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Theoretical investigation of noise characteristics of double-barrier resonant-tunneling systems

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A theory of noise characteristics of a double-barrier resonant-tunneling system is established. The noise-current power density has a Lorentzian form with a characteristic frequency given by the resonance-level broadening. The low-frequency characteristics which are dependent on structure parameters and bias voltage can be directly compared with experimental measurements.

Modern fabrication technology enables experimentalists to produce various microstructures which evidently exhibit quantum interference and nonequilibrium statistics. The double-barrier resonant-tunneling structure (DBRTS) is typical of the microstructures that have been the focus of many experimental and theoretical investigations. Since its conception by Tsu and Esaki[1] and the first realization of significant negative differential resistance by Sollner *et al.*,[2] many aspects of this system have been intensively studied, e.g., dc characteristics,[3-7] phonon- and laser-assisted tunneling,[8,9] time-dependent processes,[5,7,10-12] and frequency response.[13-15] The noise characteristics in DBRTS's have also been systematically investigated in a recent experiment,[16] which shows strong dependence of noise power density upon the structure parameters and the bias voltage. In the present paper, we shall develop a theory of the noise characteristics for DBRTS's and compare the results with the experimental measurements.[16]

In the linear-response region when the applied voltage is very small, the thermal noise dominates. The noise-current power density $S_i(\Omega)$ is given by the well-known Einstein relation $S_i(\Omega)=4kT\sigma(\Omega)$ for frequency $\Omega \ll kT$ with $\sigma(\Omega)$ being the dynamic conductance. The noise-current power density is completely determined by the equilibrium-state properties. Once the DBRTS is biased in the resonant-tunneling region, i.e., when the applied voltage $eV \gg \gamma$ (resonance-level width), the system moves far from equilibrium. Each electron in the states within the resonance energy range has a certain probability to tunnel through the double-barrier region from one electrode to the other. Then the shot noise becomes dominant and $S_i(\Omega)$ will be directly related to the bias current. However, the tunneling event of one electron is correlated to those of other electrons because, even in the coherent tunneling limit, every electron needs to stay in the central quantum well for a finite time (lifetime of the resonant state) when getting across the double-barrier region. When one electron proceeds to tunnel through, the other electrons (fermions) of the same quantum number cannot follow up within a correlation time given by the lifetime of the resonance state. This correlation yields a nonwhite spectrum (Lorentzian) and suppresses the shot noise. In this paper we shall study both the equilibrium and nonequilibrium behaviors of the noise characteristics by employing the nonequilibrium Green's-function approach.[15]

We consider the coherent tunneling processes and neglect any possible scattering of tunneling electrons. The Hamiltonian to describe a one-dimensional DBRTS is chosen as[9,12]

$$H=\sum_k \varepsilon_k^L a_k^\dagger a_k+\varepsilon_c c^\dagger c+\sum_p \varepsilon_p^R b_p^\dagger b_p +\sum_k (T_{Lk} c^\dagger a_k + T_{Lk}^* a_k^\dagger c) +\sum_p (T_{Rp} b_p^\dagger c + T_{Rp}^* c^\dagger b_p)\,, \tag{1}$$

with $a_k(a_k^\dagger)$, $c(c^\dagger)$, and $b_p(b_p^\dagger)$ being, respectively, the annihilation (creation) operators of electrons in the left electrode, in the central quantum well and in the right electrode. $\varepsilon_c=\varepsilon_0-\alpha eV$ (α is structure dependent) is the resonance level as affected by the bias voltage. $\varepsilon_k^L=k^2/2m$ and $\varepsilon_p^R=p^2/2m-eV$ are the single-particle energies of the left and the right electrodes. The energy starting point is chosen to be the conduction-band bottom of the left electrode and αeV and eV are, respectively, the potential drops of the resonance level and of the conduction-band bottom of the right electrode caused by the bias voltage V. The fourth and fifth terms describe the coupling between quantum-well electrons and the reservoirs. The tunneling matrices T_{Lk} and T_{Rp} depend on the barrier profile including the effect of the bias V. The left- and the right-electrode subsystems are assumed to be separately in their own equilibrium states with chemical potentials μ_L and μ_R, respectively ($\mu_L-\mu_R=eV$).[15] This assumption is practically correct because the two electrode subsystems respond to an applied field much faster than the quantum-well electrons (in other words, the whole system). The central-quantum-well electrons are in a nonequilibrium state, to be determined by their coupling to the two reservoirs and to the applied field.

Since the number of electrons in the central quantum well fluctuates, the current flowing into the well

$$I_L(t)=-ie\left[H,\sum_k a_k^\dagger(t)a_k(t)\right]$$

is generally different than that flowing out of the well

$$I_R(t)=ie\left[H,\sum_p b_p^\dagger(t)b_p(t)\right].$$

The terminal current is given by $I=(I_L+I_R)/2$, according to Ramo and Shockley.[14,15,17] The power density of

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the tunneling current fluctuation then can be written as the Fourier transform $S_i(\Omega)$ of the following correlation function:

$$S_i(t-t') = \langle \{[I(t)-\langle I\rangle],[I(t')-\langle I\rangle]\}_+\rangle . \quad (2)$$

In Eq. (2), the quantum statistical (nonequilibrium) average has been expressed in terms of the path integrals[18] along the closed time path.[15,19,20] The path integrals can be carried out exactly to yield the results in combinations of steady-state Green's functions.[12,15] After this procedure and the Fourier transform, the power spectrum of noise current

$$S_i(\Omega) = \tfrac{1}{2}\xi_\alpha\xi_\beta S_{\alpha\beta}(\Omega) , \quad (3)$$

$$\begin{aligned} S_{\alpha\beta}(\Omega) = \frac{e^2}{4}\int \frac{d\omega}{2\pi}\{ & [A+B]_{\alpha\beta}(\omega+\Omega)G_{\beta\alpha}(\omega)+G_{\alpha\beta}(\omega+\Omega)[A+B]_{\beta\alpha}(\omega) \\ & +G_{\alpha\beta}(\omega+\Omega)[A-B]_{\beta\mu}(\omega)\eta_\mu G_{\mu\nu}(\omega)\eta_\nu[A-B]_{\nu\alpha}(\omega) \\ & +[A-B]_{\alpha\mu}(\omega+\Omega)\eta_\mu G_{\mu\nu}(\omega+\Omega)\eta_\nu[A-B]_{\nu\beta}(\omega+\Omega)G_{\beta\alpha}(\omega) \\ & -[A-B]_{\alpha\mu}(\omega+\Omega)\eta_\mu G_{\mu\beta}(\omega+\Omega)[A-B]_{\beta\nu}(\omega)\eta_\nu G_{\nu\alpha}(\omega) \\ & -G_{\alpha\mu}(\omega+\Omega)\eta_\mu[A-B]_{\mu\beta}(\omega+\Omega)G_{\beta\nu}(\omega)\eta_\nu[A-B]_{\nu\alpha}(\omega)\} . \end{aligned} \quad (4)$$

In Eqs. (3) and (4), $\xi_\alpha=1$ for $\alpha=+,-,\eta_+=-\eta_-=1$. The subscripts α,β in Eq. (3) and μ,ν in Eq. (4) are summed over the + and − branches. $A_{\alpha\beta}$, $B_{\alpha\beta}$, and $G_{\alpha\beta}$ are, respectively, the Green's functions of the left-electrode subsystem, the right-electrode subsystem, and the tunneling electrons.[15] Equation (3) with Eq. (4) is our central expression for the noise spectrum from which various concrete results can be extracted without any difficulty by evaluating the frequency (ω) integrations of those combinations of the known Green's functions.[12,15]

When the system is biased in the linear-response regime and in the resonant tunneling region, we have, from Eq. (4),

$$\begin{aligned} \frac{S_i(\Omega)}{e^2/2} = \int \frac{d\omega}{2\pi}(& -2[F(+)+F-2FF(+)]\gamma(\omega)\mathrm{Im}[G_r(+)+G_r] \\ & -16[f_L(+)-F(+)](f_L-F)\gamma_L^2(\omega)\mathrm{Im}[G_r(+)]\mathrm{Im}(G_r) \\ & -4[f_L(+)+f_L-2f_Lf_L(+)]\gamma_L^2(\omega)\mathrm{Re}[G_r(+)G_a] \\ & -16[f_R(+)-F(+)](f_R-F)\gamma_R^2(\omega)\mathrm{Im}[G_r(+)]\mathrm{Im}(G_r) \\ & -4[f_R(+)+f_R-2f_Rf_R(+)]\gamma_R^2(\omega)\mathrm{Re}[G_r(+)G_a]+4\gamma_L(\omega)\gamma_R(\omega)\mathrm{Re}[G_r(+)G_a] \\ & -2\{[1-2f_L(+)](1-2f_R)+(1-2f_L)[1-2f_R(+)]\}\gamma_L(\omega)\gamma_R(\omega)\mathrm{Re}[G_r(+)G_r] \\ & +4\{(1-2F)[2f_L(+)+2f_R(+)-1-2F(+)]+[1-2F(+)](2f_L+2f_R-1-2F)\} \\ & \times\gamma_L(\omega)\gamma_R(\omega)\mathrm{Im}[G_r(+)]\mathrm{Im}(G_r)) , \end{aligned} \quad (5)$$

where $G_{r(a)}$, $f_{L(R)}$, and F represent, respectively, the retarded (advanced) Green's functions $G_{r(a)}(\omega)$, the Fermi-Dirac distributions of the left (right) electrode $f_{L(R)}(\omega)$, and the nonequilibrium distribution of tunneling electrons $F(\omega)$.[15] $G_{r(a)}(+)$, $f_{L(R)}(+)$, and $F(+)$ stand for $G_{r(a)}(\omega+\Omega)$, $f_{L(R)}(\omega+\Omega)$, and $F(\omega+\Omega)$, respectively. $\gamma(\omega)=\gamma_L(\omega)+\gamma_R(\omega)$ is the resonance-level width broadened by tunneling with $\gamma_L(\omega)=\Sigma_k|T_{Lk}|^2\times\pi\delta(\omega-\varepsilon_k^L)$ and $\gamma_R(\omega)=\Sigma_k|T_{Rp}|^2\pi\delta(\omega-\varepsilon_p^R)$.

In the linear-response region when the applied voltage $eV\ll\gamma$, the noise-current power density $S_i(\Omega)$ is related to the linear conductance $\sigma(\Omega)$ (Ref. 15) by

$$S_i(\Omega) = \frac{e^{\beta\Omega}+1}{e^{\beta\Omega}-1}2\Omega\sigma(\Omega) , \quad (6)$$

which produces the well-known Einstein relation if the frequency is much smaller than that given by temperature, i.e., $\Omega\ll 1/\beta$.

When the temperature $1/\beta$ is zero and the system is biased in the resonant tunneling region ($eV\gg\gamma$), the system of tunneling electrons is driven far from equilibrium. Then the shot noise becomes dominant. After working out the integrations in Eq. (5), the noise-current power density can be expressed as

$$S_i(\Omega) = e\langle I\rangle\left[1+\left(1-\frac{4\gamma_L\gamma_R}{\gamma^2}\right)\frac{4\gamma^2}{\Omega^2+4\gamma^2}\right] , \quad (7)$$

where $\langle I\rangle$ is the steady-state tunneling current induced by the bias.[12] In contrast to the full shot noise in a single-barrier tunneling structure which has a white spectrum, the power spectrum of noise current in a DBRTS $S_i(\Omega)$ has a Lorentzian distribution form with characteristic frequency $\Omega_0=2\gamma$. Moreover, the shot noise in a DBRTS is not full; instead, it is suppressed to a certain degree that depends on the symmetry parameter of the structure.

Usually, one may expect that the coherent tunneling in a DBRTS yields full shot noise because each tunneling electron sees the double-barrier structure as a whole.[16] Actually this expectation is not physically true. To understand the unexpected behavior of nonwhite, suppressed shot noise in DBRTS as predicted in Eq. (7), let us examine how the full shot noise is produced in a single-barrier tunneling system. In a single-barrier tunneling structure under a finite bias voltage V, the states in the left electrode are filled up to Fermi energy μ_L while the right are filled up to μ_R with $\mu_L - \mu_R = eV$. In the (μ_R, μ_L) region, an electron in any of the states has a certain probability to tunnel through the barrier from the left to the right. The time an electron takes to get through the barrier is negligibly small and there is no correlation between different events of electron tunneling. These two key points, (i) no transit time, and (ii) complete randomness of tunneling events, lead to the full shot noise in a single-barrier system.

In a DBRTS, however, the main contribution to the charge transport is from the electrons in the states within the $(\varepsilon_c - \gamma, \varepsilon_c + \gamma)$ energy range. The two key points leading to full shot noise in a single-barrier structure are all broken to a certain degree: It takes an electron a finite time $1/\gamma$ to tunnel through a double barrier; when one electron proceeds to tunnel through the structure, the other electrons of the same quantum number cannot follow simultaneously, i.e., one event of tunneling an electron through a DBRTS is correlated to other tunneling events. The finite correlation time gives a nonwhite spectrum (Lorentzian) and the correlation between different events, of course, suppresses the shot noise. In extremely asymmetric structures, the current is mainly controlled by one barrier and thus the shot noise should be full [see Eq. (7)]. For less and less asymmetric structures, the correlation becomes stronger and stronger and the shot noise is more and more suppressed. In a symmetric structure, it becomes only half of the full shot noise.

The dependence of the low-frequency characteristics upon the structure parameters and the applied bias voltage is mainly through the symmetry factor $4\gamma_L\gamma_R/\gamma^2$:

$$\frac{S_i(0)}{e\langle I\rangle} = 2 - \frac{4\gamma_L\gamma_R}{\gamma^2} . \tag{8}$$

γ_L and γ_R depend, respectively, on the left and the right barrier width and height including the effect of the bias voltage. Thus a symmetrical structure ($\gamma_L = \gamma_R$ at zero bias) could become very asymmetrical, e.g., $\gamma_L \ll \gamma_R$ under a certain reverse bias (by convention,[16] reverse bias means the left barrier is used as an emitter and the right barrier as a collector). Accordingly, the noise characteristics [Eq. (8)] will change with an increasing bias voltage. Actually, our theoretical result for the dependence of low-frequency noise characteristics upon the structure parameters and the bias voltage, given by Eq. (8), can be used directly to interpret the experimental measurements.[16]

Take sample 1 of Ref. 16 as the first example. The left barrier is a 105-Å layer of $Al_{0.5}Ga_{0.5}As$, the right barrier 85-Å of $Al_{0.4}Ga_{0.6}As$. The left barrier is higher and thicker than the right one and thus $\gamma_L \ll \gamma_R$. Under a reverse bias, the right barrier will be lowered further which means γ_R is becoming even larger than γ_L. Therefore $S_i(0)/e\langle I\rangle$ [Eq. (8)] is always near the constant 2. This behavior agrees well with the experimental result in Fig. 3 of Ref. 16.

For sample 2 of Ref. 16, the left barrier is an 85-Å layer of $Al_{0.5}Ga_{0.5}As$, the right barrier is an 85-Å layer of $Al_{0.4}Ga_{0.6}As$. The left barrier is higher and has the same width as the right one and thus $\gamma_L \ll \gamma_R$. Under a forward bias, the left barrier will be relatively lowered and thus γ_L will be growing closer to γ_R. Therefore, Eq. (8) shows that $S_i(0)/e\langle I\rangle$ is at first close to 2 for small bias and becomes suppressed with the increasing forward bias, which is also in agreement with experiment in Ref. 16.

Sample 3 of Ref. 16 has 70 Å of $Al_{0.5}Ga_{0.5}As$ as the left barrier and 85 Å of $Al_{0.4}Ga_{0.6}As$ as the right one. While the left barrier is higher, the right barrier is thicker and, as a result, γ_L and γ_R are close to each other. Thus $S_i(0)/e\langle I\rangle$ from Eq. (8) has a value close to 1 for small bias. When the bias voltage (either reverse or forward) is increased, the system may become more and more asymmetrical and the value of $S_i(0)/e\langle I\rangle$ will increase and eventually get close to 2. This is exactly what has been observed experimentally.[16]

In summary, we have established an analytical theory of noise characteristics of DBRTS's. The theoretical result of low-frequency characteristics which is dependent on structure parameters and bias voltage can be directly compared with experimental measurements. Finally, the sequential tunneling component[5,6] may be important in more realistic systems and needs to be incorporated by taking into account the scattering effects of tunneling electrons.

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[1]R. Tsu and L. Esaki, Appl. Phys. Lett. **22**, 562 (1973).

[2]T. C. L. G. Sollner, W. D. Goodhue, P. E. Tannenwald, C. D. Parker, and D. D. Peck, Appl. Phys. Lett. **43**, 588 (1983).

[3]B. Ricco and M. Ya. Azbel, Phys. Rev. B **29**, 1970 (1984).

[4]P. J. Price, Phys. Rev. B **38**, 1994 (1988); M. Büttiker, IBM J. Res. Dev. **32**, 63 (1988); A. R. Bonnefoi, T. C. McGill, and R. D. Burnham, Phys. Rev. B **37**, 8754 (1988); G. Kim and G. B. Arnold, *ibid.* **38**, 3252 (1988).

[5]S. Luryi, Appl. Phys. Lett. **47**, 490 (1985).

[6]T. Weil and B. Vinter, Appl. Phys. Lett. **50**, 1281 (1987); M. Jonson and A. Grincwajg, *ibid.* **51**, 1729 (1987).

[7]W. R. Frensley, Phys. Rev. Lett. **57**, 2853 (1986); Phys. Rev. B **36**, 1570 (1987).

[8]V. J. Goldman, D. C. Tsui, and J. Cunningham, Phys. Rev. B **36**, 7635 (1987).

[9]N. S. Wingreen, K. W. Jacobsen, and J. W. Wilkins, Phys. Rev. Lett. **61**, 1396 (1988); D. Sokolovski, Phys. Rev. B **37**, 4201 (1988); M. Jonson, *ibid.* **39**, 5924 (1989); P. Johnsson, *ibid.* **41**, 9892 (1990).

[10]G. Garcia-Calderon and A. Rubio, Phys. Rev. B **36**, 4462 (1987); J. D. Bruno, T. B. Bahder, and C. A. Morrison, *ibid.* **37**, 7098 (1988); H. Guo, K. Diff, G. Neofotistos, and J. D.

Gunton, Appl. Phys. Lett. **53**, 131 (1988).

[11]W. Trzeciakowski, D. Sahu, and T. F. George, Phys. Rev. B **40**, 6058 (1989); B. Gu and L. Gu, *ibid.* **40**, 6126 (1989).

[12]L. Y. Chen and C. S. Ting, Phys. Rev. B **41**, 8533 (1990).

[13]J. M. Gering, D. A. Crim, D. G. Morgan, P. D. Coleman, W. Kopp, and H. Morkoc, J. Appl. Phys. **61**, 271 (1987).

[14]W. R. Frensley, Superlattices Microstruct. **4**, 497 (1988); N. S. Wingreen, Appl. Phys. Lett. **56**, 253 (1990).

[15]L. Y. Chen and C. S. Ting, Phys. Rev. Lett. **64**, 3159 (1990).

[16]Y. P. Li, A. Zaslavsky, D. C. Tsui, M. Santos, and M. Shayegan, Phys. Rev. B **41**, 8388 (1990).

[17]S. Ramo, Proc. IRE **27**, 584 (1939); W. Shockley, J. Appl. Phys. **9**, 639 (1938).

[18]A. R. Hibbs and R. P. Feynman, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965); R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).

[19]Z. B. Su, L. Y. Chen, and C. S. Ting, Phys. Rev. Lett. **60**, 2323 (1988); Phys. Rev. B **38**, 3866 (1988).

[20]J. Schwinger, J. Math. Phys. (N.Y.) **2**, 407 (1961); L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)]; L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjiamin, New York, 1962); for recent review and the path integral formulation, see K. C. Chou, Z. B. Su, B. L. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985); Z. B. Su, L. Y. Chen, X. T. Yu, and K. C. Chou, Phys. Rev. B **37**, 9810 (1988).