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Evaporation of a single vortex line in a weakly coupled multilayered superconductor

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A single Abrikosov line is considered in a field perpendicular to the layers in superconductors with Josephson coupling of the layers. It is shown that the vortex line evaporates above some critical temperature T^* in compounds with perpendicular superconducting correlation length $\xi_z(0) \ll d/\kappa$ where κ is the Ginzburg parameter and d is the interlayer distance. In a system with identical layers, T^* coincides with the critical temperature of the Kosterlitz-Thouless transition.

I. INTRODUCTION

The problem of thermal fluctuations in type-II superconductors presents, in the case of layered structures, quite striking and fascinating features. For rather strongly coupled superconducting layers, the anisotropy confers on the three-dimensional (3D) Abrikosov vortex lattice peculiar elastic properties: easy shear of the lattice for fields perpendicular to the layers, that leads to the possibility of a liquid vortex state.[1] For weakly coupled superconducting layers in the limit of Josephson coupling,[2] other phenomena may occur due to the weak superconducting coupling of 2D vortices in different layers.

A first question is raised in comparison to thin films where bound vortex-antivortex pairs are generated by thermal fluctuations below the Kosterlitz-Thouless (KT) temperature $T_{\rm KT}$, and free vortices above $T_{\rm KT}$. Such a mechanism operates in layered superconductors if the interlayer coupling is weak enough. In the limiting case of vanishing Josephson interaction, the temperature $T_{\rm KT}$ is given by the usual 2D expression because electromagnetic coupling of 2D vortices in different layers can be neglected,[3,4] see below. The effect of Josephson interactions on the KT transition has not yet been clarified; the dependence of $T_{\rm KT}$ on the strength of such coupling was estimated by use of heuristic arguments.[3,5] Some evidence of KT behavior has indeed been obtained in copper-based layered superconducting oxides.[6–8]

A second question concerns the validity of the concept of a vortex line crossing the layers (the usual Abrikosov 3D vortex). For layered structures, the correct picture is that of a linear chain of coupled 2D vortices, and one should ask whether such an object is stable against thermal fluctuations. This question concerns the stability of the Abrikosov vortex line against less-ordered structures, such as a gas of point 2D vortices. The answer to this question, would help us to understand, at least in weak fields, the nature of the irreversibility line (or melting line) observed in superconducting oxides (for a review, see Ref. 9).

We address in the following the second question and show that the usual Abrikosov vortex line does not exist in layered superconductors above some critical temperature T^* if the interlayer coupling is weak enough. In a simple layered structure made of identical layers, T^* coincides with $T_{\rm KT}$. To obtain such a picture we start from the interesting limit of vanishing Josephson coupling, realized, for instance, in long-scale superconducting superlattices.[10–12] In that case, the magnetic field is screened only by currents flowing inside the layers, as in the thin-film geometry. As a consequence, a long-range coupling of 2D vortices through the 3D magnetic field remains in the perpendicular direction.[13,6,14] In such an electromagnetic (EM) model a real evaporation of the Abrikosov line happens, i.e., a phase transition takes place from a line to a gas of free 2D vortices. Josephson coupling leads to confinement of these 2D vortices, and as a result this gas fills the tube, the thickness of which grows as the Josephson coupling diminishes.

II. ENERGY OF A VORTEX LINE

We consider a single vortex line in a field perpendicular to the layers. In a weakly coupled layered structure, this line is indeed a chain of 2D vortices in each layer. Thermal fluctuations introduce some distortions along the line, and the final structure can be obtained in the usual way, starting from the energy functional that describes the coupling of 2D vortices in different layers. The functional of interest is the Lawrence-Doniach free-energy functional which depends on the phase of the order parameter $\Psi_n(\mathbf{r}) = \Psi_0 \exp(i\phi_n)$ and on the vector potential:

$$F\{\chi_n(\mathbf{r}),\mathbf{B}(\mathbf{R})\} = \frac{H_{c0}^2}{8\pi}\tau\sum_n\int d\mathbf{r}\left[\xi_\parallel^2(0)\left(\frac{\partial\chi_n}{\partial\mathbf{r}}\right)^2 + \rho[1-\cos(\chi_n-\chi_{n+1})]\right] + \int d\mathbf{R}\frac{\mathbf{B}^2}{8\pi}\ , \tag{1}$$

where $\mathbf{r}=(x,y)$, $\mathbf{R}=(\mathbf{r},z)$, $\mathbf{B}=\mathrm{rot}\mathbf{A}$, $\rho=2\xi_z^2(0)/d^2$. The z axis is perpendicular to the layers. H_{c0} is the thermodynamical critical field extrapolated to T=0, and $\tau=(T_c-T)/T_c$. Here d is the interlayer distance, $\xi_\parallel(0),\xi_z(0)$ are, respectively, the parallel and perpendicular correlation lengths extrapolated to $T=0$, $\chi_n(\mathbf{r})=\phi_n(\mathbf{r})-(2\pi/\Phi_0)\int_0^{\mathbf{R}}\mathbf{A}\cdot d\boldsymbol{l}$ is the gauge-invariant phase, $\phi_n(\mathbf{r})$ is the phase of the order parameter, and $\Phi_0=hc/2e$. The dimensionless parameter ρ describes the strength of the Josephson coupling in comparison with intralayer condensation energy. Josephson coupling of the layers corresponds to the limit $\rho\ll 1$.

Considering one vortex in each layer we need to calculate the free energy of the system with functional (1), under the condition

$$\oint\nabla\phi_n(\mathbf{r})\cdot d\boldsymbol{l}=2\pi \tag{2}$$

for a path integral around some point defined by coordinates $\mathbf{r}_n$ (the center of a 2D vortex) in layer n. The functional integration in the partition function involves the integration over variations of the shape of vortices, i.e., the functions $\phi_n(\mathbf{r},\mathbf{r}_n)$ and the coordinates $\mathbf{r}_n$. The set of equilibrium values $<\mathbf{r}_n>$ describes the Abrikosov line if the correlations of $\mathbf{r}_n$ and $\mathbf{r}_{n+1}$ are strong enough.

III. THE CRITICAL TEMPERATURE OF EVAPORATION IN THE EM MODEL

Let us consider the limit of vanishing Josephson coupling, $\rho=0$. The corresponding equation for the mean-field phase reads $\Delta\phi_n(\mathbf{r},r_n)=0$ where $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2$, using the gauge $\mathrm{div}\mathbf{A}=0$. Thus, neglecting unimportant fluctuations in the shape of 2D vortices we get $\phi_n(\mathbf{r},\mathbf{r}_n)=\arctan[(x-x_n)/(y-y_n)]$. The vector potential therefore obeys the linear equation

$$\Delta\mathbf{A}(\mathbf{R})=\frac{1}{\Lambda}\sum_n\left(\mathbf{A}(\mathbf{R})-\frac{\Phi_0}{2\pi}\frac{\partial\phi_n}{\partial\mathbf{r}}\right)\delta(z-nd)\ , \tag{3}$$

where $\Lambda=\lambda_L^2/d$ is the effective penetration depth of a single layer. Solving (3) and calculating the free-energy functional (1) for a given set of coordinates, $\mathbf{r}_n$, one obtains the EM interaction energy of 2D vortices:[14]

$$F_{\mathrm{EM}}(\mathbf{r}_1,\ldots,\mathbf{r}_N)=\sum_{m\neq n}G_{m-n}(\mathbf{r}_n-\mathbf{r}_m)+F_{\mathrm{EM}}(0),$$
$$G_n(\mathbf{r})=\frac{\Phi_0^2}{4\pi\Lambda}\int\frac{d\mathbf{q}}{(2\pi)^2}\frac{\sinh(qd)}{2\Lambda q^3\sqrt{G_q^2-1}}(e^{i\mathbf{q}\cdot\mathbf{r}}-1)\times(G_q-\sqrt{G_q^2-1})^{|n|}\ , \tag{4}$$

where $G_q=\cosh(qd)+\sinh(qd)/2\Lambda q$. The quantity $F_{\mathrm{EM}}(0)$ is the ground-state energy which corresponds to the straight line $\mathbf{r}_n=0$.

The energy (4) describes the interaction of supercurrents associated with vortices in different layers m and n via a magnetic field screened by currents in all other layers. This screening gives a small value for the interaction (an additional factor of d/λ_L in the limit $d\ll\lambda_L$ in comparison with the intralayer interaction of vortices) as well as an exponential decay of the interaction with the distance between the layers m and n. At the same time the asymptotic behavior of the interaction (4) at large distances along layers $|\mathbf{r}_n-\mathbf{r}_m|\gg\lambda_L$ is logarithmic as in the 2D case because only the currents inside layers are counted in the limit $\rho=0$. We now obtain a system of point particles (one in each layer) with coordinates $\mathbf{r}_n$, the particles' interaction being given by (4). Thermal fluctuations cause the displacements of these particles from the straight vortex line.

We first consider small long-wave distortions in the harmonic approximation. Due to the 1D character of the system of vortices, such fluctuations remove long-range order, preserving only the short-range one, which is actually sufficient to define a vortex line. We obtain in this approximation the effective line stiffness with respect to bending of the line, which is similar to that of a usual bulk vortex line. As a result one finds a random wandering of the line, described by a diffusionlike expression:

$$\frac{<(\mathbf{r}_n-\mathbf{r}_0)^2>}{\lambda_L^2}=\frac{32\pi^2 dT}{\Phi_0^2}n\ . \tag{5}$$

Since the dimensionless diffusion coefficient in the right-hand side of (5) is very small, essential deviations $\langle(\mathbf{r}_n-\mathbf{r}_0)^2\rangle$ of order λ_L^2 occur at a distance of the order of 10^5 layers at $T\sim 100$ K (taking $\lambda_L=1500$ Å and $d=10$ Å). Such distortions are unimportant and can be neglected. We note that the Josephson interaction gives an additional contribution to the stiffness of line, that can be neglected in comparison with the EM contribution if $\rho\ll\kappa^{-2}$, where κ is the Ginzberg parameter.

We now consider the short-wavelength distortions. They correspond to a relative motion of particles in neighboring layers, i.e., values of $|\mathbf{r}_n-\mathbf{r}_{n+1}|$ are of interest. The Abrikosov line exists if values of $\boldsymbol{l}_n=\mathbf{r}_n-\mathbf{r}_{n+1}$

that are small in comparison with λ_L are the most probable. Fixing as a reference the position of the first vortex, one can say that the line exists if the probability distribution function $P(\mathbf{r}_2, \mathbf{r}_3, \ldots, \mathbf{r}_N)$ (for an N-layer system) can be normalized, otherwise we get a gas of uncorrelated particles instead of a line. This criterion is similar to the normalization of probability density in quantum mechanics being the criterion for a bound (localized) state to exist. The normalization condition depends on the behavior of $P\{\mathbf{r}_n\}$ for large $\boldsymbol{l}_n$, which is determined by the asymptotic form of the interaction G_{n-m} at large $|\mathbf{r}_n - \mathbf{r}_m| \gg \lambda_L$. This form is obtained from (4):

$$F_{\mathrm{EM}}(\mathbf{r}_n, \mathbf{r}_m) = \frac{\Phi_0^2 d^2}{16\pi^2\lambda^3} \ln\frac{|\mathbf{r}_n - \mathbf{r}_m|}{\lambda_L} e^{-|n-m|d/\lambda_L} . \tag{6}$$

We note that the parameter λ_L/d is very large and the logarithmic interaction (6) is very weak. However, due to the very slow decay of the interaction along the z axis the number of effectively interacting particles is large.

Using the same arguments as in the case of KT transition, we came to the conclusion that a critical temperature T^* should exist above which particles cannot be localized near the line, the entropy gain being larger than the energy lost if the line were to evaporate into a gas of particles.

To obtain T^* we use a scaling argument. Let us consider the region of variables $d\mathbf{r}_2 \cdots d\mathbf{r}_N$ with large values $|\mathbf{r}_n - \mathbf{r}_m|$. The probability of finding the system in this region is given by the expression

$$dP = P(\mathbf{r}_2, \ldots, \mathbf{r}_N) d\mathbf{r}_2 \cdots d\mathbf{r}_N = \frac{1}{Z} \exp\left(-\frac{\alpha d}{4\lambda_L} \sum_{n \neq m} \ln\frac{|\mathbf{r}_n - \mathbf{r}_m|}{\lambda_L} e^{-|n-m|d/\lambda_L} \right) d\mathbf{r}_2 \cdots d\mathbf{r}_N , \tag{7}$$

where $\alpha = \Phi_0^2/8\pi^2\Lambda T$. In (7), the quantity Z is the partition function (for normalization of P). Rescaling the variables by a factor C, i.e., $\mathbf{r}'_n = C\mathbf{r}_n$, we get the new region $d\mathbf{r}'_n \cdots d\mathbf{r}'_N$. One finds from (7) the probability $dP' = C^S dP$ with $S = (-\alpha/2 + 2)N - 2$. For large N and $C > 1$ the probability grows with C if $T > T^*$, where

$$T^* = \frac{\Phi_0^2 d}{32\pi^2\lambda_L^2(T^*)} . \tag{8}$$

Thus for $T > T^*$ the probability cannot be normalized. This means that the particles are delocalized above T^* and an Abrikosov line does not exist: we have instead a free gas of 2D vortices. The temperature T^* coincides with T_{KT} in the model under consideration.

IV. DISTRIBUTION FUNCTION FOR THE DISTORTIONS OF 2D VORTICES

Taking the line $\mathbf{r}_n = 0$ as the center of the vortex line, we can calculate the distribution function $p_n(\mathbf{r})$ for distortions of 2D vortices in layer n from the ground-state position (straight line). Actually deviations exist due to long-wavelength fluctuations; these are governed by the diffusion law (5). Such deviations are indeed very small up to T^*: the dimensionless coefficient is of the order of $(d/\lambda_L)^2$ near T^* and on the scale λ_L of the interaction, deviations from the ground-state line are less than d.

Due to the long-range type of the interaction between vortices (of order λ_L), each vortex interacts with about (λ_L/d) other vortices. It is therefore reasonable to perform a mean-field (MF) calculation of the free energy and assume the vortex positions in different layers to be uncorrelated. For a given distribution function $p_n(\mathbf{r})$ for the vortex position at coordinate $\mathbf{r}$ in layer n, one can obtain the energy $f_n(\mathbf{r})$ at coordinate $\mathbf{r}$ of a 2D vortex, and the equations for self-consistency of the functions $p_n(\mathbf{r})$ as

$$\begin{gathered} f_m(\mathbf{r}) = \sum_n \int d\mathbf{r}' G_{m-n}(\mathbf{r} - \mathbf{r}') p_n(\mathbf{r}'), \\ p_n(\mathbf{r}) = Z_n^{-1} e^{-f_n(\mathbf{r})/T}, \quad Z_n = \int d\mathbf{r}\, e^{-f_n(\mathbf{r})/T} . \end{gathered} \tag{9}$$

The vortex line exists if all functions $p_n(\mathbf{r})$ can be normalized to unity.

Inside the crystal (at distances $z \gg \lambda_L$ from the surface) all the $f_n(\mathbf{r})$ are similar and

$$f(\mathbf{r}) = \int d\mathbf{r}' G(\mathbf{r} - \mathbf{r}') p(\mathbf{r}'), \quad G(\mathbf{r}) = \sum_n G_n(\mathbf{r}) . \tag{10}$$

When $p(\mathbf{r})$ is normalized, the asymptotic behavior of $f(\mathbf{r})$ and $G(\mathbf{r})$ is the same. The latter has logarithmic behavior at large distances $r \gg \lambda_L$:

$$G(\mathbf{r}) = \frac{\Phi_0^2}{8\pi^2\Lambda} \ln\frac{r}{\lambda_L} . \tag{11}$$

Using (9) and (11) we see that the distribution function has a power-law behavior, $p(\mathbf{r}) \propto r^{-\alpha}$ and exponent α diminishes with temperature.

Near the surface the interaction (6) should be changed. For layers $n, m > 0$ the factor $e^{-|n-m|d/\lambda_L}$ in (6) should be replaced by $(e^{-|n-m|d/\lambda_L} + e^{-|n+m|d/\lambda_L})$. So the asymptotic behavior of the distribution function near the surface is the same as in the bulk.

The distribution function is normalized below the temperature T_{MF} which is given by the equation $\alpha(T_{\mathrm{MF}}) = 2$. Thus T_{MF} is higher than the critical temperature obtained by the scaling argument which gives the condition $\alpha(T^*) = 4$.

The result is thus a power-law asymptotic behavior for the distribution function of 2D vortices, as well as for the magnetic induction $\mathbf{B}(\mathbf{r})$ in the model under consideration, while in the usual Abrikosov vortex the asymptotic behavior of $\mathbf{B}(\mathbf{r})$ is exponential.

V. THE ROLE OF JOSEPHSON COUPLING

In contrast to electromagnetic coupling, Josephson coupling is short range and couples only adjacent layers. Let us evaluate the Josephson energy $F_J(\mathbf{r})$ for two 2D vortices of the same sign, each in one neighboring layer and separated by $\mathbf{r}$ in the direction parallel to the layers. Now although the form of vortices depends on $\mathbf{r}$, they are not cylindrical as in the case of EM coupling. The phase difference $\phi = \phi_n - \phi_{n+1}$ obeys the equation[3,15]

$$\xi_{\parallel}^2(0)\frac{\partial^2\phi}{\partial \mathbf{r}^2} + \rho\sin\phi = 0\ . \tag{12}$$

Thus ϕ decays exponentially on a characteristic length $r_J = \xi_{\parallel}(0)/\sqrt{\rho}$. This allows us to evaluate $F_J(\mathbf{r})$ in two limiting cases. First, if $r \ll r_J$, the main contribution comes from the region of integration $r \ll r_J$ where vortices are nearly cylindrical, which gives[3]

$$F_J(r) = \rho\frac{H_{c0}^2\tau}{32\pi}dr^2\ln\frac{r_J}{\min(\xi_{\parallel}, r)}\ . \tag{13}$$

On the other hand, if $r \gg r_J$, the phase is perturbed in a one-dimensional region of length $r \geq r_J$, which yields

$$F_J(r) = \sqrt{\rho}\frac{H_{c0}^2\tau}{8\pi}d\xi_{\parallel}(0)r. \tag{14}$$

We see from (13) and (14) that Josephson coupling provides confinement and prevents the complete evaporation of 2D vortices. At $T > T^*$ the vortices indeed evaporate into a thicker tube of radius r_0 which is determined by the condition $F_J(r_0) \approx T$. Such a picture is valid provided $r_0 \gg \lambda_L$. As a result the expansion of the line to a tube of Josephson radius $r_0 \approx \pi r_J \gg \lambda_L$ takes place in compounds if $\rho \ll \kappa^{-2}$.

The lower critical field H_{c1} above T^* is given by the expression

$$H_{c1}(T) = \frac{\Phi_0}{4\pi\lambda_L^2(T)}\ln\kappa - \frac{4\pi T}{\Phi_0}\ln\frac{\xi_{\parallel}^2(0)}{\rho\xi_{\parallel}^2(T)}. \tag{15}$$

At T^* we get

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda_L^2(T^*)}\ln(\kappa\rho^{1/2}) \tag{16}$$

and $H_{c1}(T^*)$ is zero at that temperature, if $\rho \leq \kappa^{-2}$.

In conclusion we have proved that in multilayer compounds with very weak Josephson coupling the vortex line expands as temperature grows, and the distribution of the field in a vortex has an asymptotic power-law behavior instead of the standard exponential one. The expansion of vortex lines can be observed experimentally. The Bi- and Tl-based oxides are possible candidates for such a study, together with artificial superlattices of Y-Ba-Cu-O/Pr-Ba-Cu-O type. In a system with identical layers, the vortex line evaporates into a gas of 2D vortices at the temperature $T^* = T_{\mathrm{KT}}$ and at the same point the lower critical magnetic field tends to zero as T approaches T^*.

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[1] D.R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).

[2] W.E. Lawrence and S. Doniach, in *Proceedings of the 12th International Conference on Low Temperature Physics, Kyoto, 1971*, edited by E. Kanda (Keigaku, Tokyo, 1971), p. 361.

[3] M.V. Feigelman, V.B. Geshkebein, and A.I. Larkin, Physica C **167**, 177 (1990).

[4] B. Horovitz (private communication).

[5] L.I. Glazman and A.E. Koshelev, this issue, Phys. Rev. B **43**, 2835 (1991).

[6] P.C.E. Stamp, L. Forro, and C. Ayache, Phys. Rev. B **38**, 2847 (1988); P.C.E. Stamp, Phys. Rev. Lett. **63**, 582 (1989).

[7] S. Martin, A.T. Fiory, R.M. Fleming, G.P. Espinosa, and A.S. Cooper, Phys. Rev. Lett. **62**, 677 (1989).

[8] D.H. Kim, A.M. Goldman, J.H. Kang, and R.T. Kampwirth, Phys. Rev. B **40**, 8834 (1989).

[9] A.P. Malozemoff, in *Physical Properties of High Temperature Superconductors–I*, edited by D.M. Ginsberg (World Scientific, Singapore, 1989).

[10] I.K. Schuller and C.M. Falco, in *Inhomogeneous Superconductors–1979 (Berkeley Springs, WV)*, Proceedings of the Conference on Inhomogeneous Superconductors, edited by D.V. Gubser *et al.*, AIP Conf. Proc. No. 58 (AIP, New York, 1979), p. 197.

[11] J.-M. Triscone, M.G. Karkut, K. Antognazza, O. Brunner, and O. Fischer, Phys. Rev. Lett. **63**, 1016 (1989); J.-M. Triscone, O. Fischer, O. Brunner, L. Antognazza, A.D. Kent, and M.G. Karkut, *ibid.* **64**, 804 (1990).

[12] D.M. Lowndes, D.P. Norton, and J.D. Budai, Phys. Rev. Lett. **65**, 1160 (1990).

[13] K.B. Efetov, Zh. Eksp. Teor. Fiz. **76**, 1781 (1979) [Sov. Phys. JETP **49**, 905 (1979)].

[14] A. Buzdin and D. Feinberg, J. Phys. (Paris) **51**, 1971 (1990).

[15] S. Doniach, in *High Temperature Superconductivity*, Los Alamos Symposium 1989, edited by K.S. Bedell *et al.* (Addison-Wesley, Redwood City, 1990), p. 406.