Electron-phonon scattering times in thin Sb films

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(Received 13 August 1990)

Electron heating measurements have been used to study the electron-phonon scattering time in thin Sb films in the temperature range 1–4 K. The magnitude of the scattering time is in reasonable accord with the theory, while the temperature dependence is of the form $\tau_{e\text{-ph}} \sim T^{-\alpha}$, with $\alpha \sim 1.4$. The value of α appears to be significantly smaller than predicted by the theory, and is not understood.

I. INTRODUCTION

The past decade has seen much progress in the understanding of the transport properties of disordered systems, especially in one and two dimensions.[1–4] The contributions of weak localization and electron-electron interactions to the conductivity at low temperatures have been much studied, both theoretically and experimentally. An important result of this work has been the determination of various electron scattering times, which can be extracted with precision from the weak-localization magnetoresistance in small magnetic fields.[1] Of particular interest to date has been electron-electron scattering, with theory and experiment being in fairly good agreement with regard to both the magnitude and temperature dependence of this scattering rate.[2–4]

There has also been a good deal of theoretical work on electron-phonon scattering in disordered systems,[5–7] but much less is known experimentally about this process. The relative lack of experimental information concerning electron-phonon scattering is caused by the following fact. For the purposes of comparison with the theory, it is preferable to work at as low a temperature as possible, which in practice means below about 4 K. At these temperatures the electron-electron phase-coherence time, $\tau_{e\text{-}e}$, is generally comparable to or shorter than the electron-phonon time, $\tau_{e\text{-ph}}$. Since the most convenient experimental probe, measurement of the weak-localization magnetoresistance, yields the sum of the scattering rates $\tau_{e\text{-}e}^{-1} + \tau_{e\text{-ph}}^{-1}$, it is difficult, at best, to determine $\tau_{e\text{-ph}}$.[8] Nevertheless, some results for $\tau_{e\text{-ph}}$ have been obtained from magnetoresistance work,[1,7,9] mainly above ~4 K, and these are in reasonable agreement with the theory.[7,9]

While many experimental techniques measure the sum of the electron-electron and electron-phonon scattering rates, this is not always the case. Consider a process, such as Joule heating with an electric field, through which energy is deposited in the electron system. Electron-electron scattering redistributes this energy among the electrons. In contrast, electron-phonon scattering removes this energy from the electron gas, and hence $\tau_{e\text{-ph}}$ alone determines the amount that the electron temperature increases above the lattice temperature in response to the joule heat. By measuring this temperature increase it is possible to determine $\tau_{e\text{-ph}}$, even when the electron-phonon scattering is much weaker than the electron-electron scattering. This method and others like it have been used several times in recent years to measure $\tau_{e\text{-ph}}$ in a variety of different systems,[10–15] but there have been no systematic attempts to compare the results for disordered metals with recent theories.

In the present work we use electron heating measurements to study $\tau_{e\text{-ph}}$ in thin Sb films. While the magnitude of $\tau_{e\text{-ph}}$ is in reasonable accord with the theory, the temperature dependence appears to be much weaker than predicted.

II. THEORY

It is well known[1–4] that the weak-localization magnetoresistance is sensitive to the electron phase-coherence time, τ_ϕ, and it is sometimes important to distinguish this quantity from the inelastic-scattering time. In disordered systems the electron-electron phase-coherence time is shorter than the electron-electron inelastic time. However, electron-phonon scattering events at low temperatures are strongly inelastic, so in this case the phase-coherence and inelastic-scattering times are the same.[7,9]

For a three-dimensional system in the so-called clean limit, which means $k_F \ell \gg \theta_D/T$, where ℓ is the elastic mean free path, the theory predicts[7,6]

$$\tau_{e\text{-ph}}^{-1} \sim \lambda_{\text{ph}} \frac{T^3}{\theta_D^2}, \tag{1}$$

where θ_D is the Debye temperature and λ_{ph} is a material-dependent constant that measures the strength of the electron-phonon coupling. For a disordered system the prediction is[7,6]

$$\tau_{e\text{-ph}}^{-1} \sim \lambda_{\text{ph}} \frac{T^3}{\theta_D^2} B(T), \tag{2}$$

which differs from (1) only by the factor $B(T)$. For

a simple jellium model with a spherical Fermi surface $B(T) \sim T$ at low temperatures, and decreases as T is increased at high temperatures. In addition, $B(T)$ is a function of the degree of disorder through $k_F\ell$. The nonmonotonic temperature dependence of $B(T)$ means that $\tau_{e\text{-ph}}$ cannot be described by a simple power law over the entire temperature range, although over the limited range accessible in a typical experiment one would expect to observe an approximate power law with an effective exponent. That is, one would find

$$\tau_{e\text{-ph}}^{-1} \sim T^{\alpha}, \tag{3}$$

with the exponent α characterizing the temperature dependence of $\tau_{e\text{-ph}}$.

It is important to note that the electron-phonon rate in (1) is simply proportional to the number of phonons with energies of order of the thermal energy, so the exponent of T in (1) and (2) is just the effective spatial dimensionality of the phonons.[7] For a thin metal film on a substrate, it is not at all clear that this dimensionality should be 3 [as is assumed in (1) and (2)]. For the case of interest to us here, the film thickness is generally comparable to or smaller than the wavelength of a thermal phonon. In this regime, the problem of constructing a realistic theoretical model for the electron-phonon scattering in a metal film is quite difficult, since the coupling between the film and the substrate must clearly play a key role. Belitz and Das Sarma[9] have argued that the relevant dimensionality that characterizes electron-phonon scattering in this case should be 2. In two dimensions, one would expect behavior similar to that given in (1) and (2), but with the exponent of T having a value of 2, since as noted above, this exponent is simply the spatial dimensionality of relevant phonons. Thus, for a clean two-dimensional system the theory predicts $\tau_{e\text{-ph}}^{-1} \sim T^2$, while in the disordered case the exponent can be larger or smaller than 2, depending on the behavior of $B(T)$. A careful evaluation of $\tau_{e\text{-ph}}$ in two dimensions[9] using parameters appropriate for metal films typical of those studied in recent experiments yields a temperature dependence that can be described by a power law (3) with $\alpha \sim 2.5$. In addition, Belitz and Das Sarma have shown that the experimental results for $\tau_{e\text{-ph}}$ for temperatures above about 4 K are in good agreement with this temperature dependence, and that the observed magnitudes of $\tau_{e\text{-ph}}$ are also in good accord with the theory. Thus, it appears that the behavior of $\tau_{e\text{-ph}}$ in a variety of thin metal films can, at least at relatively high temperatures, be described reasonably well by the theory if one assumes that the electron-phonon scattering is effectively two dimensional.

III. EXPERIMENTS

Our experiments were performed using Sb films prepared by thermal evaporation onto glass substrates. They were patterned by mechanical scribing into strips typically 7 cm long, with resistances of ~10 kΩ. The films had resistivities of $\rho \approx 200\ \mu\Omega\,\text{cm}$, and thicknesses in the range 50–900 Å. The measurements were carried out in a standard ^{4}He cryostat, with the samples immersed directly in the liquid helium. The resistance was measured with an ac bridge operated at 200 Hz. A second ac electric field, which had a frequency of 500 Hz, could be applied independently of the 200 Hz measuring field. The 500 Hz field was used to heat the electrons; the 200 Hz field was always sufficiently small that it did not heat the electrons appreciably.

At low temperatures there are two main contributions to the temperature-dependent part of the resistance, R, of a thin metal film; these are weak localization and electron-electron interactions.[2–4] In order to simplify the analysis, all of the measurements reported here were performed with a magnetic field of 1 T applied perpendicular to the plane of the film. A field of this magnitude quenches the contribution of weak localization, leaving only electron-electron interaction effects to produce a change in R with temperature.[2,3] The contribution from electron-electron interactions is predicted to depend only on the electron temperature, so it provides a convenient thermometer.[16]

The relationship between the electric field responsible for Joule heating and the resultant electron temperature has been discussed by a number of workers,[17,10–14] who have shown that for small fields a simple heating model predicts

$$T_e - T_{\text{lattice}} = \tau_{e\text{-ph}} \frac{E^2}{\rho C_v}, \tag{4}$$

where T_e is the electron temperature, T_{lattice} is the lattice (phonon) temperature, E is the Joule heating field (in our case, the 500 Hz electric field[18]), ρ is the electrical resistivity, and C_v is the electronic heat capacity. Note also that in (4) it is assumed that E is small, so that the electron temperature is not far from T_{lattice}.

Our experiments consisted of simply measuring R as a function of the Joule heating (500 Hz) field, E, at a fixed lattice temperature (and with the magnetic field applied). By combining these results with measurements of R as a function of T with $E = 0$, the variation of T_e with E was obtained at a number of different temperatures. For all of the results reported below, it was verified that E was sufficiently small that T_e varied as E^2, as expected from (4). Finally, (4) was used to derive $\tau_{e\text{-ph}}$ as a function of T.[19]

Figure 1 shows results for R as a function of E. It is seen that $\Delta R \sim E^2$, as expected from (4). The inset in Fig. 1 also shows results for the temperature dependence of R, for a fairly thin (i.e., high $R_\square$) sample, in the absence of a heating field. As expected, R varied logarithmically with T, and the results are in good agreement with the theoretical predictions for electron-electron interactions.[2–4] A logarithmic variation was observed below 4 K for all samples with thicknesses below about 150 Å, which corresponds to $R_\square \gtrsim 100\ \Omega$. For thicker samples the contribution of electron-electron in-

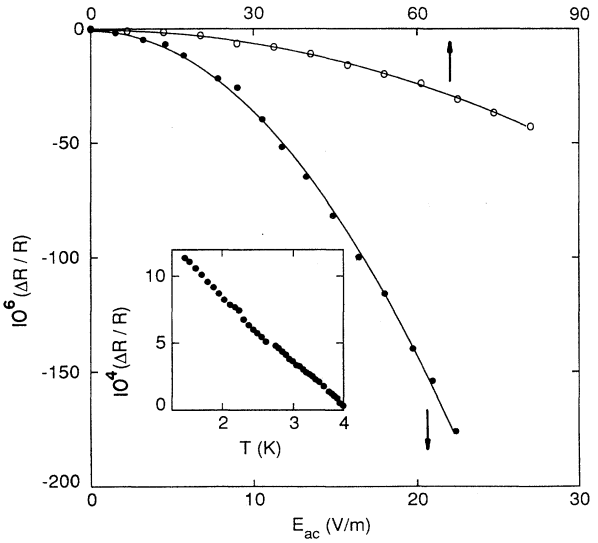

FIG. 1. Typical results for R as a function of E at two different temperatures, as indicated in the figure. These results are for an Sb sample with $R_\square = 233\ \Omega$. The solid lines are fits to the form $\Delta R \sim E^2$, as implied by (4). The inset shows typical results for the temperature dependence of R with the heating field $E = 0$. This sample had $R_\square = 110\,\Omega$. Note that all of the results shown here were obtained with a magnetic field $B = 1$ T applied perpendicular to the plane of the film. The uncertainties are no larger than the symbols.

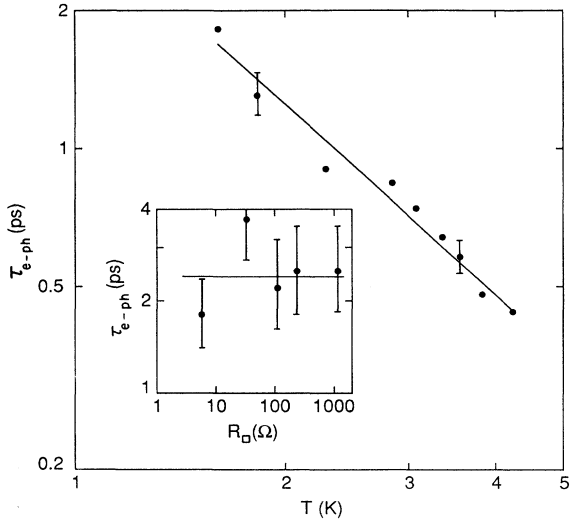

FIG. 2. Results for $\tau_{e\text{-ph}}$ as a function of T for an Sb film with $R_\square = 110\,\Omega$, as derived from the heating measurements using (4). Some typical uncertainties are also indicated. The solid line is a fit to the power law form (3), with $\alpha = 1.35 \pm 0.15$. The inset shows $\tau_{e\text{-ph}}$ at 1.3 K as a function of $R_\square$. The uncertainties shown here are our best estimates of the systematic errors.

teractions was smaller (it scales with $R_\square$), and the direct electron-phonon (i.e., Drude) contribution was significant, leading to a positive dR/dT at the highest temperatures.[14] In this region the resistance cannot be employed to measure the electron temperature, since the electron-phonon contribution depends on a combination of the electron and lattice temperatures.[15,14] As a result, heating measurements cannot be used to extract $\tau_{e\text{-ph}}$ in this range. This restriction limited us to temperatures below about 2 K for our thickest samples, and hence it was only possible to measure the temperature dependence of $\tau_{e\text{-ph}}$ (i.e., the value of α) for samples with thicknesses less than about 150 Å. However, it was still possible to measure the magnitude of $\tau_{e\text{-ph}}$ for all samples at our lowest temperature (~ 1.3 K).

Results for $\tau_{e\text{-ph}}$ as a function of T are given in Fig. 2. The behavior is seen to be consistent with a power law of the form (3). It is important to note that there is no anomaly in the behavior of $\tau_{e\text{-ph}}$ at the superfluid transition (~ 2.17 K). This indicates that the thermal coupling of the phonons to the liquid-helium bath was strong enough that it was not a bottleneck in the overall transfer of heat from the electrons to the helium, and hence that the temperature rise observed (i.e., in Fig. 1) was indeed determined by $\tau_{e\text{-ph}}$. The values of α found for several different samples, all with thicknesses below 150 Å, were in the range 1.3–1.5 with uncertainties typically of ± 0.2. It is clear from Fig. 2 that an exponent of 2.5, as was found in a previous analysis[9] of data for other metal films, is certainly *not* consistent with our results.

The inset in Fig. 2 shows the variation of $\tau_{e\text{-ph}}$ at 1.3 K as a function of $R_\square$; i.e., film thickness. To within the uncertainties, $\tau_{e\text{-ph}}$ is independent of $R_\square$. This result is a bit surprising, since the elastic mean free path, ℓ, increases by more than a factor of 10 in going from the low to high $R_\square$ samples.[20] From (2) we would have expected $\tau_{e\text{-ph}}$ to vary significantly with ℓ.

The magnitude of $\tau_{e\text{-ph}}$ that we find is in reasonably good accord with that found for other metal films.[9,7] For example, for Cu at 4 K the theory[9] predicts $\tau_{e\text{-ph}} \sim 1 \times 10^{-10}$ s, which is not far from the value we find for Sb, Fig. 2. (Unfortunately, the value of $\tau_{e\text{-ph}}$ has not yet been calculated for the specific case of Sb.) In addition, we have also measured the phase-coherence times, τ_ϕ, in our films using the weak-localization magnetoresistance.[1,21,14] The results show that τ_ϕ is about an order of magnitude smaller than $\tau_{e\text{-ph}}$ in Fig. 1, which is consistent with the general expectation that at these temperatures τ_ϕ should be dominated by electron-electron scattering, as discussed above.[2–4] The temperature dependence of τ_ϕ was consistent with a power-law form, with an exponent $\alpha = 1.3 \pm 0.2$. This is in rough accord with the theory, which predicts that the electron-electron phase breaking time is characterized by the form (3) with $\alpha = 1$. The magnitude[14] of τ_ϕ was also in good agreement with the theoretically predicted value of $\tau_{e\text{-}e}$.

The surprising result of our work is that the temperature dependence of $\tau_{e\text{-ph}}$ is much weaker than expected. Admittedly, there are significant uncertainties in

the analysis, largely in the estimates of quantities such as C_v in (4). However, it seems very unlikely that these uncertainties would affect the temperature dependence we obtain for $\tau_{e\text{-ph}}$, i.e., the value of α. Moreover, the magnitude of $\tau_{e\text{-ph}}$ is very much in line with theoretical expectations, which suggests that these uncertainties are probably not substantial.

IV. DISCUSSION AND COMPARISON WITH PREVIOUS WORK

Previous experimental results for $\tau_{e\text{-ph}}$ in two-dimensional conductors have come essentially from two types of experiments. As described above, measurements of the weak-localization magnetoresistance have been used to obtain $\tau_{e\text{-ph}}$ at relatively high temperatures; i.e., when $\tau_{e\text{-ph}} \lesssim \tau_\phi$, which means typically $T \gtrsim 4$ K. These results have been reviewed and compared with the theory of Belitz and Das Sarma,[9] who showed that, for a number of different materials (but not Sb), $\alpha \sim 2.5$. They also showed that this value of α is consistent with a careful theoretical evaluation of $\tau_{e\text{-ph}}$ with the assumption that the electron-phonon scattering is effectively two-dimensional.

To the best of our knowledge, essentially all of the results for $\tau_{e\text{-ph}}$ at lower temperatures have come from heating measurements. Early work on thin AuPd films[10] found an exponent $\alpha \sim 3.5 \pm 0.5$. Roukes *et al.*[12] studied relatively clean Cu films, and found $\alpha \sim 3$ over a wide temperature range below 1 K. In a somewhat different system, Wennberg *et al.*[13] used heating measurements to study $\tau_{e\text{-ph}}$ in GaAs-$Al_xGa_{1-x}As$ heterostructures, with the result $\alpha \sim 3.0 \pm 0.2$. These values of α are all consistent with the value predicted for two-dimensional electron-phonon scattering in a disordered system in the low-temperature limit.[22]

In view of the previous experimental measurements of α, the value we find for Sb is seen to be anomalously low. The calculated behavior[7] of $B(T)$ for the values of $k_F\ell$ appropriate for our Sb films[23] appears to yield a value of α that is ~ 2.5, similar to that found for other materials. However, these calculations assume a simple jellium model, with a spherical Fermi surface, and a Debye model for the phonons. It is certainly conceivable that these simple models may not accurately describe Sb.[24] A more realistic calculation of $\tau_{e\text{-ph}}$ for Sb would be most welcome. We should also recall that our method for extracting $\tau_{e\text{-ph}}$ is based on using the contribution of electron-electron interactions to the conductance as a thermometer for the electron gas. It is possible that this thermometer is defective,[15] although it is hard to see why this method would fail for Sb, while it seems to work for other systems. Finally, the simple heating model (4) may, for some reason, break down for Sb, although if this is the case, it is again difficult to see why it works well for other materials.

In conclusion, we have measured the electron-phonon scattering time in thin Sb films. The temperature dependence of $\tau_{e\text{-ph}}$ appears to be much weaker than predicted by the theory.

ACKNOWLEDGMENTS

We thank P. Muzikar and R. Sladek for helpful discussions. This work was supported by the National Science Foundation through Grant No. DMR-8915574.

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ically with $R_\square$. The sample with the $R_\square = 1.1$ kΩ had a thickness $t = 45$ Å and $\rho = 500\ \mu\Omega$ cm, which corresponded to $\ell \sim 50$ Å, while the sample with $R_\square = 5.7\ \Omega$ had $t = 910$ Å, $\rho = 50\ \mu\Omega$ cm, and $\ell \sim 500$ Å. The variation of ρ with t was consistent with that expected from boundary scattering, and thus implies that the films are fairly uniform (as opposed to being extremely granular), even when $t = 45$ Å.

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[22] This value of α is also consistent with scattering from three-dimensional phonons in the clean limit, which was the interpretation given in Ref. 12.

[23] It is difficult to estimate $k_F\ell$ precisely, since this estimate implicitly assumes a spherical Fermi surface, etc., which is only a rough approximation for Sb. Nevertheless, we estimate that $k_F\ell$ lies in the range ~2.5–25, with the lower value appropriate for our thinnest (highest $R_\square$) films, and the higher value appropriate for the thickest (lowest $R_\square$) samples.

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