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Electrodynamic response of a harmonic atom in an external magnetic field

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We consider theoretically the long-wavelength magneto-optical response of an electron system, harmonically confined in two (quantum wire) or three (quantum dot or disk) spatial dimensions by external parabolic potentials. In particular, we prove explicitly that the resonance frequencies of such systems are independent of electron number and exactly the same as the corresponding bare resonance frequencies. This is the generalization of Kohn's theorem to harmonically confined structures. We discuss a number of recent experimental results in light of this exact result.

The electrodynamic response of an interacting-electron system (IES) in the presence of a confining potential is generally dominated by the collective motion of the electrons. In the microscopic description of such systems the resonance frequencies are shifted from the one-electron transition energies by many-body effects such as self-energy corrections, depolarization effects, and vertex or excitonic corrections. In the classical limit, the natural description is in terms of plasma effects and is treated by hydrodynamic theory. In either case, the electrodynamic response reveals little of the underlying one-electron states of the IES and the theories are messy, containing uncontrolled approximations. We present here a proof that, for a special class of IES's, the electrodynamic response to a uniform electric field can be solved exactly and contain direct information about the bare one-electron levels in the system.

The most well-known example of this type is Kohn's theorem[1] where the long-wavelength cyclotron-resonance frequency of a three-dimensional *translationally invariant* IES in a *jellium background* is known to be exactly the *bare* cyclotron resonance frequency $\omega_c = eB/m$, where B is the uniform external magnetic field and m the bare one-electron band mass (i.e., unrenormalized by the electron-electron interaction) of the system. The translational invariance of the IES (i.e., there can be no impurities or phonons breaking the translational invariance) ensures that the total Hamiltonian can be divided into a center-of-mass part and a relative coordinate part with the electron-electron interaction entering *only* the relative coordinate-dependent part of the Hamiltonian. Since long-wavelength external radiation couples only to the center-of-mass part of the motion, it follows directly that electron-electron interaction cannot affect the resonance frequency. In the many-body language, Kohn's theorem asserts the *exact* cancellation between the self-energy and the vertex diagrams for long-wavelength response of the translationally invariant three-dimensional IES. Of more relevance to the current context is a recent paper[2] by Brey, Johnson, and Halperin, who showed that a variant of Kohn's theorem exists for electrons confined in a one-dimensional parabolic potential (the parabolic potential well case) with free motion in the other two dimensions. In Ref. 2 it was shown that the response of this parabolically confined two-dimensional IES is at the bare harmonic-oscillator frequency, independent of electron-electron interaction effects. Experimentally,[3] the resonance frequencies in parabolic quantum wells are observed to be independent of the two-dimensional electron density, consistent with the theorem. In this paper, we obtain a generalized Kohn's theorem in the context of atomlike quantum dot or wire structures, where the confining parabolic potential is either three or two dimensional. There is current interest[4–13] in these systems as magneto-optical properties of various types of quantum dot and wire structures are being increasingly investigated with advances in growth and lithographic techniques. We first provide an explicit proof of a generalized Kohn's theorem for harmonically confined atoms (i.e., quantum dots) and then discuss a number of recent experimental results in this context. We note that the original Kohn's theorem[1] applies to a manifestly translationally invariant system whereas our generalization (and, that of Ref. 2) makes it valid for specific inhomogeneous (i.e., translationally *noninvariant*) situations where the background potential is quadratic. This generalization may be intuitively expected since a quadratic potential corresponds to a uniform jelliumlike charge background via the Poisson's equation.

In an *empty* isotropic three-dimensional harmonically confined atom, the one-electron transition energies are at the bare harmonic frequency ω_0. In the presence of finite number of electrons in this "atom," the one-particle confining potential will be substantially renormalized by self-consistent (mean-field) potential arising from the electrons themselves, and, consequently, the one-electron energy levels will change due to this Hartree potential. There will be additional many-body (self-energy) corrections (beyond the mean-field effect) due to the short-range exchange-correlation part of the electron-electron interaction. In fact, in the IES the single-particle energy levels do not strictly exist except in the quasiparticle sense. The question being addressed here is the following: what are the electromagnetic resonance response frequencies of the IES in the long-wavelength limit? The answer to this question, as we show here, is unique for a harmonic atom, namely, that the electrodynamic response frequency of a harmonic IES is the *bare* harmonic frequency, irrespective of electron-electron interaction effects. The application of a magnetic field to the system gives rise to another frequency ω_c in the usual way, leading to mode mixing effects. This conclusion is exact and completely independent of the number of electrons in the system or their mutual interactions.

We first consider an isotropic, harmonic atom (three-dimensional "quantum dots") confined by a three-dimensional potential $V(x,y,z)=\frac{1}{2}m\omega_0^2(x^2+y^2+z^2)$ in the presence of a uniform, external magnetic field B. Here, m is the bare, one-particle, electronic effective mass in the system. The Hamiltonian for the system can be written as

$$H=\sum_i \left[(1/2m)(p_{ix}^2+p_{iy}^2+p_{iz}^2)+\frac{\omega_c}{2}(x_i p_{iy}-y_i p_{ix}) + \frac{m\omega_0^2 z^2}{2}+\frac{m\Omega^2}{8}(x_i^2+y_i^2)\right] + \sum_{i<j} u(\mathbf{r}_i-\mathbf{r}_j) , \tag{1}$$

where u denotes the electron-electron interaction, $\mathbf{r}_i \equiv (x_i,y_i,z_i)$ is the position of the ith electron in the atom, $\mathbf{p}_i=(p_{ix},p_{iy},p_{iz})$ is the momentum of the ith electron, and, with no loss of generality, we choose the external magnetic field to be along the z axis described by a symmetric gauge $\mathbf{A}=(-By/2,Bx/2,0)$ in the x-y plane. The frequency Ω is given by

$$\Omega=[\omega_c^2+(2\omega_0)^2]^{1/2} . \tag{2}$$

At first, we neglect the electron-electron interaction [the last term in Eq. (1)] and solve the single-particle problem. The z motion is decoupled from the x-y motion and can be trivially solved. We concentrate now on the x-y part of the single-particle Schrödinger's equation defined by the term within the large parentheses of the Hamiltonian in Eq. (1). The problem can be solved in a cylindrical coordinate system giving the energy spectrum

$$E_{n,l}=\hbar\Omega\left[n+\frac{|l|+1}{2}\right]+\frac{l}{2}\hbar\omega_c , \tag{3}$$

where n is a non-negative integer, and l is the quantum number for the z component (l_z) of the angular momentum. Using the dipole selection rule ($\Delta l=\pm 1$) for the long wavelength absorption of electromagnetic radiation we obtain the resonance frequencies

$$E_\pm=\frac{\hbar}{2}(\Omega\pm\omega_c)=\hbar\omega_\pm \tag{4}$$

which are the same as the corresponding classical result.[6]

In the presence of the electron-electron interaction, $U=\sum_{i,j}u(\mathbf{r}_i-\mathbf{r}_j)$, we now prove that $\omega_\pm$ are still the resonance frequencies of the system. We introduce operators $\hat{a}_\pm$ and $\hat{b}_\pm$ defined by

$$\hat{a}_\pm=\sum_j\left[m\frac{\Omega}{2}(x_j\pm iy_j)\mp i(p_{jx}\pm ip_{jy})\right] , \tag{5}$$

$$\hat{b}_\pm=\sum_j\left[m\frac{\Omega}{2}(x_j\mp iy_j)\mp i(p_{jx}\pm ip_{jy})\right] . \tag{6}$$

It is easy to show that

$$[H,\hat{a}_\pm]=\pm\hbar\omega_+\hat{a}_\pm \tag{7}$$

and

$$[H,\hat{b}_\pm]=\pm\hbar\omega_-\hat{b}_\pm . \tag{8}$$

If $\Psi_{s,t}$ is an eigenstate of the interacting Hamiltonian with the eigenenergy $E_{s,l}$, then from Eqs. (7) and (8) we get

$$H\hat{a}_\pm\Psi_{s,t}=(\pm\hbar\omega_+ + E_{s,t})\hat{a}_\pm\Psi_{s,t} \tag{9a}$$

and

$$H\hat{b}_\pm\Psi_{s,t}=(\pm\hbar\omega_- + E_{s,t})\hat{b}_\pm\Psi_{s,t} . \tag{9b}$$

We infer from Eq. (9) that $\Psi_{s\pm1,t}=\hat{a}_\pm\Psi_{s,t}$ and $\Psi_{s,t\pm1}=\hat{b}_\pm\Psi_{s,t}$ are also eigenstates of H with the eigenvalues $E_{s\pm1,t}=E_{s,t}\pm\hbar\omega_+$ and $E_{s,t\pm1}=E_{s,t}\pm\hbar\omega_-$, respectively.

In the presence of external electromagnetic radiation propagating along the z direction, we must add the following perturbation to the Hamiltonian:

$$H'=\sum_j Ee^{-i\omega t}(x_j+iy_j) = Ee^{-i\omega t}\left[\frac{1}{m\Omega}\right](\hat{a}_+ + \hat{b}_-) , \tag{10}$$

where E is the electric field, ω the frequency of the external radiation, and we have considered right-handed circular polarization. The perturbation H', defined by Eq. (10), can only connect the state $\Psi_{s,t}$ with the states $\Psi_{s+1,t}$ and $\Psi_{s,t-1}$ because it is a linear combination of a and b operators. Similarly, left-handed circularly polarized radiation will only connect the state $\Psi_{s,t}$ with $\Psi_{s,t+1}$ and $\Psi_{s-1,t}$. Since an arbitrary electromagnetic field can be decomposed into a combination of right-handed and left-handed polarization, it is clear that an arbitrary electromagnetic perturbation will only connect the system $\Psi_{s,t}$ with states $\Psi_{s\pm1,t}$ and $\Psi_{s,t\pm1}$, which are also eigenstates of the unperturbed interacting Hamiltonian H with energies shifted by $\pm\hbar\omega_\pm$. We, therefore, conclude that the resonance frequencies *for the interacting system* will be exactly the same as those of the noninteracting system, namely, $\omega_\pm$ defined by Eq. (4). As is clear from the above discussion, this result depends only on the commutation properties of the electromagnetic perturbation H' [of Eq. (10)] with the interacting Hamiltonian H (through the operators $\hat{a}_\pm$ and $\hat{b}_\pm$) and is exact. Thus, independent of the number of electrons in the harmonic atom, it can absorb long-wavelength electromagnetic radiation only at the *bare* frequencies.

Similarly we can show that the same result holds for an anisotropic harmonic atom (where the harmonic frequencies in the x,y,z directions are different) as well as for a situation (of current experimental relevance) where the confinement in the x-y plane is harmonic while that along the z direction, $V(z)$, is arbitrary but with the magnetic field oriented *exactly* along the z direction. For the anisotropic harmonic atom, the commutation properties discussed above still hold because the classical energy can still be separated out into a center-of-mass (the one-particle part) and a relative coordinate (the interacting part) part, but the eigenmodes cannot be written down explicitly because it involves the following cubic equation in ω^2:

$$\omega^6-\omega^4(\omega_c^2+\omega_x^2+\omega_y^2+\omega_z^2) + \omega^2[\omega_c^2(\omega_x^2\sin^2\theta\cos^2\phi+\omega_y^2\sin^2\theta\sin^2\phi+\omega_z^2\cos^2\theta) + (\omega_x^2\omega_y^2+\omega_z^2\omega_x^2+\omega_y^2\omega_z^2)]-\omega_x^2\omega_y^2\omega_z^2=0 , \tag{11}$$

where $\omega_{x,y,z}$ is the harmonic frequency in the three directions and (θ,ϕ) are the spherical polar angles for the orientation of the magnetic field.

For an anisotropic two-dimensional system, or, an asymmetric quantum disk (i.e., when the confinement in the z direction is so strong, that the z width can effectively be considered zero) this equation can be solved to give the following resonance frequencies (with $\omega_c = eB\cos\theta/mc$) for the system:

$$\omega_{\pm}^2 = \frac{\omega_x^2+\omega_y^2+\omega_c^2 \pm [\omega_c^4+2\omega_c^2(\omega_x^2+\omega_y^2)+(\omega_x^2-\omega_y^2)^2]^{1/2}}{2} . \tag{12}$$

This result also applies to the case of an arbitrary confining potential $V(z)$ in the z direction and a "tilted" magnetic field $\mathbf{B}$ directed at an angle θ with respect to the z direction, provided that the confining length in the z direction is much shorter than the other relevant lengths—in particular, the width of the confining wave function in the z direction must be substantially smaller than the magnetic length and the wave-function widths in the harmonic x-y directions. Clearly, the theorem breaks down in high magnetic field in this tilted situation.

For the sake of completeness, we briefly discuss the case of one-dimensional parabolic quantum wires in a perpendicular external magnetic field. We assume the wires are built on zero-thickness (in the z direction) x-y plane by adding a confinement in the y direction. The confining potential is $\frac{1}{2}m\omega_0^2 y^2$. Thus, the electrons are free in the x direction, and completely (i.e, δ-function-like) or harmonically confined in the z direction as well as being harmonically confined in the y direction with the external magnetic field along the z direction. We use the Landau gauge. The Hamiltonian of the system is then given by

$$H = \sum_i [(1/2m)(p_{ix}^2+p_{iy}^2) - \omega_c y_i p_{ix} + (m/2)\tilde{\omega}^2 y_i^2] + U , \tag{13}$$

where $\tilde{\omega} = (\omega_c^2+\omega_0^2)^{1/2}$. We define operators,

$$\hat{c}_{\pm} = \sum_j [m\tilde{\omega} y_j \mp i p_{jy} - (\omega_c/\tilde{\omega}) p_{jx}] . \tag{14}$$

It is straightforward to prove,

$$[H,\hat{c}_{\pm}] = \pm\hbar\tilde{\omega}\,\hat{c}_{\pm} . \tag{15}$$

Applying a similar argument as before, we can show that if Ψ_n is an eigenstate of the Hamiltonian (13) with energy E_n, then $\hat{c}_{\pm}\Psi_n$ is also an exact eigenstate of the Hamiltonian with the energy $e_n \pm \hbar\tilde{\omega}$. The resonance frequencies for the IES are then at $\tilde{\omega} = (\omega_c^2+\omega_0^2)^{1/2}$ which does not depend on the electron density of the system.

In the rest of this paper, we briefly discuss a number of recent magneto-optical experimental results in IES in light of these exact results. Obviously, the validity of the theorem in real systems depends on how close the confining bare potential is to quadratic. It is possible at present to obtain a two-dimensional electron gas confined in a nearly perfect (one-dimensional) parabolic potential in $Ga_xAl_{1-x}As$ heterostructures as demonstrated in Ref. 3 (and references therein). In these systems, the far-infrared optical excitation is found[3] to obey the theorem as discussed by Brey, Johnson, and Halperin.[2] In the case of two- and three-dimensional (i.e., quantum wire, quantum disk, and, quantum dot) confinement, however, the bare potential is generally not exactly quadratic and only in specific cases[11] it can be approximated as such.

Three types of three-dimensional confinements are currently being investigated. Electrons on the surface of liquid He are not charge compensated and external electrostatic fields are used[4,5] to confine the carriers. The electron systems obtained in this way are generally large and contain on the order of 10^7-10^8 electrons in pools of 1 cm diameter. Selective etching of a semiconductor heterojunction obtained by photo- or electron-beam lithography is used to define geometrical confinement whose size varies from μm (Ref. 6) to nm (Ref. 7). Finally, confinement is also obtained by the use of cross-grid gates formed at the surface of $GaAs/Al_xGa_{1-x}As$ heterojunctions,[8,13] of InSb (Ref. 9) and Si metal-oxide-semiconductor[10] (MOS) devices. These devices provide a selective depletion field with the same spatial periodicity as the gate and allow a continuous tunability of the confining potential.[13] In all these cases, and when isotropy in the plane can be assumed, the confining electrostatic potentials can be expanded in the series

$$V(R) = \sum_k \alpha_k (r/a)^{2k} \tag{16}$$

which converges for $r \le a$, where a is the physical radius of the confinement. It is obvious from the above equation that the generalized Kohn's theorem remains valid in situations where the radius of the electron distribution R (which depends on the total number of electrons N_0) is such that $R \ll a$. Sikorski and Merkt[9] have recently studied quantum dots in InSb with $N_0 \le 20$ electrons (where N_0 is the number of electrons in the dot). Their experimental results are completely consistent with the assumption of a two-dimensional harmonic atom. They observed a resonant mode with frequencies given by the functional form of Eq. (4) *and,* to within their experimental accuracy, the frequencies are *independent* of N_0. These two findings strongly suggest that in the region where the electrons are confined, the potential is essentially quadratic for this structure. Demel *et al.*,[7] Liu *et al.*,[8] and Alsmeier, Batke, and Kottaus[10] have explored situations with $20 < N_0 < 350$ bridging the region between quantum dots and classical discs of electrons. In these studies, a dominant mode was observed with resonant frequencies consistent with Eq. (4). But, in all three of these studies striking deviations from the above theorem are observed either in the form of an additional resonance[7,8,13] or in the form of a dependence[10] of the main resonance on N_0.

An additional mode with a weaker oscillator strength was found at higher frequencies, bearing a similar field dependence as described by Eq. (4).[7,8] The very fact that this exact mode was observed[7,8,13] demonstrates that in these systems the lateral confinement was not strictly parabolic. However, assuming that the nonparabolic component can be considered a small perturbation, the main observed mode can still be viewed, in the context of the generalized Kohn's theorem, as "the center-of-mass mode" which corresponds to the oscillation of the *static* electron distribution. The extra mode, then, corresponds to an internal excitation of the electron distribution. The above assumption is, in fact, reasonable for cross-grid gated systems as shown numerically by Kumar, Laux, and Stern.[11] In the case where the quantum dots are defined physically by selective etching, the electrostatic potential depends on the remote ionized donors, and, on electrons trapped on the surface in unintentional perimetric defects. It is likely that these defects deplete a region around the perimeter, making it possible to have $R \ll a$ as observed in Ref. 7. Demel *et al.*[7] interpreted all their observed modes as magnetoplasmon modes confined in a disk whose frequencies can be calculated using classical magnetohydrodynamic theories.[5,12] They observed a coupling between the center-of-mass and the internal mode which manifests itself in the lifting of the excitation degeneracy at mode crossing. They interpret the mode coupling in terms of nonlocal effects (and, in view of the generalized Kohn's theorem, nonparabolic confinement is also required). If the observed modes can be related to magnetoplasmon theory (as is argued in Ref. 7), then these modes bear different angular momentum quantum numbers.[5,12] For this reason, this coupling is not allowed in systems with circular symmetry.

In the work of Alsmeier, Batke, and Kottaus[10] no extra mode was detected. However, the frequency ω_0 was found to be *dependent* on N_0. This fact by itself signals a breakdown of the generalized Kohn's theorem. They also observed a small deviation of the ω_- mode from the functional dependence of Eq. (4). Such a deviation is also not expected in the classical magnetoplasmon[5,12] theories (where the bare potential is far from parabolic).

Finally it is interesting to explore the case of large electron disks ($N_0 > 20\,000$) (Refs. 4–6) since in that case a clear breakdown of Kohn's theorem is expected for $R \simeq a$. Mast, Dahm, and Fetter[4] and Glattli *et al.*[5] have studied such large confined systems on the liquid-He surface. Reference 5, for example, gives a dramatic illustration of this breakdown with the observation of more than fifteen magnetoplasmon modes. It is important to note, however, that these systems are probed with nonuniform electromagnetic fields and the long-wavelength approximation of the theorem does not apply. Thus, the breakdown is due to both the nonparabolic confinement as well as to the nonuniformity of the probe. The nonuniformity of the exciting field by itself allows 11 of the 15 resonances observed in Ref. 5. (In a system with circular symmetry, a uniform field can only excite normal modes with angular momentum quantum number equal to ± 1.) For this reason it is interesting to consider large disks of electrons on semiconductors since they are usually probed with uniform far-infrared radiation. Allen, Stormer, and Hwang[6] carried out such measurements on 3-μm-diameter disks containing about $N_0 = 26\,000$ electrons. They observed[6] a resonant mode following the dependence of Eq. (4). No obvious observation of the breakdown of the theorem could be observed in their results. It is important to note, however, that the functional dependence of the resonant frequency by itself does not guarantee the parabolicity of the well. The breakdown in this case would be signaled through the observation of higher-frequency magnetoplasmon modes. The first additional mode is expected around a frequency of $3\omega_0$.[5,12] Allen, Stormer, and Hwang[6] interpreted their observed mode in terms of a classical depolarization field of a uniformly charged oblate ellipsoid. It is interesting to note that this analysis corresponds exactly to the case of the quantum harmonic atom and, therefore, is fully consistent with the generalized Kohn's theorem. For this case the areal electron density distribution is $n_s(r) = \frac{3}{2} N_0/(\pi R^2)(1 - r^2/R^2)^{1/2}$. This analysis, which is apparently quantitatively quite satisfactory, does not lead to the correct density profile in a large charge compensated electron disk (the density should be uniform). The observed mode in Ref. 6 is better described as a magnetoplasmon in a two-dimensional uniform electron gas in a disk geometry.[12]

In conclusion, we have generalized Kohn's theorem to quantum dots, disks, and wires by showing explicitly that the long-wavelength magneto-optical response of harmonically confined IES occurs exactly at the bare resonance frequencies of the system, independent of the electron density. This result sheds light on a number of recent experimental results in low-dimensional systems.

Note added: We have become aware of several papers published subsequent to the submission of our manuscript with similar results: P. A. Maksyn and T. Chakraborty, Phys. Rev. Lett. **65**, 108 (1990); F. M. Peeters, Phys. Rev. B **42**, 1486 (1990); A. V. Chaplik and A. V. Govorov (unpublished).

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