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Tunneling measurement of the electron inelastic-scattering rate in a strongly disordered superconductor

Deuk Soo Pyun and Thomas R. Lemberger
Department of Physics, Ohio State University, Columbus, Ohio 43210
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Tunneling measurements show that the inelastic-scattering rate for single-electron excitations in strongly disordered superconducting indium oxide films at $T \ll T_{c0}$ is approximately proportional to T^3, rather than to $\exp(-\Delta/k_BT)$ as expected for electron-electron scattering. A recent theory indicates that electron-phonon scattering may be the explanation.

Disorder in conductors localizes electron wave functions, decreases electronic screening of Coulomb interactions, increases the electron inelastic-scattering rate, suppresses the electron density of states $N_n(E)$ near the Fermi energy E_F, and ultimately causes a metal-insulator (M-I) transition.[1,2] The present study focuses on the inelastic-scattering rate, $1/\tau_{in}$, because it is an important microscopic parameter (e.g., it limits electron localization[3-6] and rounds BCS singularities in superconductors[7,8]) and because theory[5,9-12] of inelastic electron scattering in disordered conductors is developed well enough to make comparison with experiment useful.

While theory of disordered conductors in general is incomplete, theory of inelastic scattering agrees with several experimental results in the normal state and just below the superconducting transition at T_{c0}. These include the inelastic rate determined from the magnetoresistance of disordered two-dimensional (2D) films of Al, Mg, Au, and Ag (Refs. 4-6) and 3D granular Al,[9] the inelastic rate determined just below T_{c0} in 2D and 3D disordered Al films from the enhancement of the critical supercurrent by microwave currents,[13,14] and the relation between the disorder-induced suppressions in T_{c0} and $N(E_F)$.[15,16] These successes motivate us to test the theory further by measuring $1/\tau_{in}$ well below T_{c0} where superconducting modifications are likely to be substantial.

Existing studies of inelastic scattering in the superconducting state do not present a unified picture. Tunneling measurements on Pb-Bi films showed that $1/\tau_{in}(T)$ could be determined accurately by fitting the (normalized) superconducting density of states $N_1(E,T)=N_s(E,T)/N_n(E,T)$ near $E=\Delta$ with a phenomenological inelastically broadened BCS density of states, at least for small inelastic rates $\hbar/\tau_{in}\Delta \ll 1$ (Δ is the order parameter).[8] Using this technique, Dynes *et al.*[17] found in 3D granular Al at 60 mK ($T/T_{c0} \ll 1$) a rate $1/\tau_{in}$ proportional to resistivity $\rho_{4.2}$, which they ascribed to e-e scattering. In 2D granular Sn films at 1.6 K ($T/T_{c0} \approx 0.5$), White, Dynes, and Garno[18] found a rate $1/\tau_{in}$ proportional to resistance per square $R_\square$, which they ascribed to e-e scattering. In strongly disordered 2D Pb films at 1.6 K ($T/T_{c0} < 0.5$), Dynes *et al.*[19] found no evidence for inelastic broadening in $N_1(E,T)$. Theory predicts a very rapid decrease in electron-electron scattering below T_{c0}.[10,12] Lee and Lemberger[20] observed this rapid decrease in charge-imbalance relaxation measurements in weakly disordered 2D Al films. These latter results suggest that the inelastic scattering seen well below T_{c0} in granular Al and Sn is much too large to be e-e scattering. The discrepancy is another reason to examine the T dependence of $1/\tau_{in}$ below T_{c0} in very disordered superconductors.

This paper presents $1/\tau_{in}(T)$ determined by tunneling technique in two films of superconducting amorphous-composite indium oxide, a-InO_x. Data on samples with a wider range of disorder but a smaller range of T/T_{c0} will be presented elsewhere.[21,22] In a-InO_x, disorder is beyond the simple "dirty limit" because the disorder-induced suppression in $N_n(E)$ is significant.[23] However, the samples are not very close to the M-I transition which occurs at resistivity $\rho_{4.2} \approx 9$ mΩ cm.[24]

The microstructure of a-InO_x is amorphous indium oxide (≈ 50 wt.% In) with some small crystallites[24,25] of semiconducting In_2O_3 (region B of Fig. 1 in Ref. 24). We follow the reactive ion-beam sputtering deposition procedure of Ref. 24, and we obtain[21] similar results regarding the effects of electron-electron interactions on resistivity and T_{c0}, namely, $\rho(T) \approx 1/(\sigma_0+\alpha T^{1/4})$ and $T_{c0} \approx 5K\rho(295K)/\rho(0)$.[26,27] For our films, $\rho_{4.2} \approx 1/\sigma_0$. A difference is that for a given resistivity the effects of e-e interactions are smaller in our films than those reported in Ref. 24. The differences may be related to the difference in substrates, as discussed elsewhere.[21,22] Resistivities and T_{c0}'s of our films deposited onto oxidized Al(1 wt.% Mn) counterelectrodes are, within a few percent, identical to our films deposited onto bare glass substrates.

Al(1 wt.% Mn)/AlO_x/a-InO_x normal-insulator-superconductor tunnel junctions ($R \approx 500$ Ω) are formed by evaporating Al(1 wt.% Mn) strips onto glass substrates, depositing SiO to define the junction area to about 300×300 μm^2, oxidizing for 30 min in air, then depositing a cross strip of a-InO_x. 1 wt.% of Mn reduces T_{c0} of AlMn to zero. Measurements are made down to 0.5 K in a ^{3}He probe.

The tunneling procedure and analysis are designed to allow separation of the energy dependence of $N_1(E,T)$ from the total density of states $N_s(E,T)=N_1(E,T) \times N_n(E,T)$ with minimal assumptions about the T and E dependence of $N_n(E,T)$ below T_{c0}. This is important because $N_n(E,T)$ in a-InO_x is observed to depend on T above T_{c0} and because $N_n(E>\Delta,T)$ has an anomalous energy dependence[23] so extrapolation to $T<T_{c0}$ and/or $E<\Delta$ is perilous.

We measure the junction conductance $G_j(V,T) \equiv \partial I/\partial V|_{T,I_s}$ as a function of voltage V across the junction with no supercurrent I_s in the a-InO_x film. The experimental configuration and typical $G_j(V,T,0)$'s are in Fig. 1(a). Table I lists sample and fitting parameters. G_j is assumed to be given by[28,29]

$$G_j(V,T) = C\int_{-\infty}^{\infty} dE N_1(E,T) N_n(E,T) \times \partial f(E-eV)/\partial(eV)|_T \,, \qquad (1)$$

where $f(E)$ is the Fermi function and C is a constant.

We also measure the change $\delta G_j(V,T,I_s)$ in $G_j(V,T)$ caused by a supercurrent I_s in the a-InO_x film. Specifically, $\delta G_j(I_s)$ vs I_s is measured for fixed current I through the junction; $\delta G_j(V,T,I_s)$ vs I_s at fixed voltage V is obtained in a straightforward fashion.[21] We observe that $\delta G_j \propto I_s^2$ and that δG_j is independent of the absolute and relative directions of I and I_s to within 1 wt.%.

A small supercurrent reduces Δ and reduces the energies of quasiparticle states near Δ slightly so that the peak in $N_1(E)$ is broadened and moved down in energy.[30] For given E and T, the change $\delta N_1(E,T,I_s)$ is linear in the "extrinsic" pair-breaking rate,[30,31] $1/\tau_{s,\mathrm{ext}} = \frac{1}{2}D(p_s/\hbar)^2 \propto I_s^2$, where $p_s \propto I_s$ is the superfluid momentum and D is the diffusion constant. $\delta G_j(V,T,I_s)$ is given by Eq. (1) with N_1 replaced by δN_1. Thus, except for thermal smearing, $\delta G_j(V,T,I_s) \propto \delta N_1(eV,T,I_s) \propto I_s^2$ is predicted and observed. Dividing $\delta G_j(V,T,I_s)$ by $G_j(V,T)$ normalizes out $C \times N_n(E)$ with data from the same temperature:

$$\frac{\delta G_j(V,T,I_s)}{G_j(V,T)} = \frac{\int dE\,\delta N_1(E,T,I_s) N_n(E,T)[-\partial f(E-eV)/\partial E]}{\int dE N_1(E,T) N_n(E,T)[-\partial f(E-eV)/\partial E]}\,, \qquad (2)$$

$$\approx \frac{\int dE\,\delta N_1(E,T,I_s)[-\partial f(E-eV)/\partial E]}{\int dE N_1(E,T)[-\partial f(E-eV)/\partial E]}\,. \qquad (3)$$

Numerical calculations with reasonable approximations for $N_n(E)$, such as $G_j(V,T)$ measured at $T/T_c = 0.97$ [Fig. 1(a)] where superconducting effects are small [cf. dotted line in Fig. 1(a)], show that the step from Eq. (2) to Eq. (3) is quite accurate.

To obtain $1/\tau_{\mathrm{in}}$, we need a model for $N_1(E,T)$ which includes the broadening effects of inelastic scattering and the elastic pair-breaking effect of a supercurrent. The Eliashberg equations are unsuitable because they are extremely difficult to solve for N_1 except near $T=0$ and $T=T_{c0}$. Instead, we use a phenomenological model[32] based on Beyer-Nielsen's approximation[33] to the Eliashberg equations. For vanishing supercurrent, the model matches the successful phenomenological model of Dynes, Narayanamurti, and Garno,[8] which includes inelastic scattering via an energy independent $1/\tau_{\mathrm{in}}$. For vanishing inelastic scattering, the model matches Maki's theory for the effect of a supercurrent on N_1.[30] Because the inelastic rate is small, $\hbar/\tau_{\mathrm{in}}\Delta \ll 1$, in our samples, we also calculate $\Delta(I_s)$ from Maki's theory. Tunneling measurements on Al and Sn films agree well with the model.[31] The model has many weaknesses, especially that it neglects the energy dependence of $1/\tau_{\mathrm{in}}$. However, because inelastic scattering is weak for our films, we believe the model serves as a physically reasonable interpolation between the two limits. A better analysis will be possible when theory can calculate the tunneling density of states in strongly disordered superconductors, including a supercurrent.

The parameters of the model are the normalized inelastic rate $\Gamma_{\mathrm{in}} \equiv \hbar/2\tau_{\mathrm{in}}\Delta$ and elastic pairbreaking rate $\Gamma_s \equiv \hbar/\tau_s\Delta$, the order parameter Δ, and the critical current $I_c(0)$ at $T/T_{c0}=0$. Intrinsic elastic pair breaking by phase fluctuations[20] should be included in Γ_s, in principle. Calculations show that intrinsic *elastic* pair breaking has a negligible effect on $\delta G_j/G_j$, so we neglect it. Hence, Γ_s is determined entirely by the applied supercurrent.

Mathematical details of the model are discussed elsewhere.[21,22,32] Results are shown in Fig. 2 for illustration. Solid lines show $N_1(E/\Delta)$ calculated for two cases: (1) The BCS limit of no inelastic scattering and no supercurrent ($\Gamma_{\mathrm{in}}=0$, $\Gamma_{s,\mathrm{ext}}=0$); and (2) a small inelastic rate

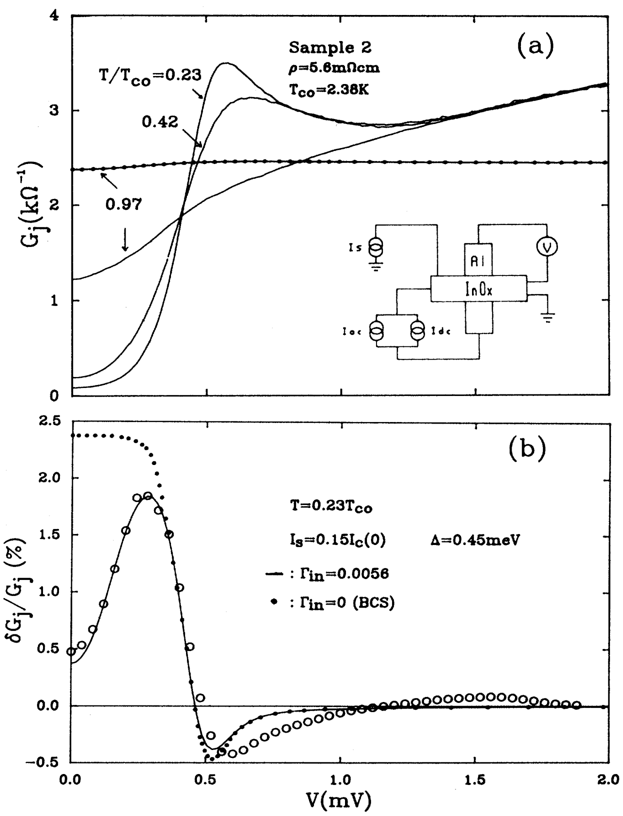

FIG. 1. (a) G_j vs V for sample 2. Calculation for BCS superconductor (dotted curve) shows smallness of superconducting effects at $T/T_c = 0.97$. Inset shows schematic sample geometry and wiring. (b) $\delta G_j/G_j$ vs V for sample 2. Dotted and solid curves are calculated for the BCS case $\Gamma_{\mathrm{in}}=0$ and for $\Gamma_{\mathrm{in}} = 0.0056$, respectively.

TABLE I. Parameters for a-InO$_x$ films: resistance per square $R_\square(4.2\text{ K})$, thickness d, resistivity $\rho_{4.2}=R_\square/d$, mean-field transition temperature T_{c0}, order parameter $\Delta(0)$, $2\Delta(0)/kT_{c0}$, fitted and calculated critical current $I_c(0)$, and $\hbar/\tau_{in}\Delta(0)$ extrapolated to T_{c0}.

Sample	$R_\square$ (kΩ)	d (Å)	$\rho_{4.2}$ (mΩ cm)	T_{c0} (K)	$\Delta(0)$ (meV)	$\frac{2\Delta(0)}{kT_{c0}}$	$I_c^{\rm fit}(0)$ (mA)	$I_c^{\rm cal}(0)$ (mA)	$\hbar/\tau_{in}\Delta(0)$ (Extrapolated to T_{c0})
1	0.68	350	2.4	3.28	0.63	4.46	33.5	24.5	0.60
2	1.60	350	5.6	2.38	0.45	4.39	11.4	9.6	2.28

and no supercurrent ($\Gamma_{in}=0.1$, $\Gamma_{s,\rm ext}=0$). The dotted curves show N_1 when a small supercurrent is applied ($\Gamma_{s,\rm ext}=0.02$) for the same values of Γ_{in}. The supercurrent shifts all quasiparticle energies downward, thus moving $N_1(E)$ toward its normal-state value of unity, i.e., increasing N_1 for $E<\Delta$, and decreasing N_1 for $E>\Delta$. Thus, $\delta N_1(E/\Delta)$ (inset) changes sign near $E=\Delta$.

Figure 1(b) shows $\delta G_j[V,I_s=0.15I_c(0)]/G_j(V,T)$ vs V for sample 2 with calculated curves for $\Gamma_{in}=0$ and for the best fit, $\Gamma_{in}=0.0056$. Clearly, in our model, the minimum in $\delta G_j/G_j$ at $V=0$ is from inelastic scattering. This feature can be traced to the low-energy tail in N_1 induced by inelastic scattering. At higher voltages where G_j is relatively constant, $\delta G_j/G_j$ resembles δG_j, changing sign near $eV=\Delta$ as expected.

The parameters Γ_{in} and Δ are determined quite accurately from fits to the minimum in $\delta G_j(V,T,I_s)/G_j(V,T)$ at $V=0$ and the zero crossing at $V\approx 0.5$ mV, respectively. Uncertainty in the fitted value of Γ_{in} is about 10% for $T\leq 0.6T_{c0}$, i.e., where thermal smearing is small. The uncertainty in Δ is about 2%. $\Delta(I_s=0,T)/\Delta(0,0)$ has the BCS form within 2% for both samples over the measured range, 0.2–0.7T_{c0}.[21]

$I_c(0)$, the ideal zero-temperature depairing critical current, is determined by fitting the overall magnitude of $\delta G_j/G_j$. A small $I_c(0)$ means that a small supercurrent produces a large pair-breaking effect. In the weak-coupling dirty limit, $I_c(0)$ is related to the residual resistivity $\rho_{4.2}$, film width w and thickness d, and $\Delta(0)$ through[20]

$$I_c^2(0)=0.466d^2w^2 2N_n(0)\Delta(0)^3/\hbar\rho_{4.2}\,. \tag{4}$$

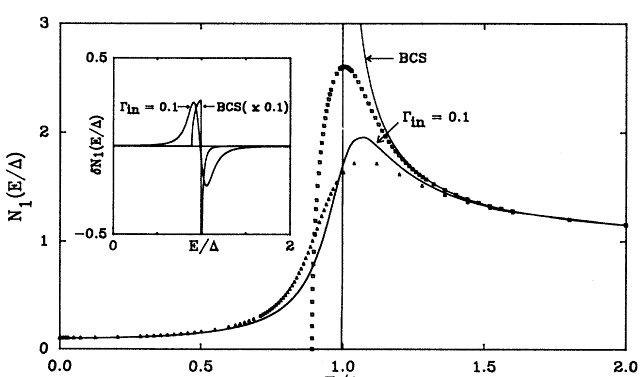

FIG. 2. Calculated $N_1(E/\Delta)$ vs E/Δ for BCS case, $\Gamma_{in}=0$, and for $\Gamma_{in}=0.1$ (solid curves). Calculated effect of a supercurrent shown by dotted curves with $\Gamma_{s,\rm ext}=0.02$. Inset: $\delta N_1(E/\Delta)\equiv N_1(E/\Delta,I_s)-N_1(E/\Delta,I_s=0)$ for the same two cases.

We use the measured $\Delta(0)$ and $N(0)\approx 1.7\times 10^{27}$/eV m^3, which follows from the electron density[23] $n\approx 4\times 10^{20}$ cm^{-3}. Modifications to this relation due to strong coupling and disorder-induced suppression of $N_n(E)$ are unknown. Fitted values agree well with Eq. (4) (Table I). Moreover, a single value of $I_c^{\rm fit}(0)$ for each sample fits data at all temperatures, meaning that the superfluid density, $n_s(T)$, which relates j_s to p_s, has the weak-coupling dirty-limit dependence on T despite strong coupling and the suppression in $N_n(E_F)$.

Figure 3 shows $\delta G_j/G_j$ for sample 1. Agreement with the model (solid curves) is good, especially for $eV\lesssim\Delta$. The inset of Fig. 3 shows $\hbar/\tau_{in}(T)\Delta(0)$ vs $(T/T_{c0})^3$ for both samples, showing that $\hbar/\tau_{in}\propto T^3$ even for $\Gamma_{in}\ll 1$. This is our major result. Despite the crudeness of our model, we believe that $1/\tau_{in}$ is representative of the magnitude and T dependence of an appropriate average over microscopic rates which depend somewhat on energy.

Inelastic scattering comes from electron interactions with electrons, phonons, and possibly two-level systems

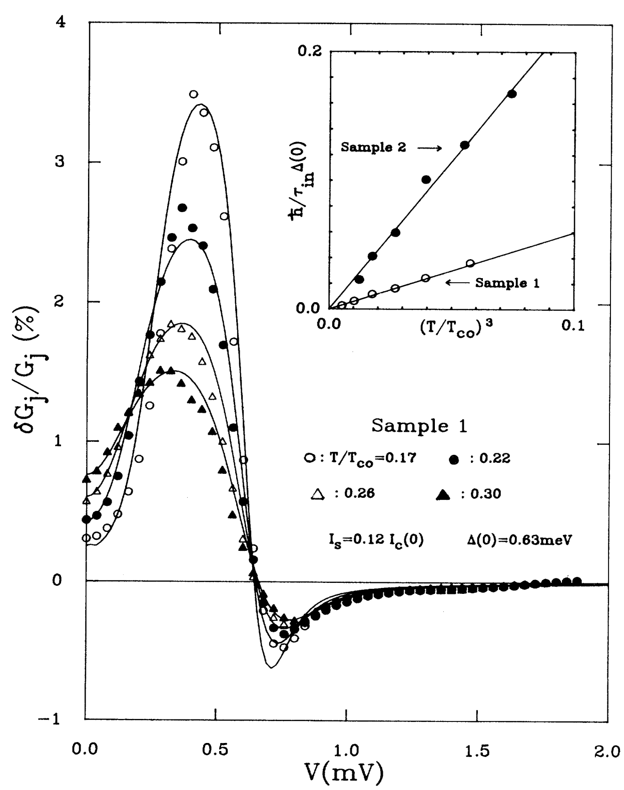

FIG. 3. Measured $\delta G_j/G_j$ vs V of sample 1. Phonon structure appears centered about 1.7 mV. Inset shows $\hbar/\tau_{in}\Delta(0)\propto T^3$ for both samples.

(TLS).[34,35] Very recently, Devereaux and Belitz[12] calculated the imaginary part of the quasiparticle self-energy at $E=\Delta$ for $kT/\Delta \ll 1$ including e-e and e-ph coupling. They find that electron scattering from phonons, as opposed to recombination due to phonon emission, dominates over electron-electron scattering. Furthermore, they find that their calculation accounts for the observed magnitude of $1/\tau_{\mathrm{in}}$ and its dependence on T and $\rho_{4.2}$. This agreement is very encouraging. Devereaux and Belitz[12] discuss the interpretation of inelastic scattering in granular Al and Sn in light of their new results.

We emphasize that our results indicate that the low-energy density of states vanishes as a power of T, not exponentially. This should lead to power-law behavior in many physical quantities at low temperatures.

Returning to the data, note that $\delta G_j/G_j$ becomes positive again above 1 mV. We interpret this as evidence for a phonon mode at $\hbar\omega_{\mathrm{ph}} \approx 0.7$ meV which would move quasiparticle states from just above to just below $E=\hbar\omega_{\mathrm{ph}}+\Delta$, thereby reducing and augmenting N_1 at these energies, respectively.[36] This strong-coupling effect is not included in our model. It is likely that it is responsible for discrepancies between the data and the model at $eV \gtrsim \Delta$. It also may be responsible for our inability to obtain a good fit to $G_j(V,T)/G_j(V,T_c)$ with the model of Dynes *et al.*[8]

Film homogeneity is a strong concern. TEM measurements on films similar to ours show them to be mostly amorphous InO with occasional crystallites of In_2O_3 from 40 to 400 Å across.[25] The crystallites occupy roughly 1% of the total surface area. If these crystallites were normal metal and were responsible for the low-energy tail in $N_1(E)$, then one might expect $N_1(E)$ to be a BCS density of states plus a constant, ≈ 0.01, for $E \leq \Delta$, rather than an inelastically broadened BCS density of states, which describes the measured density of states much better. Also, $N_1(0)$ decreases to much less than 0.01 as T decreases and never saturates. Furthermore, the size and number of crystallites *decreases*[25] with increasing ρ while $N_1(0)$ *increases*. Thus we believe that these visible inhomogeneities are not the source of our results.

In summary, the inelastic scattering rate in heavily disordered superconducting a-InO_x is proportional to T^3 even at low temperature where $\hbar/\tau_{\mathrm{in}} \ll \Delta$. A plausible explanation is electron-phonon scattering. Confirmation of our results in other materials and extension to films ranging from 2D to 3D is needed. Extension of theory to calculation of the density of states is essential for detailed interpretation of the data.

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