Resonant tunneling via quantum bound states in a classically unbound system of crossed, narrow channels

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We consider the ballistic transport of noninteracting electrons in intersecting narrow channels of finite length. The structure we study consists of two perpendicular channels, one of which connects two reservoirs of two-dimensional (2D) electrons. When a weak potential difference is applied to the 2D-electron-gas regions, ballistic transport of electrons occurs within subbands as for a single channel. In addition to the subband transport, we predict that there are sharp resonances associated with bound quantum states located at the intersection of the channels. For longer channels the conductance G associated with this kind of resonant tunneling appears to be quantized as $G=2e^2N/h$, where $N=1$ or 2. In addition to the peaks in G associated with resonant tunneling, there are also deep "antiresonances" that severely distort the quantized plateaus in G obtained for a single channel.

I. INTRODUCTION

Current lithographic and etching techniques make it possible to shape quasi-two-dimensional (2D) electrons at the interface of semiconductors into nanopatterns such as very narrow channels, grids, rings, dots, etc.[1–4] Because of the smallness of such structures quantum-mechanical effects are strongly manifested. For example, in narrow channels the de Broglie wavelength can be of the same order as the width of the channel itself. Hence quantization of the transverse motion becomes an important issue. Electrons propagate in one-dimensional subbands, of which only a few may be occupied. Transport properties of electron waveguides must therefore be discussed in terms of such subbands. In high-mobility samples like $Al_xGa_{1-x}As/GaAs$ the mean free path exceeds several μm. In short channels, connecting, e.g., two 2D reservoirs, one therefore encounters ballistic transport and a remarkable quantization of the conductance, $G=2e^2N/h$, where N is the number of occupied 1D subbands.[5,6] N may be altered by varying the width of the channel via an applied gate voltage.

Another remarkable feature of narrow channels has recently been studied by Peeters[7,8] and Schult, Ravenhall, and Wyld,[9] who considered two perpendicular, perfect channels of infinite length. Quantum-mechanical calculations show that bound states reside at the intersection of the two channels. From a classical point of view the potential is open. The quantum-mechanical trapping of electrons in such a potential may thus seem less obvious and appears to be a phenomenon that has attracted little attention in the past. On the other hand, the existence of trapped modes in waveguides for microwaves has been known for many years. With the new possibilities of also fabricating narrow electron waveguides it is natural that the problem of trapped waves now also surfaces in a quantum-mechanical context. The purpose of the present work is to explore the possibility of observing, at least in principle, the bound states in two intersecting electron waveguides by resonant tunneling. From a conceptual point of view it is interesting that one may encounter resonant tunneling also in a situation where there are no classical potential barriers to be penetrated. In passing we will also recover how the transport in crossed channels differs very much from the case of a single channel.

In Sec. II we present the model and the theoretical framework. In principle the formalism is elementary, but we prefer to present it in quite some detail because we believe it to be useful to general readers. Section III gives calculational aspects and numerical results. In particular it is shown that the bound states give rise to sharp peaks in the current. These peaks are associated with tunneling. Section IV, finally, contains a brief summary and comments.

II. THEORETICAL MODEL

There are a number of recent theoretical studies[10] of charged-particle motion in intersecting narrow wires (four-terminal junctions). The emphasis is on magnetotransport and the quenching of the low-field Hall resistance. Channels are assumed to be infinitely long and there is a finite magnetic field B. In the present case, however, we wish to probe bound states by means of tunneling. It is then essential that channels are of finite length. We will also let $B=0$ since we wish to focus attention to open potentials. The geometry that we will consider is given in Fig. 1, in which the cross should have a size consistent with some "effective" size of the bound-state wave functions, i.e., these functions should fit well into the crossbar region.

The detailed shape of the potential which confines the

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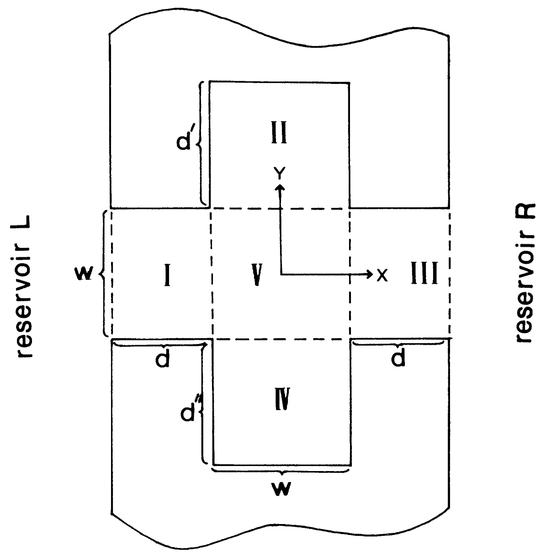

FIG. 1. Schematic representation of the crossbar structure connecting left and right reservoirs of 2D electrons with equal Fermi energy. When the potential difference is applied electrons flow from L to R. The regions I–V are used in the expansion of wave functions. The size of the crossbar region is assumed to be roughly the same as the "effective" extension of the bound-state wave functions of Schult, Ravenhall, and Wyld (Ref. 9). Therefore the cross may appear to electrons at the bound-state energies more or less to be infinite. As drawn here the crossbar geometry could, of course, also be referred to as a quantum dot or cavity.

electron to the channels is not important in determining the very presence of bound states at the intersection and the associated resonant tunneling. We will therefore assume an inifinite square-well lateral-confinement potential of width w and well bottom at energy E_0. For simplicity we let E_0 take the same value as the bottom of the surrounding 2D-electron-gas regions. This is a reasonable assumption except for close to pinchoff, but suffices for the considerations here. Choosing $E_0=0$ we therefore have the Hamiltonian $H=-\hbar^2(\partial^2/\partial x^2+\partial^2/dy^2)/2m^*$ outside the barriers, with the boundary condition that the wave function vanishes at the onset of the infinite barriers; m^* is the effective mass.

When a weak potential difference is applied to the 2D reservoirs we have to consider the process by which electrons are injected into the channel connecting the 2D regions and emitted from it. Doing so we will use Kirczenow's[11] analogous ballistic model for a single short channel in between two 2D reservoirs. Consider a free electron with wave vector $\mathbf{k}=(\kappa,k)$ and energy $\varepsilon(\mathbf{k})=\hbar^2(\kappa^2+k^2)/2m^*$ incident on the channel opening from the left. For $x<-(d+w/2)$, its wave function can be written as

$$\psi_L(x,y)=e^{i\kappa x+iky}+\int_{-\infty}^{+\infty}dk'\;A_L(k')e^{-i\kappa' x+ik'y}\;, \tag{1}$$

where κ' may be either real or imaginary, i.e., evanescent waves are included. The quantity $\kappa'=[2m^*\varepsilon(\mathbf{k})/\hbar^2-(k')^2]^{1/2}$ is real and positive for $|k'|<(2m^*\varepsilon)^{1/2}/\hbar$, while for $|k'|$ greater than this value we choose κ'/i to be positive.

For the emitted electron in $x>(d+w/2)$, the wave function is

$$\psi_R(x,y)=\int_{-\infty}^{+\infty}dk'\;A_R(k')e^{i\kappa' x+ik'y}\;. \tag{2}$$

In the channel region I in Fig. 1 the wave function may be expanded in terms of exact solutions as

$$\psi_{\rm I}(x,y)=\sum_n (B_{{\rm I},n}e^{q_n x}+C_{{\rm I},n}e^{-q_n x})\times\sin\left[\frac{n\pi}{w}(y+w/2)\right] \tag{3}$$

with $n=1,2,3,\ldots$. If the energy of the injected electron is less than a sublevel $E_n=\hbar^2(n\pi/w)^2/2m^*$ the quantity q_n is real, $q_n=[E_n-2m^*\varepsilon(\mathbf{k})/\hbar^2]^{1/2}$. In the opposite case, $\varepsilon(\mathbf{k})>E_n$, we write $q_n=i[2m^*\varepsilon(\mathbf{k})/\hbar^2-E_n]^{1/2}$, i.e., exponential states turn into freely propagating states. The wave function $\psi_{\rm III}(x,y)$ associated with channel region III follows from Eq. (3) by replacing index I by III. In the usual way ψ_L is now matched to $\psi_{\rm I}$ at $x=-(d+w/2)$ and ψ_R to $\psi_{\rm III}$ at $x=(d+w/2)$ by requiring that amplitudes and derivatives with respect to x are equal. $A_L(k')$ and $A_R(k')$ may be eliminated[11] which results in

$$\sum_n\left[\left[T_{m,n}+\frac{q_n w}{2}\delta_{n,m}\right]e^{-q_n(d+w/2)}B_{{\rm I},n}+\left[T_{m,n}-\frac{q_n w}{2}\delta_{n,m}\right]e^{q_n(d+w/2)}C_{{\rm I},n}\right]=2i\kappa e^{-i\kappa(d+w/2)}M_{k,m}\;, \tag{4a}$$

$$\sum_n\left[\left[T_{m,n}-\frac{q_n w}{2}\delta_{n,m}\right]e^{q_n(d+w/2)}B_{{\rm III},n}+\left[T_{m,n}+\frac{q_n w}{2}\delta_{n,m}\right]e^{-q_n(d+w/2)}C_{{\rm III},n}\right]=0\;, \tag{4b}$$

where

$$M_{k,n}=\int_{-w/2}^{+w/2}dy\;\sin\left[\frac{n\pi}{w}\left[y+\frac{w}{2}\right]\right]e^{iky} \tag{5}$$

and

$$T_{m,n}=i\int_{-\infty}^{+\infty}\frac{dk'}{2\pi}\kappa' M_{k',m}M_{-k',n}\;. \tag{6}$$

In the case of a single channel, i.e., if we remove the stubs in Fig. 1 ($d'=d''=0$), Eqs. (4a) and (4b) are identical with Kirczenow's expressions.[11]

We now turn to the interior of the cross. When expanding and matching wave functions we may then follow Schult, Ravenhall, and Wyld[9] who considered the rather similar case of two infinitely long, perpendicular channels. The difference is that both types of exponential solutions, decreasing and increasing, must be allowed for here as in Eq. (3). In regions II and IV the combination of the two types of solutions is, however, fixed by the condition that $\psi_{\rm II}(x,y)$ and $\psi_{\rm IV}(x,y)$ must vanish at the end of the stubs, i.e.,

$$\psi_{\rm II}(x,y)=\sum_n D_{{\rm II},n}(e^{q_n[y-(d'+w/2)]}-e^{-q_n[y-(d'+w/2)]})\sin\left[\frac{n\pi}{w}(x+w/2)\right] \tag{7}$$

and similarly for $\psi_{\rm IV}(x,y)$. In the central region V, finally, the expansion reads[9]

$$\psi_{\rm V}(x,y)=\sum_n \sin\left[\frac{n\pi}{w}(y+w/2)\right](E_n e^{q_n x}+F_n e^{-q_n x})+\sum_n \sin\left[\frac{n\pi}{w}(x+w/2)\right](G_n e^{q_n y}+H_n e^{-q_n y}) \; . \tag{8}$$

Equating amplitudes at $x=-w/2$ we have from Eqs. (3) and (8)

$$B_{{\rm I},n}e^{-q_n w/2}+C_{{\rm I},n}e^{+q_n w/2}-E_n e^{-q_n w/2}-F_n e^{q_n w/2}=0 \; . \tag{9}$$

When matching the derivatives it is convenient to Fourier expand the exponentials $e^{\pm q_n y}$ in the interval $-w/2<y<w/2$ and then equate Fouirer coefficients, which gives

$$q_n(B_{{\rm I},n}e^{-q_n w/2}-C_{{\rm I},n}e^{q_n w/2}-E_n e^{-q_n w/2}+F_n e^{q_n w/2})-\sum_m \left[\frac{m\pi}{w}\right][G_m f_n(q_m)+H_m f_n(-q_m)]=0 \; , \tag{10}$$

where

$$f_n(\pm q_m)=\frac{2}{w}\int_{-w/2}^{w/2}dy\, e^{\pm q_m y}\sin\left[\frac{n\pi}{w}(y+w/2)\right] \; . \tag{11}$$

Similar relations as in Eqs. (9) and (10) are obtained from matching at the remaining boundaries. If $N_{\rm max}$ transverse modes are included in the expansions of ψ there are $10N_{\rm max}$ linear equations determining the (complex) expansion coefficients $\mathbf{C}=(B_{{\rm I},n};C_{{\rm I},n};B_{{\rm II},n},\ldots;H_n)$.

The current J at $T=0$ K is evaluated as outlined by Kirczenow.[11] Assume that there is a weak potential difference V between the two 2D reservoirs, which causes a flow of electrons from left to right. Because of the Pauli principle only incident waves in the narrow window (E_F-eV, E_F) at the Fermi energy E_F will contribute to the current. The conductance G is then

$$G\left[\frac{h}{2e^2}\right]=-\frac{m^*}{eh}\int_{-\pi/2}^{\pi/2}d\varphi\, j(\mathbf{k})\Big|_{|\mathbf{k}|=k_F} , \tag{12}$$

where $j(\mathbf{k})$ is the current associated with an incident wave with $\mathbf{k}=k_F(\cos(\varphi),\sin(\varphi))$ and k_F is the Fermi wave number. If we use $\psi_{\rm I}$ of channel I to evaluate $j(\mathbf{k})$ we have

$$\begin{aligned} j(\mathbf{k})&=\int_{-w/2}^{w/2}dy\frac{e\hbar}{m^*}{\rm Re}\left[\psi_{\rm I}^* i\frac{\partial}{\partial x}\psi_{\rm I}\right]\\ &=-\frac{e\hbar}{m^*}\frac{w}{2}\sum_n {\rm Im}[q_n(B_{{\rm I},n}+C_{{\rm I},n})^*(B_{{\rm I},n}-C_{{\rm I},n})] \; . \end{aligned} \tag{13}$$

III. NUMERICAL CALCULATIONS AND RESULTS

We now turn to calculational aspects and actual results. In practice the number of basis functions in the expansions of $\psi_{\rm I},\ldots,\psi_{\rm V}$ must be finite. In general we have found good convergence, which means that the inclusion of $N_{\rm max}\simeq 6-8$ transverse modes suffices quite well for our purposes. If the two stubs in Fig. 1 are removed by choosing $d'=d''=0$ in the present formalism we return to the case of a single ballistic channel already analyzed in detail by Kirczenow.[11] We have therefore used this limit to test our numerical results against his and found good agreement in general. The computed conductance is essentially steplike and in gross agreement with observations. Superimposed on the "ideal" conductance plateaus $G=2e^2N/h$ there is an oscillatory pattern due to longitudinal resonant states. For finite d' and d'' this picture is considerably modified.

The calculated conductance in the crossbar geometry in Fig. 1 is displayed in Figs. 2(a)–2(d) as a function E_F while $d^{(',")}$ and w are kept constant. In practice one would vary w rather than E_F but the qualitative picture of filling successive subbands would remain the same. The calculations have been performed with $m^*=0.067m_0$ which is appropriate for the $Al_xGa_{1-x}As/GaAs$ interface. We first discuss the two sharp peaks at energies E' and E''. These are located at the energies at which bound states at the intersection are to be expected according to Schult, Ravenhall, and Wyld,[9] namely $E'=0.66E_1$ and $E''=0.93E_2$, where $E_1=\hbar^2(\pi/w)^2/(2m^*)$ and $E_2=4E_1$ are the two lowest sublevels. The two peaks thus correspond to resonant tunneling via such bound states. At the lowest peak the conductance is $G=2e^2/h$. Also at the second peak the conductance takes a quantized value, $G=4e^2/h$, provid-

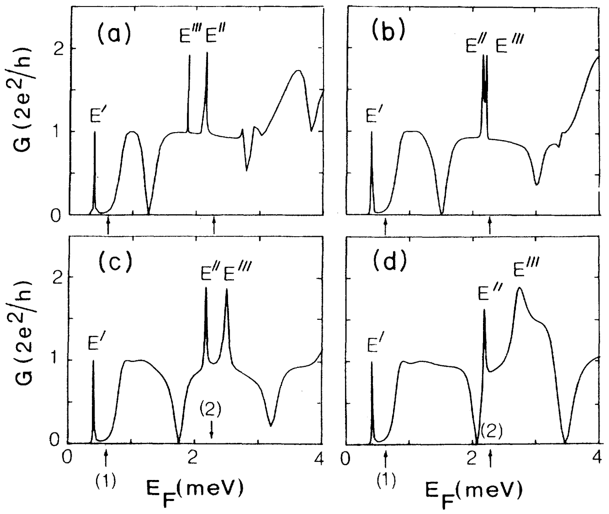

FIG. 2. Calculated conductance in units of $(2e^2/h)$ as a function of the Fermi energy (in meV). The length of the channel connecting the 2D regions and its width are kept constant, $w=100$ nm and $d=50$ nm, and the effective mass is $m^*=0.067m_0$. Cases (a)–(d) refer to $d'=d''=75$, 60, 50, and 40 nm, respectively. The positions for the two lowest sublevels E_1 and E_2 are indicated by arrows labeled (1) and (2). E', E'', and E''' denote bound-state energies.

ed the stubs are sufficiently extended, e.g., $d'=d''=90$ nm. In addition to the resonance peaks we notice how the quantized plateaus associated with a single channel have now been washed out. Strong interference effects, of a kind already found for four-terminal junctions[10] and for T-shaped electron waveguides,[12] give rise to a vanishing conductance at $E_F\sim 1.2$ meV.

The positions of the peaks marked by E' and E'' in Figs. 2(a)–2(d) are insensitive to the extension of the sidearms. The reason for this is that the wave functions are well localized at the center of the cross and thus hardly extend to the far end of the stubs. Consequently, electrons in these states effectively behave as for an infinite cross. Our main objective has been to show how these kinds of states[9] can give rise to resonant tunneling. In addition to the resonance peaks at E' and E'' there is, however, a third peak at E''' in Figs. 2(a)–2(d). This peak behaves quite differently. Thus it is sensitive to variations in d' and d''. The reason for this is that the corresponding wave function is mainly localized in the stubs. To a very rough approximation we have $E'''=\hbar^2/2m^*[(\pi/d')^2+(\pi/w)^2]$, i.e., simple quantization in a box. Presumably there are more states of this kind at higher energies, but we will not pursue this issue here, since our main concern is with bound states related to an infinite cross. The state E''' would not appear in a four-terminal probe.

While the resonance peaks at E' and E'' remain sharp as d' and d'' decrease, the peak at E''' becomes considerably broadened and eventually fades away as the sidearms are made progressively shorter. The reason for this is that E''' develops into a quasibound or resonant state when it is pushed into the second subband. We have investigated this point by considering the possibility of bound states in an infinitely long channel with stubs as in Fig. 1 ($d\rightarrow\infty$). The expansions of $\psi_{\rm I}$ and $\psi_{\rm III}$ then have to be modified such that only exponentially decaying or outgoing waves are included. Equations (1), (2), (4a) and (4b) will not enter the discussion at all. Matching of wave functions as above, but now at the intersection only, leads a set of linear homogeneous equations for the expansion coefficients. The requirement for nontrivial solutions and bound states is that the corresponding determinant vanishes. Proceeding in this way we have reconfirmed that the two sharp peaks at E' and E'' actually correspond to the bound states predicted by Schult, Ravenhall, and Wyld.[9] Our calculations also show that for shorter wings, $d'=d''\sim 50$–60 nm, the state at E''' ceases to be a bound state in the sense that the determinant cannot be made to vanish in the appropriate energy range. Rather it transforms into a quasibound state. Also the remaining bound states will eventually experience the same fate as d' and d'' get sufficiently small. This brings previous theories of magnetotransport in four-terminal junctions into mind.[10] In these theories only freely propagating electrons could carry a current so that resonant rather than bound states at the intersection were probed. Apart from being at higher energies, these states are reminiscent of the bound states discussed here.

IV. BRIEF SUMMARY

In summary we have shown by an exact model calculation that the bound states in a crossbar geometry in between two 2D reservoirs give rise to resonant tunneling somewhat akin to transverse tunneling through double barriers in semiconductor heterostructures[13,14] and resonant transmission through zero-dimensional states in a one-dimensional electron interferometer at high magnetic fields.[15] For sufficiently long sidearms the conductance is quantized as $G=2e^2N/h$ where $N=1$ or 2. We therefore suggest that measurements of the tunneling current could be a way of observing such bound states. We recall, however, that the calculations here are based on a very idealized model potential. For example, in a real semiconductor structure corners are likely to be rounded rather than sharp.[16] For this reason it would be of interest to improve the present model by introducing a smoother lateral confinement. While interference patterns may turn out to be more smeared, peaks associated with resonant tunneling should remain intact. Only the position should be shifted.

Other factors that would complicate measurements are, of course, disorder due to scattering centers and variations in the channel width. Close to pinch off such factors may lead to badly defined channels and low mobility. Inaccurate lithography could also complicate the picture in the sense that the second bound state at E'' requires that the widths of the two channels are equal. If the restriction to equal widths is relaxed, the state with E'' thus disappears, but at the same time new ones turn up under appropriate conditions.[8] These new models also give rise to resonant tunneling in the way we have discussed here. In summary one may thus remain optimistic about the possibility of measuring resonant tunneling in the present type of semiconductor structures.

Note added in proof. The quantum-mechanical trapping of electrons has also been studied theoretically by J. L. D'Amato, H. P. Pastawski, and J. F. Weisz [Phys. Rev. B **39**, 3554 (1989)], who considered T-shaped circuits and finite rings with leads in a tight-binding model. Most recently structures consisting of a short channel with one to four stubs (a finite lattice of T:*s*) have been studied experimentally by R. J. Haug, K. Y. Lee, and J. M. Hong [in *Nanostructures: Fabrication and Physics,* Proceedings of Symposium Y, 1990 Fall Meeting of the Materials Research Society, edited by S. D. Berger, H. G. Craighead, D. Kern, and T. P. Smith III (Materials Research Society, Pittsburgh, 1990)]. The measured conductance shows a rich structure which indicates that resonant tunneling of the kind discussed here might be a real possibility.

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