PHYSICAL REVIEW B VOLUME 43, NUMBER 4 1 FEBRUARY 1991

ac absorption in the high-T_c superconductors: Reinterpretation of the irreversibility line

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(Received 30 May 1990)

A simple physical picture for the ac absorption in high-T_c superconductors in the mixed state is proposed. We argue that the transition in the real part and the peak in the imaginary part of the ac susceptibility are due to the skin size effect and occur when the skin penetration depth is of the order of the size of the sample. The frequency of the peak is proportional to the resistivity $\rho(H,T)$, excluding other field- or temperature-dependent factors. Experiments to determine the true irreversibility line associated with the transition between the vortex-glass and the vortex-liquid state are suggested.

The unusual behavior of the high-T_c superconductors in the mixed state attracts considerable attention. One of the most intriguing phenomena is the existence of the so-called irreversibility line in the H-T plane. This line separates the regions of reversible (high temperature T and high field H) from irreversible (low T and H) behavior of the system.

The irreversibility line was observed by means of magnetization measurements:[1] above the irreversibility line the magnetization showed reversible behavior, while at $T<T_{\rm irr}(H)$ it became irreversible and hysteresis loops were observed.

Another characteristic line, the so called $H_{c2}(T)$ line, was found from resistivity measurements.[2] For a given value of $\rho_{\rm min}$, this line is defined by the relation $\rho(H,T)=\rho_{\rm min}$ (usually $\rho_{\rm min}\sim 10^{-1}\,\rho_n$-$10^{-5}\,\rho_n$, where ρ_n is the normal-state resistivity). The shape of this line depends on the chosen value of $\rho_{\rm min}$ and one observes that the lower $\rho_{\rm min}$, the more this line approaches the irreversibility line $H_{\rm irr}(T)$ obtained from the magnetization measurements.[3]

Recent transport measurements[4,5] showed a remarkable change in the current-voltage (I-V) characteristic at some line $T_g(H)$. Above $T_g(H)$ a linear resistance was found, whereas below $T_g(H)$ the voltage V exhibited an extremely nonlinear current dependence: $V\propto \exp(-A/j^{\alpha})$. This behavior was attributed[4] to the transition from a vortex-liquid state at $T>T_g(H)$ to a vortex-glass state at $T<T_g(H)$. Again this line is close to the irreversibility line $T_{\rm irr}(H)$ mentioned above.

Another technique which is used to define the irreversibility line is ac susceptibility measurements.[6–8] In such experiments one usually superimposes a small ac field ≤ 1 Oe on a large dc field and measures the real and imaginary parts of the susceptibility $\chi=\chi'+i\chi''$. One finds a steplike change in χ' and a peak of absorption (χ'') at some line $T_{\rm peak}(H)$ in the H-T plane. This peak is rather narrow in temperature, and its position depends logarithmically upon the frequency of the superimposed ac field.

It is usually assumed that the transition in χ', which is accompanied by a peak in the dissipation χ'', occurs when the measuring frequency ω_m is of the order of the inverse relaxation time τ^{-1} of the vortex system.[9] The nature of these anomalies in the ac susceptibility was discussed by different authors.[9–12] Two essentially different regimes of vortex motion can be distinguished: (i) the phononlike oscillations of the vortex lines near their equilibrium positions *within* the potential wells produced by the random potential (so-called *intravalley* oscillations), and (ii) thermally activated hops between *different* metastable states in the random potential (so-called *intervalley* transitions). Hebard *et al.*[10] attributed the peak of dissipation to the intravalley oscillations of the vortex lines. However, the microscopic frequency ω_0 (of the order of 10^9–10^{11} Hz) of these oscillations is much higher than the measuring frequency ω, which is usually in the range between 10^5 and 10^7 Hz. In fact, the experimental data are better explained if one assumes that the absorption of the ac field is mainly governed by the hopping of the vortices over the barriers between different metastable states.[9,11] The characteristic time scale for such intervalley transitions is exponentially larger than ω_0^{-1} by a factor $\sim\exp(U_0/r\,k_BT)$, where U_0 is the activation barrier between the different metastable states. The frequency of the absorption peak is roughly given by $\omega_{\rm peak}\sim\omega_0\exp(-U_0/k_BT)$, where the involved activation energy U_0 is consistent with the one in resistivity.[9,11]

Inui, Littlewood, and Coppersmith[11] made an attempt to incorporate both processes [(i) and (ii)] and found that the frequency of the peak $\omega_{\rm peak}$ is proportional to the macroscopic resistivity $\rho(T)$ which, in turn, depends exponentially on temperature. This result is in qualitative agreement with experiments,[9,11] but the way the χ'' behavior was derived is not entirely consistent: Considering the overdamped intravalley motion of the vortex lines (i.e., oscillations within the isolated potential wells) Inui *et al.*[11] used the exponentially large damping constant resulting from the thermally activated vortex motion over the pinning barriers separating different potential wells.

Kes *et al.*[12] developed the theory for thermally assisted flux flow (TAFF) in the limit of small driving forces and employed this theory in the calculation of the ac susceptibility. In this framework thermally activated hops over energy barriers give rise to a diffusionlike equation for the vortex motion with an exponentially small diffusion constant $D \propto \exp(-U_0/k_BT)$. On the basis of this relation, an expression which relates the temperature and magnetic field at the peak to the frequency ω of the ac field has been derived and the position of the peak has been found to depend logarithmically on the size of the sample. Unfortunately their final formulas [Ref. 12, Eqs. (12) and (13)] contain a number of microscopic parameters, which makes it rather difficult to compare their results with experiments and disguises the physical origin of the absorption peak in the original TAFF model.[12]

The aim of this paper is to relate the position of the peak in the absorption χ'' to the experimentally measurable macroscopic quantities. To do this we follow the ideas of the TAFF theory.[12] However, we do not consider any microscopic mechanism for the vortex motion and express all the quantities through the measurable dc resistivity of the superconductor. We argue that the peak in the ac absorption can be directly explained from the skin size effect and we shall show that the frequency associated with this peak is proportional to the resistivity without any other field- or temperature-dependent factors.

One of the most characteristic features of the high-T_c superconductors is the remarkable broadening of the resistive transition in the presence of a magnetic field.[2,9,13] In sufficiently strong magnetic fields, these superconductors exhibit a measurable linear dc resistivity at temperatures far below T_c. The onset of the linear resistivity is associated with the melting of the flux-line lattice at the temperature $T_g(H) < T_c$ (Refs. 14 and 15) and the TAFF process in the resulting liquid phase.[16,17] As mentioned above, in this paper we shall not consider any microscopic mechanisms of this TAFF but simply take the nonzero linear resistivity *as a fact*. In reality, we are dealing with a good (at low temperatures very good) conductor, rather than a superconductor, and we assume the validity of Ohms law,

$$\mathbf{E} = \rho \mathbf{j} . \tag{1}$$

Here $\mathbf{E}$ is the electric field, $\mathbf{j}$ the current density, and ρ is the resistivity, which we assume to be independent of frequency at low frequencies.

Combining (1) with Maxwell's equations, we obtain the complete system of equations inside the conductor:[18]

$$\mathrm{div}\mathbf{B} = 0, \quad \mathrm{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \mathrm{rot}\mathbf{B} = \frac{4\pi \mathbf{E}}{c\rho} . \tag{2}$$

In a variable field of frequency ω, all quantities depend on the time through a factor $e^{-i\omega t}$. Eliminating the electric field $\mathbf{E}$ from (2) results in the following equation for $\mathbf{B}$:

$$\Delta \mathbf{B} = -\frac{4\pi i\omega}{\rho c^2}\mathbf{B} . \tag{3}$$

This defines the skin penetration depth δ_s (Ref. 18) as

$$\delta_s = \left[\frac{c^2\rho}{2\pi\omega}\right]^{1/2} . \tag{4}$$

It is interesting to compare this skin penetration depth with the London penetration depth λ. For the dirty limit of BCS theory, one finds

$$\delta_s = \lambda \left[\frac{2\pi\Delta \tanh(\Delta/2k_BT)}{\hbar\omega}\frac{\rho}{\rho_n}\right]^{1/2} , \tag{5}$$

where Δ is the superconducting gap. The value of 2Δ corresponds to a frequency of the order of 10^{13} Hz, whereas the measuring frequency usually ranges between 10^5 and 10^7 Hz and values for $\rho(H,T)/\rho_n$ are between 1 and 10^{-5}. Therefore, one usually has the inequality $\delta_s \gg \lambda$, which justifies our approach [note that the averaged equations (1) and (2) can only be used if the characteristic depth is much larger than the microscopic lengths which determine the resistivity]. Depending on temperature, the depth δ_s can be either larger (high T) or smaller (low T) than the sample size. At the temperature where δ_s is of the order of the sample size d, the imaginary part of the ac susceptibility χ'' has a maximum.[18] Let us consider the case of the slab geometry: $0 \le x \le d$, $\mathbf{B} \| \mathbf{z}$, $\mathbf{E} \| \mathbf{j} \| \mathbf{y}$. Taking into account the boundary conditions at the surfaces ($x = 0, d$) one can find the ac component of the magnetic field inside the sample,

$$B(x,t) = H_0(t)\frac{e^{ikx} + e^{ik(d-x)}}{1 + e^{ikd}} , \tag{6}$$

where $H_0(t) = H_0 e^{-i\omega t}$ is the superimposed ac field and $k = (1+i)\delta_s^{-1}$. Then for the ac susceptibility

$$\chi = \frac{1}{4\pi}\left[\frac{\int B(x)dx}{H_0 d} - 1\right]$$

one obtains

$$4\pi\chi' = \frac{\sinh u + \sin u}{u(\cosh u + \cos u)} - 1; \quad 4\pi\chi'' = \frac{\sinh u - \sin u}{u(\cosh u + \cos u)} , \tag{7}$$

where

$$u = d/\delta_s = \left[\frac{\omega}{\rho}\frac{2\pi d^2}{c^2}\right]^{1/2} .$$

Figure 1 shows χ' and χ'' vs ρ/ω. χ'' attains a maximum at $u_{\max} = 2.25$ corresponding to the relation

$$\omega_{\mathrm{peak}} \simeq 0.8\frac{c^2}{d^2}\rho(H,T) . \tag{8}$$

The physical interpretation of the origin of the peak in χ'' is very simple:[18] if $\delta_s \to \infty$ the field penetrates the sample completely and the out-of-phase signal χ'' tends to zero. In the opposite limiting case: $\delta_s \to 0$, the screening

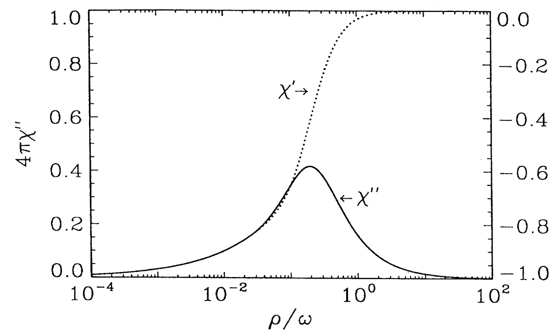

FIG. 1. Plot of the real (χ') and imaginary (χ'') parts of the ac susceptibility as a function of ρ/ω (which is given in units of $2\pi d^2/c^2$).

is complete, $\chi'\rightarrow -1/4\pi$ and $\chi''\rightarrow 0$.

Formula (8) is our main result. It only involves the measurable quantities $\omega_{\rm peak}$, ρ and d (note that the slab geometry is particularly convenient for experiments with high-T_c superconductors, since single crystals have a platelike shape and the above analysis applies quantitatively to the case when the ac field is in the *ab* plane). The position of the peak in χ'' coincides with the resistive criterion $\rho_{\rm min}\simeq 1.25\omega_{\rm peak}d^2/c^2$ or

$$\rho_{\rm min}(\mu\Omega\,{\rm cm})=7.6\times 10^{-3}d^2({\rm cm}^2)\,\nu_{\rm peak}({\rm Hz})\quad(\nu=\omega/2\pi)\,. \tag{9}$$

For $\nu\sim 1$ kHz and $d\sim 0.1$ mm, we obtain $\rho_{\rm min}\sim 10^{-3}$ $\mu\Omega$ cm. The half width of this peak corresponds to a drop of the resistivity by a factor of the order of 0.06. Because of the exponential drop of the resistivity with temperature the peak in χ'' is very sharp at low field, where one finds a very sharp resistive transition and is broadened at high field in agreement with experiment.[6] Note that by using Eqs. (8) and (9) one can also find the temperature and field dependence of the resistivity in those regions of the H-T plane where ρ is too small to be detected by direct resistive measurements.

The formulas (7) coincide with the formulas of Kes *et al.* [Eqs. (10) and (11) in Ref. 12]. This is not surprising, because the basic assumption of the TAFF theory is the existence of a linear resistivity, i.e., Ohms law. The basic equations of vortex diffusion [Ref. 12, Eq. (1)] are nothing else than Ohms law combined with Maxwell's equations. Therefore, the final result has to be the same and differs only in the terms in which it is expressed.

As we mentioned above, there is a line $T_g(H)$ in the H-T plane which separates an Ohmic regime $[T>T_g\times(H)]$ from a nonlinear regime $[T<T_g(H)]$ in which the linear resistivity is zero[16,17,19,20] and

$$E\propto\exp(-A/j^{\alpha})\,. \tag{10}$$

This line seems to represent a true phase transition from a vortex-liquid to a vortex-glass state, and for weak pinning it coincides with the melting transition of the flux-line lattice.[16,17] Various experimental techniques measure different manifestations of this true irreversibility line $T_g(H)$. Then the question arises: What is the relationship between the line $T_g(H)$ and the line of the peak $T_{\rm peak}(H)$ in χ''? Answering this question involves a subtlety concerning the role of the amplitude H_0 of the ac field which we shall explain in the following: Let us apply a fixed static field H and investigate the dependence of χ'' on temperature T. First, note that the peak in χ'' arises when $\delta_s\sim d$ corresponding to a current density of the order of $j_{\rm peak}\sim cH_0/4\pi d$. As long as the applied frequency ν (i.e., $\rho_{\rm min}$) is not too small, by decreasing the temperature we will scan through the full peak in χ'' without ever leaving the Ohmic regime. Therefore, we observe a temperature $T_{\rm peak}(H)$ which due to the sharp drop in resistivity is very close but *above* $T_g(H)$ and for small enough (such that Ohm's law is valid) values of $j_{\rm peak}$ (i.e., H_0), $T_{\rm peak}(H)$ does not depend on H_0.

Ideally, one would take the limit $\rho_{\rm min}\rightarrow 0$ while $H_0\rightarrow 0$ and find that the line $T_{\rm peak}(H)$ converges to the irreversibility line $T_g(H)$. However, in a real experiment H_0 will always be finite with the consequence that for very small $\rho_{\rm min}$ the peak will be shifted below $T_g(H)$ into the nonlinear region, where the resistivity depends strongly on H_0 via $j_{\rm peak}$. An accurate description of this situation involves the solution of Maxwell's equation based on the nonlinear current-voltage relation (10).

Nevertheless, a qualitative prediction of the behavior is possible: We can estimate the position of the peak using relation (8) with an effective resistivity $\rho_{\rm eff}(j_{\rm peak})=E(j_{\rm peak})/j_{\rm peak}$ defined from (10). The dependence of $\rho_{\rm eff}$ on $j_{\rm peak}$ has the effect that decreasing the amplitude H_0 will increase the temperature $T_{\rm peak}(H)$ such that in the limit of very small H_0, one will approach the line $T_g(H)$ from *below*. We have thus found a method to determine the true irreversibility line $T_g(H)$ from an ac susceptibility experiment by varying the frequency ν and amplitude H_0 of the superimposed ac field.

In conclusion, we argue that the transition in the real part χ' of the ac susceptibility and the peak in the imaginary part χ'' correspond to the skin size effect and occur when the skin penetration depth is of the order of the size of the sample. The frequency of the peak $\omega_{\rm peak}$ is proportional to the dc resistivity of the superconductor $\rho(H,T)$ and is given by Eq. (8). We emphasize that this formula contains only measurable parameters. From the dependence of the peak position on the amplitude of the external ac field, one can find the transition line between the vortex-glass and the vortex-liquid state.

We would like to thank G. Blatter for stimulating discussions. One of us (V.M.V.) thanks P. Kes for numerous discussions clarifying the essence of TAFF theory. We acknowledge financial support from the Swiss National Foundation.

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