic field H_a was applied to a single grain and cycled between a value well below the supercooling field ($H_{\rm sc}$) and a value well above the superheating field ($H_{\rm sh}$) of the grain. If the grain is not irradiated, phase transitions in the grain are only induced by the magnetic field. As the field is swept up, the grain flips from the superconducting into the normal conducting state at $H_{\rm sh}$; as the field is swept down, the normal conducting grain flips back into the superconducting state at $H_{\rm sc}$. For a given grain orientation in the magnetic field and at a fixed temperature, a single grain has a well-defined superheating and supercooling field, as we have seen in previous work.[4]

In our experiments the grain was irradiated with α particles of a well-defined energy of 4 MeV [0.6 MeV full width at half maximum[9] (FWHM)] during the field sweeps. Since the range of 4-MeV α particles in In and Sn [about 12 μm (Ref. 10)] is smaller than the diameter of the grains ($24 - 60\,\mu$m), most of the hits lead to a deposition of the full α-particle energy in the grain. A hit heats the grain and may therefore cause a flip into the normal conducting state during an up-sweep before $H_{\rm sh}$ is reached. The closer H_a is to $H_{\rm sh}$ the smaller is the temperature rise of the grain that is needed to reach the phase boundary and start a flip. Figure 1 illustrates this correspondence. Measuring the distance of H_a from $H_{\rm sh}$ at the moment of the flip thus provides a lower limit for the temperature rise in the grain caused by a hit.

The grains used in the experiments were produced by ultrasound disintegration in mineral oil of molten 99.99% pure In and Sn and were selected under a microscope for sphericity and smoothness. The grain size was determined with an accuracy of about 1 μm by means of a calibrated grating in the ocular of the microscope. Altogether four In grains with diameters 24, 30, 40, and 60 μm and three Sn grains with diameters 34, 40, and 46 μm were chosen for the experiments.

The experiments were carried out in a top-loading ^{3}He cryostat in the temperature range from 350 mK to 1.25 K. The grain was inserted in a pickup coil attached to the end of the top-loading insert. The α source was mounted on the axis of the pickup coil so that the distance between the grain and the source was about 1.5 mm. The pickup coil with the grain was located above the ^{3}He liquid level (but in the ^{3}He vapor) in order to avoid a degradation of the α-particle energy by the liquid ^{3}He. The energy loss of the α particles in the ^{3}He vapor was a few percent of the 4 MeV, at most. Thermal coupling was ensured by means of a cold finger immersed in the liquid ^{3}He.

During a measuring period we performed 1000–2000 field sweeps within a few hours and recorded H_a for each flip. The flux change in the pickup coil accompanying a flip induced a voltage signal which was amplified and used to trigger a multichannel analyzer (MCA) which also tracked the applied field. The activity of the source was chosen such that the probability of a grain flip due to an α-particle hit during the up-sweep was only about 50%. For about half of the sweeps the flips were therefore caused not by α hits but by reaching $H_{\rm sh}$. Thus we were able to measure the superheating field simultaneously with the distribution of radiation-induced flips. Although the cooling of the grain is much faster than the field sweep, the grain can flip just once per sweep, because it flips from a superheated (metastable) state. For each grain several spectra of counts (flips) versus external field were recorded at different temperatures T in the range $0.1 < T/T_c < 0.35$. Typical examples are shown in Figs. 2 and 3. (The supercooling flips are not shown in these figures.)

At the upper edges of the spectra the superheating "peak" is clearly seen. Its width was determined by the temperature stability and was smaller than the bin width chosen for these histograms. The distribution of radiation-induced flips extends well below the superheating peak. We note the appearance of one rather well-defined step in each histogram. We shall call the field H_a at which the step appears $H_{\rm step}$.[11] The location of this step with respect to the superheating peak varies with grain diameter and temperature. A "tail" of counts extends well below this counting threshold for both In and Sn. These steps and tails will be discussed in terms of a local-heating model in the next section.

In Figs. 4 and 5 we have plotted as points the quantities $\Delta H/H_{\rm sh} \equiv (H_{\rm sh} - H_{\rm step})/H_{\rm sh}$ taken from measurements of the selected grains at different temperatures. The error bars in these figures represent the uncertainty with which we could determine ΔH from the MCA spectrum. This distinctness of the step was basically determined by the statistics and the temperature stability during a measurement. The hatched and dotted areas in Figs. 4 and 5

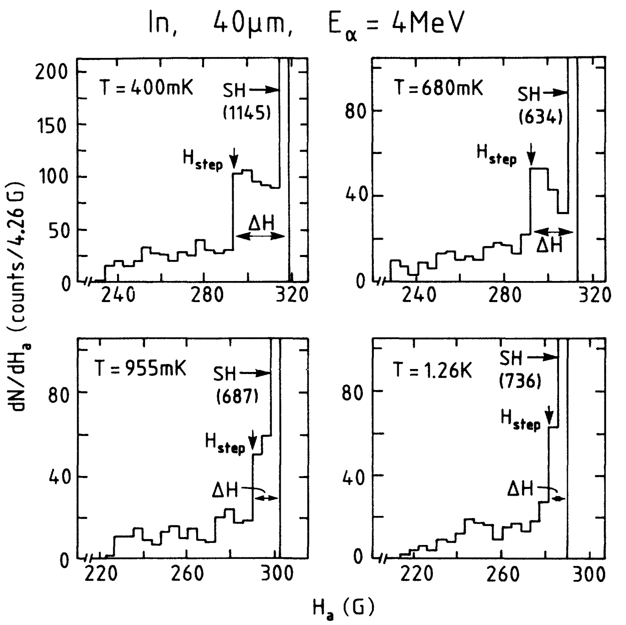

FIG. 2. The number of flips as a function of the applied field for a 40 μm diameter In grain irradiated with 4-MeV α particles at four different operating temperatures. The number of flips in the superheating peak (SH) is given in parentheses. $H_{\rm step}$ and ΔH are discussed in the text.

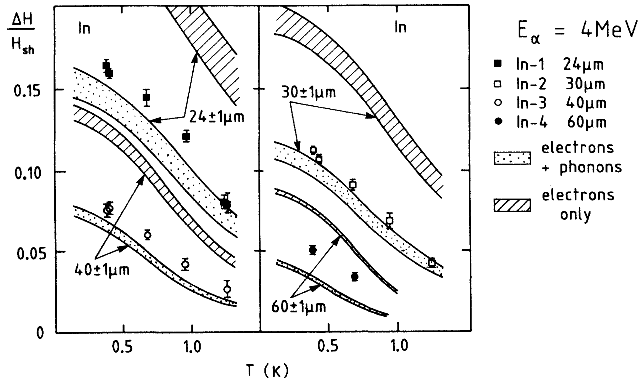

FIG. 4. The measured $\Delta H/H_{\rm sh}$ (points) as a function of operating temperature for 4-MeV α particles incident on four different In grains. The hatched and dotted areas are the expected values calculated assuming uniform heating of the grains. In the calculations the grain diameter was varied ± 1 μm. For the dotted areas the full specific heat of electrons and phonons was used ("electrons + phonons"), for the hatched areas only the electron specific heat was taken into account ("electrons only").

were calculated assuming uniform heating of the grains and are discussed in Sec. III C.

III. DISCUSSION

In our previous paper[4] we showed that the sharp counting thresholds seen for Al and Zn grains (Fig. 6) can be explained with a global-heating model. This model assumes that the energy deposited by an α particle spreads out uniformly in the grain and that the entire grain is always heated by the same amount, no matter where in the grain the α particle deposits its energy. In this case, the temperature rises across the phase boundary only if the value of the applied field is above a certain threshold value $H_{\rm step}$, so that a well-defined counting threshold is expected at $H_{\rm step}$. Although this global heating model is very simple, it describes the Al and Zn results quite well [Figs. 13(b) and 13(c) in Ref. 4].

For Sn and In there is also a counting threshold seen as a step in the histograms (Figs. 2 and 3) at $H_{\rm step}$. However, radiation-induced flips are also observed with a lower count rate well below this step. The lower count rate indicates that for field values less than $H_{\rm step}$ not ev-

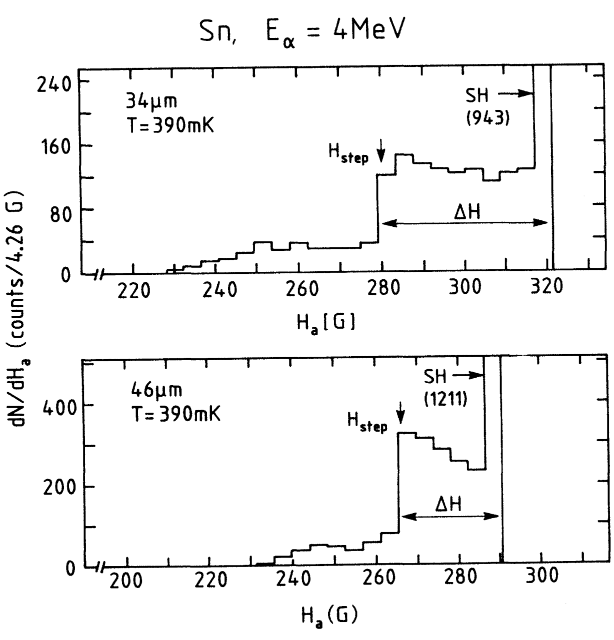

FIG. 3. The number of flips as a function of the applied field for two different Sn grains irradiated with 4-MeV α particles.

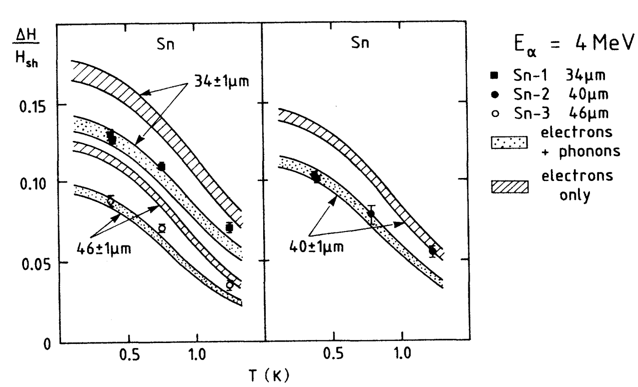

FIG. 5. The measured $\Delta H/H_{\rm sh}$ (points) as a function of operating temperature for 4-MeV α particles incident on three different Sn grains. The hatched and dotted areas are as described in Fig. 4.

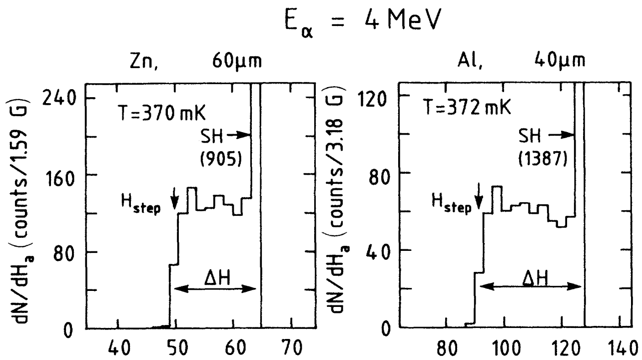

FIG. 6. The number of flips as a function of the applied field for a Zn and an Al grain irradiated with 4-MeV α particles, taken from Ref. 4 (for Al the binning was slightly altered).

ery hit causes a flip. Moreover, for fields at this distance from $H_{\rm sh}$ the energy of an incident α particle is not high enough to heat the entire grain above the phase boundary, as estimated from global heating. Therefore, the flip must occur due to some "local-heating" mechanism. In a local-heating model one assumes that it is possible to cause a flip by heating just a fraction of the grain volume above the phase boundary. In this case, whether a grain flips for a given field depends on where in the grain the energy is deposited.

In the following we present a local-heating model which is capable of explaining the shape of the histograms and we show why the step seen in the histograms is consistent with the threshold predicted by the global-heating model though the heating process is a local one. A possible explanation why Al and Zn grains exhibit global heating while Sn and In grains show local heating is also given. A discussion of our data in terms of this model follows.

A. Local-heating model

In earlier local-heating models (see, for example, Ref. 12) it was assumed that the grain flips when the normal zone, spreading after some energy deposition, reaches some point on the equator of the grain. Since, in the superconducting state, the equator of the grain is the location on its surface where the local field is highest, it was presumed to be the most sensitive part of the grain. However, this neglects the importance of nucleation centers.

We have shown previously that so-called nucleation centers, i.e., "weak points" on the surface of a grain (possibly surface defects, grain boundaries, impurities, etc.), determine the superheating field of a grain.[4,6] Each nucleation center has its own "flipping field" at a given temperature; once the field at the nucleation center reaches this critical value, the flip process begins there. For a given grain orientation in the magnetic field, the superheating field of the grain can be entirely determined by just *one* of these nucleation centers (for a detailed discussion see Ref. 4, especially Fig. 7). The one "in charge" for starting the flip is most likely located near the equator of the grain, where the local field on the surface is highest. This implies that in order to start a flip not any random spot at the equator of the grain but the site of the nucleation center has to be heated above its phase boundary.

These considerations and the assumption that the spread of energy is governed by diffusion, as commonly assumed in local-heating models, qualitatively explain why there is a step in the count rate seen in the histograms. The line of thought is illustrated below by a simple, one-dimensional picture in which the grain is represented by a line with the nucleation center at one end (Fig. 7).

A short time after a localized energy deposition, the profile of the temperature rise along this line is a Gaussian curve which spreads with time [Figs. 7(a) and 7(c)]. The temperature rise at the nucleation center as a function of time [Figs. 7(b) and 7(d)] depends on the location of the energy deposition. For an energy deposition close to the nucleation center [Figs. 7(a) and 7(b)], the temperature rise at the nucleation center reaches a maximum value $\Delta T_{\rm max}^{\rm NC}$ before decreasing to the equilibrium value $\Delta T^{\rm uniform}$ when the heat is distributed uniformly across the grain. The cooling of the grain takes place on a much longer time scale and is therefore neglected here. On the other hand, if the energy deposition is on the far side of the grain [Figs. 7(c) and 7(d)], the maximum temperature at the nucleation center is first reached when the uniform distribution of the heat is established. In this case, the maximum temperature rise at the nucleation center is equal to $\Delta T^{\rm uniform}$. In this simple one-dimensional picture this happens for all energy depositions beyond the center of the grain. No matter where the energy is de-

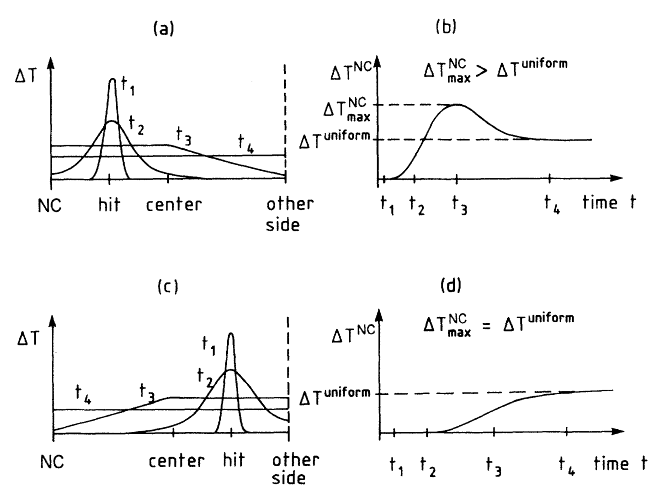

FIG. 7. One-dimensional model of grain heating. (a) Temperature rise ΔT along the grain for various times $t_1 < t_2 < t_3 < t_4$ after a hit close to the nucleation center (NC). (b) Temperature rise $\Delta T^{\rm NC}$ at the NC as a function of time after a close hit. (c),(d) as (a),(b) but for a distant hit.

posited, the temperature at the nucleation center rises by *at least* $\Delta T^{\text{uniform}}$.

The general shapes of our histograms (Figs. 2 and 3) can now be explained in the following way. Since hits close to the nucleation center lead to an "overshoot" of the temperature at the nucleation center ($\Delta T^{\text{NC}}_{\text{max}} > \Delta T^{\text{uniform}}$), they can cause flips for values of H_a relatively far below H_{sh} [Fig. 8(a)]. These flips give rise to the "tail" in the histograms. On the other hand, hits further away cause the temperature at the nucleation center to rise monotonically by $\Delta T^{\text{uniform}}$, and can therefore induce flips only for $H_a \geq H_{\text{step}}$ where $\Delta H = H_{\text{sh}} - H_{\text{step}}$ corresponds to $\Delta T^{\text{uniform}}$ [Fig. 8(b)]. While for $H_a < H_{\text{step}}$ only hits close to the nucleation center lead to flips; for $H_a \geq H_{\text{step}}$ every hit causes a flip. This explains the sharp increase in the count rate at H_{step}.

In a real experiment, the situation is more complicated than in our one-dimensional model. First, the energy deposition by an α particle is not pointlike but spread out along its track. Second, the highly excited electrons produced in the stopping process relax to lower energies, i.e., distribute their energy to many quasiparticles (broken Cooper pairs), a presupposition for a flip. The originally deposited energy will therefore be even more smeared out, before the heat diffusion starts. And third, the heat diffusion then takes place in three dimensions and is determined by boundary conditions which are not symmetric as seen from the location of energy deposition. However, the above explanation of the step remains valid since a distinction between "close" and "far" hits still exists. Note that no step would be expected if no nucleation center were present, i.e., if the grain could be flipped by heating any point on the equator.

Some additional features of the histograms (Figs. 2 and 3) should be noted:

(1) There is a lower threshold field at which the first α-induced flips are observed, indicating that $\Delta T^{\text{NC}}_{\text{max}}$ cannot be arbitrarily high. For the reasons listed in the preceding paragraph it is clear that the volume initially heated by the α particle has a certain minimum size.

(2) The count rate in the tail rises with increasing H_a. For a higher H_a the distance between the spot of energy deposition and the nucleation center can be correspondingly larger. In other words, the probability for a hit to cause the grain to flip increases due to the enlarged "sensitive grain area." For $H_a \geq H_{\text{step}}$ the entire grain is sensitive.

(3) The count rate above the step decreases slightly for increasing H_a due to a saturation effect. Since the grain can flip just once per sweep, the probability for a grain to still be in its superconducting state decreases continuously during an up-sweep. For a source of moderate strength the count rate decreases only slightly before peaking at H_{sh}.

According to this local-heating model the measured $\Delta H = H_{\text{sh}} - H_{\text{step}}$ correspond to the temperature rise of the *entire* grain after the energy has spread out. Therefore, the step seen in the histograms is consistent with the threshold predicted by the *global*-heating model (in which uniform heating is assumed) although the heating process is a *local* one (see also Sec. III C).

To show how this model might work for a spherical grain, we have performed a simple Monte Carlo calculation, whose results are shown in Fig. 9. A grain diameter of $40\,\mu$m is used, and a nucleation center is assigned to a particular spot on the equator. α particles traveling parallel to the magnetic field are randomly incident on the grain. Each α particle then penetrates $10\,\mu$m into the grain before depositing its energy at a single point, whose distance d to the nucleation center is then calculated. To simulate the initially heated volume discussed above, if d is smaller than $6\,\mu$m it is set to this minimum value. To calculate the maximum temperature at the nucleation center, we make the simplification of using the form for the spread of heat from a point source in an infinite medium[13], $\Delta T^{\text{NC}}_{\text{max}} = A\Delta T_d$ where ΔT_d is the temperature rise of a uniformly heated sphere of radius d. For an infinite medium, we find $A = 0.31$. For a finite medium, we expect A to be somewhat larger; Fig. 9 is calculated with $A = 0.35$. We have corrected for the saturation effect mentioned above, assuming a radiation-induced flip occurs in about half the sweeps. We see from Fig. 9 that the model reproduces the step and tail structure of the data quite well. The Monte Carlo simulation does *not* produce a clear step as in the data if we remove the nucleation center from the model so that a flip can be initiated at any point on the equator.[12]

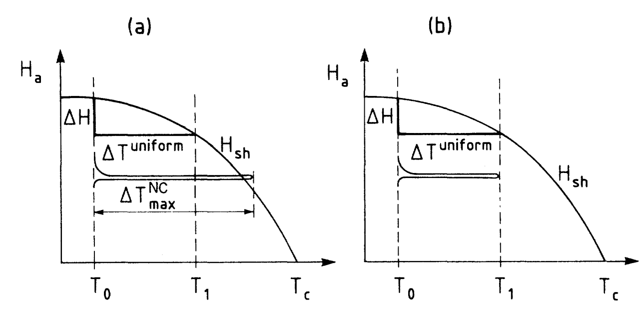

FIG. 8. Phase diagram of a grain. The temperature rise at the nucleation center is schematically indicated (the grain cools much faster than shown here). A close hit (a) can cause a flip early in the up-sweep, whereas for a distant hit (b) the field must reach the threshold value $H_{\text{sh}} - \Delta H$.

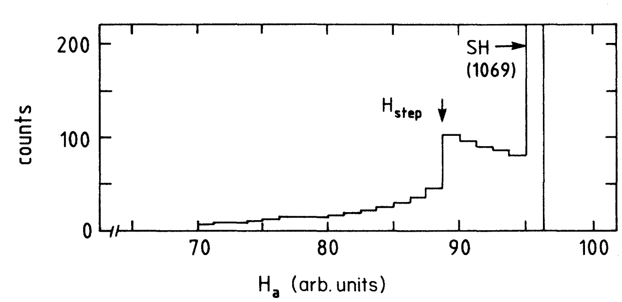

FIG. 9. Monte Carlo calculation of the number of flips as a function of the applied field as described in the text.

The success of this model supports the picture of nucleation at a definite point. This is consistent with our previous analysis of the variation of the superheating field with grain orientation.[4]

B. Global versus local heating

An approach to understanding why grains made of In and Sn exhibit a local-heating behavior while others made of Al and Zn show global heating is the following. An α particle hitting the grain excites almost exclusively electrons in its stopping process. The energy of these highly excited electrons is of order 10–100 eV. They relax rapidly to lower energy by electron-electron scattering, which creates quasiparticles by breaking Cooper pairs. In order to break the metastable superconductivity, a substantial population of quasiparticles with energy near the band gap is necessary. The last stages of quasiparticle relaxation to the band gap occur via phonon emission, which becomes the dominant relaxation process when the quasiparticles reach energies of a few meV (several times the band-gap energy). (The rate for impurity scattering of quasiparticles is higher than the rate for phonon emission. However, quasiparticles rarely lose energy by impurity scattering, so this process does not contribute to their relaxation.) These relaxation processes are discussed in detail in Ref. 14.

The relative speeds of the relaxation to the band gap and diffusion throughout the grain can explain the two cases of global and local heating.

Global heating (Al, Zn). Global heating is expected for Al and Zn, as seen by the following argument. The speed of the energy transport in the grain is set by the quasiparticles, because they spread faster than phonons. The mean free path of the quasiparticles is determined by the rate of impurity scattering, and should be of order 1000 Å for Al, In, Sn, and Zn. Their velocity is approximately the Fermi velocity ($\sim 2\times10^8$ cm/s), so that the quasiparticles spread diffusively across the grain within about 10 ns. This is short compared with the time scale for phonon emission by quasiparticles of a few meV, which is several 100 ns in Al and Zn.[15] Thus for Al and Zn, the energetic quasiparticles spread throughout the grain before relaxing to the band gap and forming an approximately thermal distribution. In other words, the energy is quickly distributed throughout the grain before the grain is really heated, so that the heating is then uniform. In this situation, the global-heating model is appropriate for predicting the flip probability.

Local heating (In, Sn). For In and Sn the time scale for phonon emission by quasiparticles of energy several times the gap energy is of the order of a nanosecond,[15] which is about 2–3 orders of magnitude smaller than for Al and Zn. This is short compared to the spreading time of the order of 10 ns. In this case, the excited quasiparticles lose their energy within a short distance and a "warm spot" of quasiparticles in local thermal equilibrium is created. (In the case of an impinging 4-MeV α particle it will be more like a "warm line" along the track of the particle.) The further spread of the heat will then more or less be governed by *heat* diffusion.[16] Thus, the maximum temperature rise at the nucleation center will be dependent on the location of an α-particle hit, as described by our local-heating model.

C. Heating of the phonon system

We now turn to a further analysis of our data which investigates the heating of the phonon system during the flip process. In Figs. 4 and 5 the measured $\Delta H/H_{\rm sh}$ determined in a series of experiments with different grains at different operating temperatures are plotted as points. As shown in Sec. III A, ΔH corresponds to the temperature rise of the whole grain after spreading of the heat. (It is therefore a measure of the sensitivity of a single grain.) The dotted and hatched areas in these figures show $\Delta H/H_{\rm sh}$ as a function of the operating temperature T as expected for uniform heating of the electron *and* phonon systems (dotted) and the electron system only (hatched). They are calculated in the following way.

For a superconductor in thermal equilibrium the amount of heat Q per volume V needed to raise its temperature from T_0 to T_1 is given by

$$\frac{Q}{V}=\int_{T_0}^{T_1} c(T)\,dT\ , \tag{1}$$

where the specific heat $c(T)$ is assumed to be the sum of the phonon (α term) and the superconducting electronic (γ term) specific heats:

$$c(T)=\alpha\,T^3+a\,\gamma\,T_c\,\exp\left(\frac{-bT_c}{T}\right)\ . \tag{2}$$

The constants a, b, α, and γ are known from experiment. The integration of Eq. (1) using Eq. (2) yields

$$\frac{Q}{V}=\alpha\,\frac{T_1^4-T_0^4}{4}+a\,\gamma\,T_c\left[T_1E_2\left(\frac{bT_c}{T_1}\right)-T_0E_2\left(\frac{bT_c}{T_0}\right)\right]\ , \tag{3}$$

where $E_2(x)$ is the exponential integral of second order.[17] Assuming the superheating phase boundary is proportional to the critical field and has therefore approximately a quadratic dependence on the temperature, i.e.,

$$H_{\rm sh}(T)=H_{\rm sh}(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]\ , \tag{4}$$

the temperature jump $\Delta T\equiv T_1-T_0$ is related to $\Delta H/H_{\rm sh}$ (Fig. 1), which is the quantity we measure:

$$\frac{\Delta H}{H_{\rm sh}(T_0)}=\frac{T_1^2-T_0^2}{T_c^2-T_0^2}\ . \tag{5}$$

The relation between Q/V and $\Delta H/H_{\rm sh}$ for a given operating temperature T (the index 0 is now dropped) is given by Eqs. (3) and (5).

Dividing the α-particle energy by the grain volume yields the energy deposition Q/V. It was calculated taking into account the small energy loss of the α particles in the ^{3}He gas. This energy loss is about 3% at 950 mK and about 7% at 1.25 K.[18] In order to include the possible error in the determination of the grain size (about 1 μm) the curves enclosing the hatched and dotted areas were calculated assuming grain diameters 1 μm larger and smaller, respectively, than the measured diameters. For the dotted areas in Figs. 4 and 5 the full specific heat of phonons *and* electrons was used ("electrons + phonons"), and for the hatched areas only the electron specific heat was taken into account ("electrons only"), i.e., $\alpha = 0$ in Eq. (2).

The parameters α, γ, a, and b used in the above calculations were taken from O'Neill and Phillips[19] and are listed in Table I. There is some evidence that for In the phonon specific heat in the superconducting state is *not* equal to the normal conducting value.[19,20] For the curves in Fig. 4 the α value for the superconducting state was used. A different choice of α affects the curves only marginally and does not qualitatively change the following interpretation. In any case, it should be noted that α is much larger in In than in Sn.

We see from Figs. 4 and 5 that the temperature dependence of the measured $\Delta H/H_{\rm sh}$ agrees qualitatively with that expected from the uniform-heating interpretation. For In, the data points for each grain are between the expectations with and without the phonon specific heat. Except for the largest grain (diameter 60 μm) the data points are obviously closer to the former ("electrons + phonons"). For Sn, the data points also fit better to the expectation including the phonon specific heat. This is different from the result we obtained previously for another Sn grain.[4] This discrepancy may be explained by the fact that the grain was made of a different raw material and by a different production process. Thus, the ratio of the diffusion velocities of quasiparticles and phonons could have been different in that grain, leading to a different splitting of the energy between electrons and phonons (see next paragraph).

These measurements indicate that the phonon system in our grains is heated at least to a large part before the superconductivity breaks down. Returning to our concept of how the deposited α-particle energy thermalizes in In and Sn grains (Sec. III B), we might pursue that interpretation further. After the initial "warm spot" of quasiparticles and phonons is created, energy is still exchanged between the quasiparticle and the phonon system during the further diffusion process. The rate of energy transfer depends on the local temperature difference between the two systems. Since the diffusion of quasiparticles is faster than the diffusion of phonons, the "warm" quasiparticles spread into regions of the grain where the phonon system is still "cold"; energy is transferred from the quasiparticles to the phonon system.[21]

When the warm quasiparticles reach the nucleation center and start the flip, the phonon system is not warmed up completely. Therefore our data points are expected to lie between the corresponding hatched and dotted areas in Figs. 4 and 5. From our data we estimate that, at the moment of the flip of an In grain, about 60 − 80% of the deposited energy is in the phonon system of the grain. The corresponding number for the Sn grains is smaller (about 20 − 50%), reflecting the smaller phonon specific heat of Sn. For Al and Zn the phonon contribution to the specific heat is much too small ($\leq 1\%$) to be resolved by our measurements.

TABLE I. Material constants used for the calculations of specific heat.

	T_c (K)	α (mJ/mole K^4)	γ (mJ/mole K^2)	a	b
In	3.405	1.22	1.69	8.25	1.34
Sn	3.71	0.246	1.78	7.85	1.42

IV. CONCLUSIONS

In order to measure the sensitivity and investigate the threshold behavior of single superconducting grains for an SSG detector, we have perfomed irradiation experiments with monoenergetic α particles on Sn and In grains.

The spectra of flips versus applied field obtained by irradiating the grains while sweeping the applied field repeatedly indicate that In and Sn grains exhibit local heating, with a step increase in the count rate at a field value corresponding to uniform heating. Both the "local-heating tail" and the "uniform-heating step" can be explained by a local heating model in which it is assumed that a certain nucleation center on the surface of the grain has to be heated above the phase boundary in order to start a flip.

In In and Sn the thermal relaxation of the initially excited quasiparticles is faster than the quasiparticle diffusion—the energy transport is slow enough that a local temperature can be established. The part of the grain containing the nucleation center can be strongly heated by an α-particle hit without heating the rest of the grain substantially before the grain flips.

For Al and Zn, which show global heating as we have seen previously in similar experiments, the thermal relaxation of the quasiparticles is 2 − 3 orders of magnitude slower than for In and Sn. Therefore, quasiparticle diffusion transports the energy throughout the grain before a temperature is established and the grain flips. In this case, the nucleation center cannot be heated more than the rest of the grain, no matter where in the grain the hit occurs.

Since there can be a well-defined energy threshold only for grains with a global-heating behavior, In and Sn are probably not suitable materials for a threshold grain detector. Zn and Al are better, since they show global heating. Of course, the problem of producing many grains with the same superheating field, which is important for a detector with a well-defined threshold, remains.

Our data also show that the "uniform-heating step" for In occurs at a slightly lower field value than expected which indicates that the phonons have not reached thermal equilibrium with the quasiparticles by the time the grain flips.

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