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Magnetoresistance oscillations in a grid potential: Indication of a Hofstadter-type energy spectrum

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We present experimental and theoretical results on magnetoresistivity oscillations of high-mobility two-dimensional electron systems in a weak superlattice potential. Experimentally, the band conductivity of a sample with a holographically produced grid potential is shown to be considerably smaller than that of the same sample with a similar linear grating potential of the same lattice constant. This is explained by a magnetotransport theory with due consideration of collision broadening effects and the peculiar subband splitting of Landau levels resulting in a Hofstadter-type energy spectrum.

The two-dimensional motion of electrons in a periodic potential of period a and a perpendicular magnetic field $\mathbf{B}$ introducing the magnetic length $l=(c\hbar/eB)^{1/2}$ leads to intricate commensurability problems. As a function of B, a complicated self-similar energy spectrum (Hofstadter-type energy spectrum) has been obtained in the two complementary, but mathematically equivalent limits of (1) a strong lattice potential and a weak magnetic field in the tight-binding approximation[1] and of (2) a weak periodic perturbation in a Landau quantized two-dimensional electron gas (2D EG).[2,3] In case (2) one finds that each Landau level (LL) splits into p subbands if

$$Ba^2/\Phi_0 \equiv a^2/2\pi l^2 = p/q\,, \tag{1}$$

i.e., if the flux Ba^2 per unit cell is a rational multiple of the flux quantum $\Phi_0=hc/e$. Up to now, this subband splitting has not been verified experimentally.

In this paper we present magnetotransport results providing strong evidence for the realization of the Hofstadter-type energy spectrum, although it is not resolved explicitly due to thermal broadening. To demonstrate this, we extend previous experimental[4,5] and theoretical[5-7] work on systems with a unidirectional modulation to structured systems with square symmetry.

In Fig. 1 we summarize typical results of a series of experiments in which a grid modulation with square-lattice symmetry was created in two steps by holographic illumination[4,5] exploiting the persistent photoconductivity effect in Si-doped $Al_xGa_{1-x}As$ at low temperatures. In a first step a split laser beam reflected from two mirrors, as sketched in the inset of Fig. 1(a), produced an interference line pattern. Thus, a grating potential with modulation in x direction was created and the anisotropic resistivity components $\rho_{xx}^{(1d)}$ [Fig. 1(b)] and $\rho_{yy}^{(1d)}$ [Fig. 1(a)] for the unidirectionally modulated ($1d$) systems were measured. The second illumination, with the sample rotated by 90°, results in a grid potential with modulation in x and in y direction. As demonstrated in Fig. 1, the resistivities $\rho_{xx}^{(2d)}=\rho_{yy}^{(2d)}$ of the bidirectionally modulated ($2d$) samples show oscillations which, at small magnetic fields ($B \leq 0.6$ T), are very similar to and in phase with the weak oscillations of $\rho_{yy}^{(1d)}$ [Fig. 1(a)], but smaller than and 180° out of phase with the large-amplitude oscillations of $\rho_{xx}^{(1d)}$ in the corresponding $1d$ situation. The data shown in Fig. 1(a) were obtained from a sample with mobility 1.4×10^6 cm^2/V s and electron density $N_s=5.1\times10^{11}$ cm^{-2} after the second illumination, those of Fig. 1(b) from one with mobility 1.2×10^6 cm^2/V s and $N_s=3.7\times10^{11}$ cm^{-2}. As has been shown previously, $1d$ samples

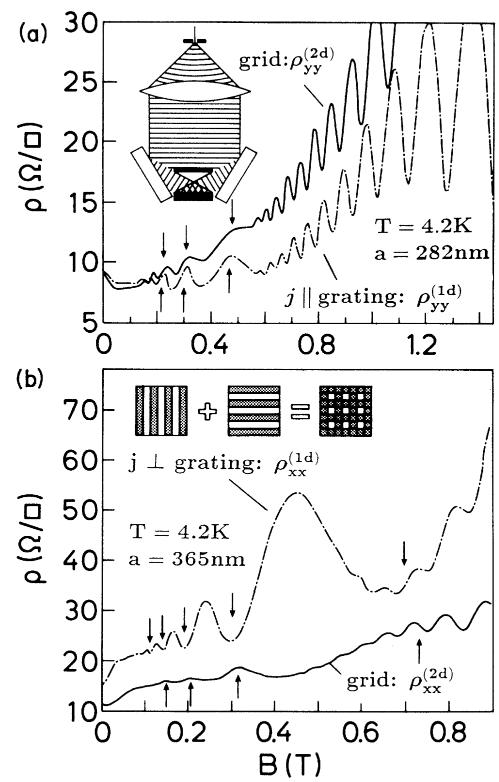

FIG. 1. Magnetoresistance in a grating (with modulation in x direction) and a grid potential for two periods and samples. The insets sketch the creation of the potential by (a) *in situ* holographic illumination, and (b) the resulting pattern. The arrows indicate the flat-band situation defined by Eq. (2) (the second illumination always increases the electron density). The grid potential, created as superposition of two gratings at right angles, suppresses the band conductivity in high-mobility samples, and the oscillations due to the scattering rate (with maxima at the arrow positions) dominate.

exhibit, in addition to the usual Shubnikov–de Haas (SdH) oscillations at higher magnetic fields ($B \geq 0.6$ T in Fig. 1), characteristic resistivity oscillations at lower fields, also periodic in $1/B$, with minima[4,5,8,9] of $\rho_{xx}^{(1d)}$ and maxima[4,5] of $\rho_{yy}^{(1d)}$ at B values for which the cyclotron radius $R_c = l^2 k_F$ of electrons at the Fermi energy ($E_F = \hbar^2 k_F^2/2m$) satisfies the commensurability condition

$$2R_c = a(\lambda - \tfrac{1}{4}),\ \lambda = 1,2,\ldots, \tag{2}$$

emphasizing the importance of the Fermi wavelength $2\pi/k_F$ (related to the density $N_s = k_F^2/2\pi$) as a third length scale determining the transport properties. The large-amplitude oscillations of $\rho_{xx}^{(1d)}$ are attributed[5-8] to an additional "band conductivity" $\Delta\sigma_{yy}$ absent in unmodulated samples, whereas the weaker antiphase oscillations of $\rho_{yy}^{(1d)}$ result from quantum oscillations of the scattering rate.[6,7] In principle, band-conductivity oscillations are also expected in bidirectionally modulated systems. It is the purpose of this paper to demonstrate that the splitting of the LLs by the grid potential into the subband structure visualized by the Hofstadter-type energy spectrum suppresses the band conductivity, leaving only the weak antiphase oscillations due to the oscillating scattering rate.

All the magnetotransport oscillations observed on modulated 2D EGs can be understood within a quantum mechanical approach using the simplest approximations for both modulation potential and collision broadening. To indicate this, we first recall some features of the energy spectrum and then briefly introduce collision broadening and the corresponding transport theory. More detailed calculations are left for a future publication.

We treat the potential $V(x,y) = V_x \cos Kx + V_y \cos Ky$, with period $a = 2\pi/K$, as weak perturbation of an unmodulated system, which lifts the degeneracy of the LLs with energy eigenstates $|nk_y\rangle$ and eigenvalues $E_n = \hbar\omega_c(n + \frac{1}{2})$, where $\omega_c = eB/mc$ is the cyclotron frequency, but which does not couple different LLs. For the $1d$ case ($V_y = 0$), this has been shown to be an extremely good approximation[6,7] for the parameter values of interest ($V_x \sim 0.3$ meV, $a \sim 300$ nm, $E_F \sim 10$ meV) and for $B > 0.1$ T, and yields Landau bands with energies $E_n(k_y) = E_n + \mathcal{L}_n V_x \cos Kx_0$, with $x_0 = -l^2 k_y$ and $\mathcal{L}_n = \exp(-\frac{1}{2}X) \times L_n(X)$, where $X = \frac{1}{2} l^2 K^2$. The Laguerre polynomials L_n lead to an oscillatory dependence of the bandwidth on the quantum number n,[5,7] reflecting that a Landau state with cyclotron radius $R_n = l(2n+1)^{1/2}$ effectively senses the average value of the periodic potential over an interval $2R_n$. The zeros of the $L_n(X)$ yield condition (2) with $R_c = R_n$ for flat bands.

With an additional modulation in y direction, the potential matrix elements

$$\langle nk_y' | V_y \cos Ky | nk_y \rangle = \tfrac{1}{2} V_y \mathcal{L}_n (\delta_{k_y',k_y+K} + \delta_{k_y',k_y-K})$$

couple Landau states with center coordinates differing by integer multiples of $l^2 K$. Since all potential matrix elements in the nth LL have the common factor $\mathcal{L}_n$, the bandwidth oscillations are the same for the $2d$ ($V_y = V_x$) and for the corresponding $1d$ ($V_y = 0$) case, and, apart from the scaling factor $\mathcal{L}_n$, the internal energy structure is the same for all LLs. If condition (1) holds, the energy eigenvalues are defined on the magnetic Brillouin zone $|k_x| \leq \pi/aq$, $|k_y| \leq \pi/a$ and form p subbands (per LL) $E_{nj}(\mathbf{k})$ ($j = 1,\ldots,p$), which are q-fold degenerate.[2,3] Contrary to the unmodulated case, the velocity now has nonzero intra-LL matrix elements $\langle n;\mathbf{k}',j'|v_\mu|n;\mathbf{k},j\rangle$ which are diagonal in $\mathbf{k}$ but not in the subband index j. In the limit $V_y \to 0$, the $E_{nj}(\mathbf{k})$ becomes independent of k_x and the p subbands merge into a single band, $E_{nj}(\mathbf{k}) \to E_n + V_x \mathcal{L}_n \cos Kx_0$, where $0 \leq x_0 \leq qa$. For further details we refer the reader to Refs. 2 and 3.

To describe the broadening of the LLs due to scattering by impurities, we follow recent work for the $1d$ case[6,7,10] and introduce into the Green's function $G_{n\alpha}^-(E) = [E - E_{n\alpha} - \Sigma^-(E)]^{-1}$ a quantum-number independent self-energy [$\alpha = (\mathbf{k},j)$, a cutoff $n \leq 2E_F/\hbar\omega_c$ is implied]

$$\Sigma^-(E) = \Gamma_0^2 \sum_{n,j} (l^2/2\pi) \int d^2k\, G_{n,\mathbf{k},j}^-(E)\,, \tag{3}$$

which in the absence of modulation reduces to the self-consistent Born approximation (SCBA) for randomly distributed short-range scatterers.[11] With the spectral function $A_{n\alpha}(E) = \pi^{-1} \mathrm{Im} G_{n\alpha}^-(E)$, this yields for the density of states (DOS) of the nth LL (per spin) $D_n(E) = (2\pi)^{-2} \sum_j \int d^2k\, A_{n\alpha}(E)$. As a typical result, Fig. 2(a) shows $D_n(E)$ for a grid with $p/q = 5$ ($B = 0.23$ T for $a = 300$ nm), and for two values of the damping, in comparison with the corresponding results for $V_y = 0$.[12] We also calculated $D_n(E)$ for other flux ratios. The general result, previously found from coherent potential approximation calculations in the strong-modulation tight-

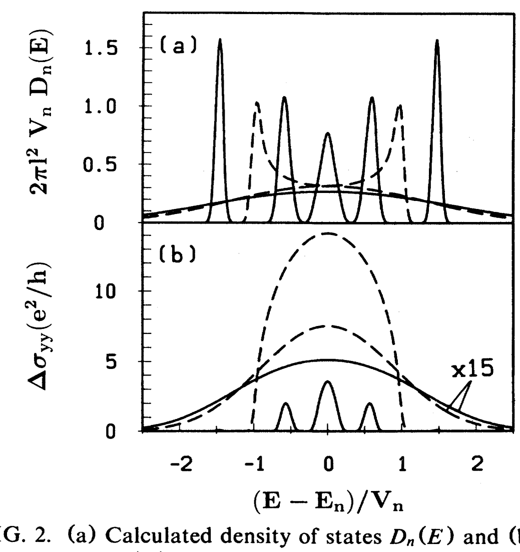

FIG. 2. (a) Calculated density of states $D_n(E)$ and (b) band conductivity $\Delta\sigma_{yy}(E)$ for one Landau level and two values of the collision broadening, $\Gamma_0/V_n = 1.0$ and 0.05. Solid (dashed) curves are for a grid (grating) potential with $V_x\mathcal{L}_n = V_y\mathcal{L}_n = V_n$ ($V_x\mathcal{L}_n = V_n$, $V_y = 0$) and $p/q = 5$. For $\Gamma_0/V_n = 1.0$ the internal band structure is not resolved, $D_n(E)$ and $\Delta\sigma_{yy}(E)$ [here $15\times\Delta\sigma_{yy}(E)$ is plotted] are similar for grid and grating. For $\Gamma_0/V_n = 0.05$, the resolved subband splitting dramatically reduces $\Delta\sigma_{yy}$ for the grid [with only tiny contributions from the narrow outer bands (near ± 1.5)].

binding limit,[13] is that, in the presence of collision broadening the fine structure (due to the many subbands for large p and q) is smeared out and only a coarse splitting into groups of subbands according to the Hofstadter-type energy spectrum[1,2] is resolved. The DOS appears as a continuous function of the magnetic field, in spite of the highly singular B dependence of the energy spectrum.

Our calculation of conductivities is based on Kubo's formulas,[11,14] which in the approximation consistent with Eq. (3) read[7,15] $\sigma_{\mu\mu}=\int dE[-f'(E)]\sigma_{\mu\mu}(E)$, with f' the derivative of the Fermi function and

$$\sigma_{\mu\mu}(E) = \frac{e^2\hbar}{2\pi}\int d^2k \sum_{n,n'}\sum_{j,j'} |\langle n';\alpha'|v_\mu|n;\alpha\rangle|^2 A_{n\alpha}(E)A_{n'\alpha'}(E)\,, \tag{4}$$

where $\alpha'=(\mathbf{k},j')$. We distinguish two contributions, $\sigma_{\mu\mu}=\Delta\sigma_{\mu\mu}+\sigma_{\mu\mu}^{sc}$, a band conductivity $\Delta\sigma_{\mu\mu}(E)$ arising from intra-LL contributions $(n'=n)$ which diverges in the absence of scatterers and vanishes for the unmodulated system, and an inter-LL $(n'\neq n)$ contribution $\sigma_{\mu\mu}^{sc}(E)$, which arises from scattering and is the only contribution in the unmodulated case. We consider here only the situation where both collision $(\sim\Gamma_0)$ and modulation $(\sim V_x\mathcal{L}_n)$ broadening are much smaller than $\hbar\omega_c$, and, consequently, the resistivities are $\rho_{xx}=\sigma_{yy}/\sigma_{yx}^2$ and $\rho_{yy}=\sigma_{xx}/\sigma_{yx}^2$ with $\sigma_{yx}=e^2N_s/m\omega_c$.

First we calculate $\Delta\sigma_{\mu\mu}$ under the assumption that the subband splitting of the LLs is not resolved. Since the intra-LL velocity matrix elements are proportional to the modulation potential, one then may replace $E_{n\alpha}$ by E_n in the spectral functions of Eq. (4) in order to calculate $\Delta\sigma_{\mu\mu}(E)$ to lowest order in the modulation. With the additional approximation

$$A_{n\alpha}(E)A_{n\alpha'}(E)\approx(\pi\Gamma_0)^{-1}\delta(E-E_n)\,, \tag{5}$$

which neglects the internal subband structure and the collision broadening of the LLs and effectively introduces a constant relaxation time $\tau=\hbar/\Gamma_0$, the sum over j and j' in $\Delta\sigma_{\mu\mu}(E)$ can be evaluated analytically. The result $\Delta\sigma_{yy}$ is independent of V_y and equals exactly the result for a unidirectional modulation in x direction obtained previously.[5,8,15] In the interesting range of temperatures, where k_BT is larger than $\hbar\omega_c$ but smaller than the energy separation $\Delta_\lambda=\frac{1}{4}m\omega_c^2a^2(\lambda-\frac{1}{4})$ of adjacent flat bands,[5,7] this result is $\Delta\sigma_{yy}\propto(V_x^2/\Gamma_0)\cos^2(2\pi R_c/a-\pi/4)$, independent of T. Extending Beenakker's[16] simplified quasiclassical calculation to the case of bidirectional modulation, one finds exactly the same result: the calculated band conductivity $\Delta\sigma_{yy}$ is independent of the modulation in y direction, in sharp disagreement with the experiment.

To understand the suppression of the band conductivity observed in the experiment, we have to take the peculiar subband splitting of the Hofstadter-type energy spectrum serious and to insert realistic values for the collision broadening. From the mobility at zero magnetic field one can estimate[7] that in the experiments shown in Fig. 1 the collision broadening is small enough to resolve the gross features of the Hofstadter spectrum well away from flat bands. If the splitting of subbands j' and j is resolved, the spectral functions do not overlap, and thus the corresponding nondiagonal matrix elements of the velocity do not contribute to $\Delta\sigma_{\mu\mu}(E)$. Then the band conductivity of the $2d$ system is considerably smaller than that of the corresponding $1d$ system, as is visualized for a typical situation in Fig. 2(b).

In the inter-LL contribution $\sigma_{\mu\mu}^{sc}$ to the conductivity we neglect the modulation effect on the velocity matrix elements, i.e., small corrections of the order $V_x\mathcal{L}_n/\hbar\omega_c$.[6,7] Then, the modulation affects $\sigma_{\mu\mu}^{sc}(E)$ only via the self-energy. To leading order in $|\Sigma^-/\hbar\omega_c|$, we obtain from Eqs. (3) and (4)

$$\sigma_{\mu\mu}^{sc}(E)=(e^2/h)\sum_n(2n+1)[2\pi l^2\Gamma_0D_n(E)]^2\,. \tag{6}$$

For zero modulation, this is a well-known result,[11] describing the SdH oscillations. In the interesting temperature range, $\hbar\omega_c<k_BT<\Delta_\lambda$, the Fermi function in the energy integral defining $\sigma_{\mu\mu}^{sc}$ can be replaced by the first term of the expansion $f'(E)=f'(E_n)+f''(E_n)(E-E_n)+\cdots$, so that $\sigma_{\mu\mu}^{sc}$ can be expressed in terms of an effective scattering rate

$$\tilde{\Gamma}_n=2\pi\int dE[2\pi l^2\Gamma_0D_n(E)]^2\,, \tag{7}$$

which oscillates as a function of n with maxima for flat bands.[17] For $k_BT\ll\Delta_\lambda$ the sum over n is easily performed to yield the T-independent Drude-type result $\sigma_{\mu\mu}^{sc}=(e^2\times N_s/m\omega^2)\tilde{\Gamma}_{n_F}/\hbar$, where $n_F=E_F/\hbar\omega_c$ and $E_F=N_s/D_0$ with the zero-B DOS $D_0=m/\pi\hbar^2$ has been inserted.

If Eq. (2) holds, the oscillating Landau bandwidth vanishes at the Fermi energy, the peaks of the DOS (near $E=E_F$) become high, $\sigma_{\mu\mu}^{sc}$ becomes maximum and the band conductivity minimum, since the intra-LL velocity matrix elements near E_F vanish. This explains the antiphase oscillations for high-mobility systems (small Γ_0)

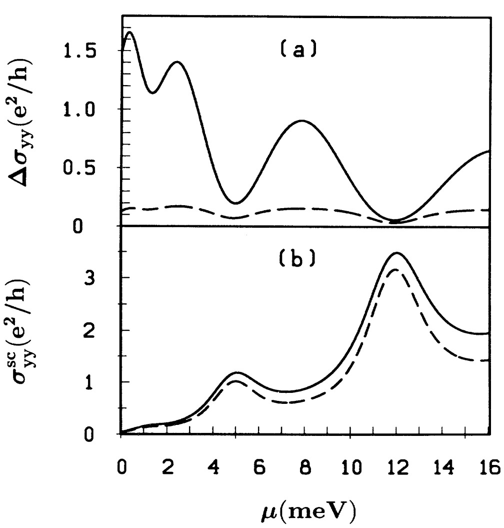

FIG. 3. (a) Calculated band and (b) scattering contribution to the conductivity vs chemical potential for temperature $T=5$ K, $B=0.23$ T, $a=300$ nm $(p/q=5)$, and $\Gamma_0=0.02$ meV. Solid (dashed) curves are for grid (grating) potential with $V_x=V_y=0.25$ meV $(V_x=0.25$ meV, $V_y=0)$.

with unidirectional grating modulation, where the band conductivity dominates, since $\Delta\sigma_{yy} \propto 1/\Gamma_0$ whereas $\sigma_{yy}^{sc} \approx \sigma_{xx}^{sc} \propto \Gamma_0$. We also obtain the same phase of the $\sigma_{\mu\mu}^{sc}$ oscillations for both unidirectionally and bidirectionally modulated systems, provided the electron density and the period of the modulation are the same. To understand the suppression of the band conductivity for systems with a bidirectional grid modulation, we have to assume that the collision broadening is so small that the subband splitting of the Hofstadter-type energy spectrum is partially resolved. This assumption is consistent with our knowledge about the strength of the modulation potential. Figure 3 demonstrates the antiphase behavior of the different contributions to the conductivity, and also the suppression of the band conductivity in the case of a grid modulation. If the collision broadening exceeds the subband splitting, nondiagonal velocity matrix elements between subbands of the same Landau level render the conductivity of the $2d$ system equal to that of the corresponding $1d$ one. We thus predict that in low-mobility samples the band conductivity will be of the same order of magnitude for both unidirectional and bidirectional modulation.

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