Charge-fluctuation effect on the critical temperature of layered high-T_c superconductors

D. Ariosa and H. Beck
Institut de Physique, Université de Neuchâtel, rue A.-L. Breguet 1, 2000 Neuchâtel, Switzerland
(Received 22 May 1990)

The value of the superconducting critical temperature (T_c) of artificially layered superconductors made out of alternating Y-Ba-Cu-O and Pr-Ba-Cu-O layers is calculated as a function of both the number of layers of pure Y-Ba-Cu-O within a unit cell of the superlattice and the thickness of the insulating (Pr-Ba-Cu-O) layer. The calculation is performed within a model of electrostatically coupled two-dimensional (2D) arrays of ultrasmall Josephson junctions. The underlying mechanism is assumed to be the depression of the Beresinskii-Kosterlitz-Thouless transition temperature ($T_{\rm BKT}$) by quantum phase fluctuations due to charging effects. The T_c value of the entire structure can then be tuned by varying the charging energy that depends on the average neighborhood of a typical site in Cu-O planes of Y-Ba-Cu-O layers. Recent experiments on such structures are described very accurately by the model. Furthermore, the ratio $(T_c^{(n)}-T_c^{(1)})/(T_c^{(2)}-T_c^{(1)})$, where $T_c^{(n)}$ is the critical temperature of a thin film made up of n Y-Ba-Cu-O unit cells, is found to be independent of the fitting parameters. This prediction is well confirmed by available experimental data. Furthermore, the model also applies for the series of Bi- and Tl-based cuprates of general formulas $Bi_2Sr_2Ca_{n-1}Cu_nO_y$ and $Tl_{2\,(1)}Ba_2Ca_{n-1}Cu_nO_y$ containing n Cu-O planes per unit cell, for which the observed values of the ratio $\rho=(T_c^{(3)}-T_c^{(1)})/(T_c^{(2)}-T_c^{(1)})$ agree within 10% with the predicted ones.

I. INTRODUCTION

Recent experiments on periodic artificially layered high-T_c superconductors (HTCS) made of alternating layers of (superconducting) Y-Ba-Cu-O and (insulating) Pr-Ba-Cu-O (Refs. 1 and 2) have shown a very peculiar dependence of T_c as a function of both the number n of Y-Ba-Cu-O pure layers and the thickness d of the insulating layer. These results strongly suggest that superconductivity can occur in a single (12 Å) layer of Y-Ba-Cu-O but the coupling between monolayers along the c axis is needed to reach $T_c=90$ K as observed in bulk materials. However, the nature of this coupling is not clear at the moment. The fact that the T_c value in the bulk is close to ten times the value in a monolayer suggests that Josephson coupling along the c axis is not a realistic explanation. Indeed, such a coupling can predict at most the value of the ratio between the critical temperatures of isotropic three-dimensional (3D) XY and 2D XY models, i.e., a ratio of order 3.[3] On the other hand, the existence of the (12 Å)/(12 Å) superlattice with very different properties than the alloy[1] and further structural characterization[4] of interfaces, suggests that interdiffusion cannot account for the observed behavior.

In HTCS, the very small coherence length makes possible the weakening of superconductivity over short distances, giving rise to Josephson barriers located at defects such as twin boundaries. In the following we will assume this picture, as many authors did (See Ref. 17), since Müller *et al.* (See Ref. 18) have suggested the existence of an intrinsic intragrain weak-link structure, in order to explain the very unusual magnetic properties (such as the $1-T/T_c\approx H^{2/3}$ "irreversibility line") observed by them on La-Ba-Cu-O samples.

In this paper each pair of Cu-O planes in the Y-Ba-Cu-O structure is taken as a 2D array of Josephson junctions. When insulated and in the absence of charging effects, this system undergoes a BKT transition at a temperature T_{c0}. Charging effects will depress the value of T_c. The diagonal charging energy contribution of an individual site of the array will be calculated in function of its relative position with respect to the conducting (Cu-O) planes of the neighboring Y-Ba-Cu-O cells. As expected, the value of T_c varies with the average spacing between adjacent Y-Ba-Cu-O unit cells, reproducing very accurately experimental data.

The paper is organized as follows. In Sec. II we introduce a linearized version of the T_c dependence on quantum effects (charging effects), in the context of a self-consistent harmonic approximation (SCHA). In Sec. III we evaluate the self-capacitance of a typical site for structures as in Refs. 1 and 2. In Sec. IV we combine results of the two preceding sections to give explicit expressions for the T_c versus d dependence for different values of n. We also present universal relationships that follow from these expressions and we compare them with experimental observations. Finally, we summarize and draw conclusions in Sec. V.

II. 2D-XY MODEL AND QUANTUM FLUCTUATIONS

A. Definition of the model

Arrays of Josephson junctions in zero magnetic field and including charging effects are usually described by the following Hamiltonian:[5]

$$H=\sum_i \frac{(2e)^2 n_i^2}{2C}-J\sum_{\langle i,j\rangle}\cos(\theta_i-\theta_j)\ . \tag{2.1}$$

In (2.1), e stands for the elementary electric charge, n_i is the number of Cooper pairs sitting at site (i), C the self-capacitance of the site, and J the Josephson coupling constant. The first summation over the sites is the diagonal contribution to the electrostatic energy of the system, which is assumed to be dominant with respect to off-diagonal terms. The second summation, over oriented bonds, is the standard exchange coupling term of the classical XY model.

An operator $\bar{n}_i$, that measures the number of Cooper pairs on a site (i), can be defined as

$$\bar{n}_i = \frac{\hbar}{i}\frac{\partial}{\partial\theta_i} . \tag{2.2}$$

$\bar{n}_i$ and θ_i are cannonically conjugate and satisfy the commutation relations

$$[\bar{n}_i, \theta_j] = \frac{\hbar}{i}\delta_{ij} . \tag{2.3}$$

The Hamiltonian (2.1) can be rewritten in the form

$$H = \sum_i \frac{\bar{n}_i^2}{2M} - J\sum_{\langle i,j\rangle}\cos(\theta_i-\theta_j) \equiv Q+U , \tag{2.4}$$

where

$$M = \frac{\hbar c}{(2e)^2} .$$

From the commutation relations (2.3) it follows, in accordance with the Heisenberg principle, that when the charge on a site is well defined, the uncertainty on the phase would be considerable, leading to quantum phase fluctuations in the system. This situation will occur at low temperatures if the coupling energy E_J is small compared with the electrostatic energy E_c. In order to characterize this situation, one introduces the parameter α as

$$\alpha = \frac{E_c}{E_J} = \frac{\hbar^2}{2MJ} = \frac{2e^2}{CJ} . \tag{2.5}$$

B. Variational method (SCHA)

The SCHA procedure consists in replacing the cosine potential in the Hamiltonian (2.4) by a variational harmonic potential of the form

$$U_{TR} = \tfrac{1}{2}K\sum_{\langle i,j\rangle}(\theta_i-\theta_j)^2 , \tag{2.6}$$

where K is an effective coupling constant that has to be adjusted in order to minimize the variational free energy given by

$$F = F_{TR} + \langle U\rangle_{TR} - \langle U_{TR}\rangle_{TR} . \tag{2.7}$$

Thermodynamic averages in (2.7) are performed as

$$\langle A\rangle_{TR} = \frac{\mathrm{tr}[A\exp(-H_{TR}/k_BT)]}{Z_{TR}} \tag{2.8}$$

with

$$H_{TR} = Q + U_{TR}$$

and

$$Z_{TR} = \prod_k \frac{1}{2\sinh(\frac{1}{2}\hbar\omega_k/k_BT)} . \tag{2.9}$$

Z_{TR} being the partition function of a set of coupled quantum harmonic oscillators. Defining the reduced variables

$$f = \frac{E}{NJ}, \quad t = \frac{k_BT}{J}, \quad \text{and} \quad \gamma = \frac{K}{J}$$

(N being the number of sites in the array) we can write for the variational free energy density, the expression

$$f = f_{TR} - 2\exp(-\tfrac{1}{2}X) - \gamma X \tag{2.10}$$

with

$$X \equiv \langle(\theta_i-\theta_j)^2\rangle_{TR} .$$

In minimizing f with respect to γ, we get

$$X = \frac{t}{2\gamma}\left[\frac{A}{3} + D_2(A)\right] , \tag{2.11}$$

where

$$A = \frac{2(\pi\alpha\gamma)^2}{t}$$

and

$$D_2(A) = \frac{2}{A^2}\int_0^A dx\frac{x^2}{\exp(x)-1} .$$

In minimizing f with respect to X, we get

$$\gamma = \exp(-\tfrac{1}{2}X) . \tag{2.12}$$

This last expression is a self-consistent equation defining the optimal coupling constant $\gamma(t)$ for each temperature. The critical temperature is reached when Eq. (2.12) admits no solution but the trivial one ($\gamma=0$). In the classical ($\alpha=0$) case,[6] this situation corresponds to $X^*=2$ and $t^*=4/e$ (e being here the basis of natural logarithms).

In this paper we will look for an approximative solution in the limit $\alpha \ll 1$. Under this approximation we can consider the following expansions:

$$D_2(A) \approx 1 - \frac{A}{3} + \frac{A^2}{24}$$

(since $A \sim \alpha^{1/2}$). Then

$$X \approx \frac{t}{2\gamma}\left[1 + \frac{\pi\alpha\gamma}{6t^2}\right] .$$

Equation (2.12) can then be approximated by

$$\gamma = \exp\left[-\frac{t}{4\gamma}\left[1+\frac{\pi\alpha}{6t^2}\gamma\right]\right] . \tag{2.13}$$

Taking

$$\frac{1}{y} \equiv \frac{t}{4\gamma}\left[1+\frac{\pi\alpha}{6t^2}\gamma\right]$$

it follows that, to the first order in α,

$$\gamma \approx t\frac{y}{4}\left[1+\frac{\pi\alpha}{24t}y\right] .$$

The self-consistent equation (2.13) becomes

$$\frac{t}{4}y+\frac{\pi\alpha}{96}y^2=\exp\left[-\frac{1}{y}\right] . \tag{2.14}$$

The critical values for $\alpha=0$ are $y^*=1$ and $t^*=4/e$. In order to find the first-order correction to t^* for $\alpha\neq 0$, we insert in (2.14) a solution of the form

$$y=1+a\alpha$$

which gives

$$t^*(\alpha)\approx\frac{4}{e}-\frac{\pi\alpha}{24}=t^*(0)-\frac{\pi\alpha}{24} . \tag{2.15a}$$

Equation (2.15a) is the linear α dependence of t_c we will use in Sec. IV.

Turning back to our initial notation, expression (2.15a) can be rewritten as

$$T_c(\alpha)\approx T_{c0}-\frac{\pi e^2}{12k_B C} . \tag{2.15b}$$

The result in Eq. (2.15b) is in excellent agreement with the fit performed by Jacobs *et al.*[5] on their Monte Carlo simulations for the same model. To see that, let us write Eq. (2.15b) as

$$\frac{T_c(\alpha)}{T_{c0}}=1-C_1\alpha+O(\alpha^2) . \tag{2.15c}$$

(The above normalization avoids that the overestimation of T_{c0} intrinsic to SCHA, masks the effect of quantum fluctuations.) Now, if we evaluate C_1 for Ref. 5 and for the present work, we find $C_1=0.090$ and $C_1=0.089$, respectively.

III. ELECTROSTATIC ENERGY CONTRIBUTION

High-T_c superconducting materials are characterized by their high degree of anisotropy. For most of HTSC, the in-plane electrical resistivity ($\rho_\perp$) is "metallic" with $d\rho_\perp/dT$ positive and close to $\rho_\perp/T$; in contrast, ρ_c (along the c axis) is larger by a factor 10^2 and usually "nonmetallic" ($d\rho_c/dT<0$).[7] We thus assume that sites on the same Cu-O plane are related by proximity effect coupling, whereas a particular site is electrostatically coupled to the Cu-O planes of adjacent Y-Ba-Cu-O unit cells. In-plane electrostatic coupling will be neglected as well as the Josephson coupling along the c axis. Our aim is to evaluate the self-capacitance C appearing in the Hamiltonian (2.1) for structures as described in Refs. 1 and 2. For a typical Cu-O plane of Y-Ba-Cu-O, there are two different kinds of environment: "internal" Cu-O planes in the center of pure Y-Ba-Cu-O, sitting at an equal distance Λ (12 Å) from the two conducting Cu-O planes of adjacent Y-Ba-Cu-O unit cells, and "edge" Cu-O planes sitting at a distance Λ (12 Å) from one of the adjacent Cu-O planes of Y-Ba-Cu-O and at a distance $(d+\Lambda)$ from the next Cu-O plane across the Pr-Ba-Cu-O insulating layer. This situation is depicted in Fig. 1.

The electrostatic potential on a small charged sphere of radius r_0 and total charge q sitting in between two parallel conducting planes can be evaluated using the image charge techniques[8] and gives

$$V_0\equiv\frac{q}{C}=\frac{q}{4\pi\epsilon_0\epsilon_c r_0}(1+r_0 S) , \tag{3.1}$$

where

$$S=\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{r_n}+\frac{1}{r_n'}\right] \tag{3.2}$$

is the infinite summation over image charges sitting on the positive half-space (label r_n) and the negative half-space (label r_n'). ϵ_c is the component along the c axis of the dielectric tensor, which is assumed, in our model, to be uniform.

If the small charged sphere is sitting at distances a and b from the negative and positive half-space conducting planes, respectively, we can write

$$S=-\frac{1}{2l}\sum_{n=1}^{\infty}\frac{n-2\eta}{n[n(n-1)+\eta]} , \tag{3.3}$$

where

$$\eta=\frac{ab}{(a+b)^2}$$

and

$$l=a+b .$$

The quantity S in (3.3) can be evaluated exactly in the symmetrical case ($\eta=\frac{1}{4}$)

$$S_0\equiv S(\eta=\tfrac{1}{4})=-\frac{2\ln 2}{l} . \tag{3.4}$$

Expression (3.4) will apply to bulk materials by taking

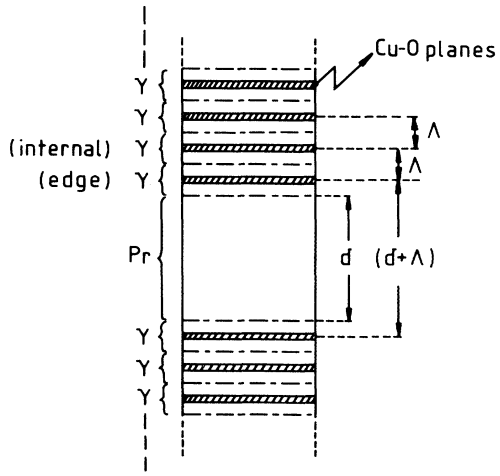

FIG. 1. Schematic representation of the Y-Ba-Cu-O/Pr-Ba-Cu-O superlattices as discussed in the text.

$l=2\Lambda$, and to samples with just one monolayer of Y-Ba-Cu-O by taking $l=2(d+\Lambda)$. For the asymmetric case ($\eta<\frac{1}{4}$) we use the following approximation:

$$S(\eta)=-\frac{1}{2l}\left[\frac{1-2\eta}{\eta}+\sum_{n=2}^{\infty}\frac{n-2\eta}{n[n(n-1)+\eta]}\right]$$
$$\approx-\frac{1}{2l}\left[\frac{1}{\eta}-4(1-\ln 2)\right] . \quad (3.5)$$

In (3.5) the dependence is dominated by the first term of the summation, since η is always smaller than $\frac{1}{4}$. The η independent part was fixed in order to get the correct symmetrical value when d goes to 0.

The value of the self-capacitance C can be extracted from (3.1) after evaluating S for each particular case.

IV. THE CRITICAL TEMPERATURE OF LAYERED SUPERCONDUCTORS

A. Y-Ba-Cu-O/Pr-Ba-Cu-O superlattices

The first simple application of the result of the two preceding sections is the calculation of the T_c of an insulated monolayer of Y-Ba-Cu-O in an infinite Pr-Ba-Cu-O medium. For this case we take the limit $d\to\infty$ in (3.4) and we get

$$V_0=\frac{1}{4\pi\epsilon_0\epsilon_c}\frac{q}{r_0}\equiv\frac{q}{C_0} ; \quad (4.1)$$

thus, from Eqs. (2.5) and (2.15) we find

$$T_c^{(1)}=T_{c0}-\frac{\pi e^2}{12k_BC_0} . \quad (4.2)$$

The critical temperature of the bulk ($d=0$) Y-Ba-Cu-O can be calculated by taking the appropriate limit

$$T_c^{\text{bulk}}=T_{c0}-\frac{\pi e^2}{12k_BC_0}\left[1-r_0\frac{\ln 2}{\Lambda}\right]$$
$$=T_c^{(1)}+\frac{\pi e^2 r_0\ln 2}{12k_BC_0\Lambda}\equiv T_c^{(1)}+\Delta T . \quad (4.3)$$

The case of the superlattice containing just one Y-Ba-Cu-O unit cell per period ($n=1$) is also symmetrical and can be computed in a similar way:

$$T_c^{(1)}(d)=T_c^{(1)}+\Delta T\frac{\Lambda}{d+\Lambda} , \quad (4.4)$$

where Δ is defined by Eq. (4.3). For all the remaining cases with $n\geq 2$, T_c can be evaluated by averaging the T_c over all the Y-Ba-Cu-O cells in the structure

$$T_c^{(n)}(d)=T_c^{(1)}-\Delta T\frac{\Lambda}{2\ln 2}\left[\frac{n-2}{n}S_0+\frac{2}{n}S(\eta)\right] , \quad (4.5)$$

where

$$\eta=\frac{\Lambda(d+\Lambda)}{(d+2\Lambda)^2} .$$

Expression (4.5) can be expressed in an alternative way by

$$T_c^{(n)}(d)=\frac{n-2}{n}T_c^{\text{bulk}}+\frac{2}{n}T_c^{(2)}(d) , \quad (4.6)$$

where it becomes clear that for $n\geq 2$, T_c is a weighted average between T_c^{bulk} ("internal") and $T_c^{(2)}$ ("edge").

The above property does not depend on the details of the model, and is confirmed with great accuracy by experiments: in Fig. 2, recent data on asymptotical values of $T_c^{(n)}(d=\infty)$ by Triscone *et al.*[9] are plotted together with the predicted curve from Eq. (4.6).

A more interesting universal feature between asymptotical values $T_c^{(1)}$ and $T_c^{(2)}$ can be expressed in units of ΔT by

$$\frac{T_c^{(2)}-T_c^{(1)}}{\Delta T}=\frac{1}{2\ln 2}\simeq 0.72 . \quad (4.7)$$

The numerical value of expression (4.7) is directly related to the electrostatic nature of the coupling between Cu-O planes. This value is confirmed within 5% by data in Refs. 1, 2, and 10. In order to compare on the same graph, experimental data of different origins and theoretical predictions, we define the following reduced quantities:

$$\tau=(T-T_c^{(1)})/\Delta T \quad (4.8)$$

and

$$\delta=d/\Lambda . \quad (4.9)$$

In Fig. 3 we plot the reduced data from the above references together with the reduced set of curves given by Eqs. (4.4) and (4.5). For Y-Ba-Cu-O/Pr-Ba-Cu-O samples we use $\Lambda=12$ Å, $\Delta T=66$ K, and $T^{\text{bulk}}=71$ K for samples from Refs. 1 and 10 and $\Delta T=80$ K for samples from Ref. 2. Notice that δ dependence is correctly followed by the theoretical curves. However, for samples at $n=2$ in Ref. 10 (full circles), one can notice an abrupt discrepancy between the fit and the data below $\delta=4$. This fact can be interpreted in terms of structural

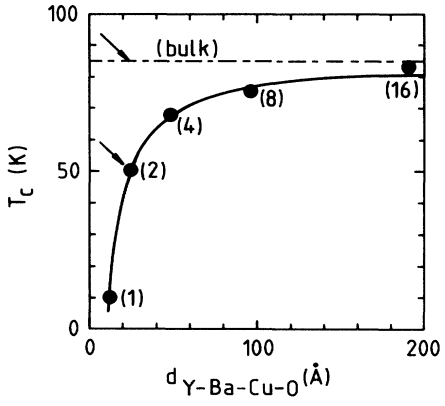

FIG. 2. Superconducting critical temperature as a function of the Y-Ba-Cu-O thickness for fixed Pr-Ba-Cu-O thickness (144 Å). The points are experimental data from Ref. 9. The full curve is the predicted behavior given by Eq. (4.6). The arrows indicate the input values $T^{(2)}$ and T^{bulk}.

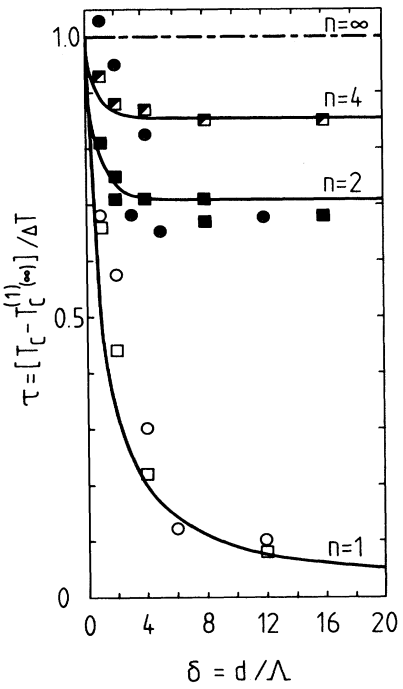

FIG. 3. Reduced superconducting critical temperatures τ as a function of the reduced thickness δ for different values of n (the number of Y-Ba-Cu-O unit cells per period in the superlattice). Solid lines are theoretical curves. Squares are taken from Ref. 2, circles from Refs. 1 and 10; open symbols (○,□) correspond to $n=1$ samples, full symbols (●,■) to $n=2$, and half squares (◩) to $n=4$.

changes at the interface between the Pr and the Y compounds due to the interplay between epitaxial strain and misfit dislocations.[11] Indeed, there is a 1% mismatch between the lattice constants on the *a-b* plane, and samples made by magnetron sputtering technique[1] are highly epitaxial. No such discrepancy is observed for samples of Ref. 2 that are made by laser beam deposition. We believe that, in this case, accommodation at interfaces is mostly achieved through misfit dislocations (see Ref. 20).

At this point, it is essential to comment about the choice we made at the beginning in taking the Cu-O bilayer in each Y-Ba-Cu-O cell as a single two-dimensional unit. In fact, in a first attempt, we tried to consider the Cu-O planes as individual metallic layers, but the fitting parameters were completely unphysical and the fit very poor. Thus we conclude that in these materials the two closest Cu-O planes behave as a single metallic sheet.

This is not the case, as we will see in the following. For Bi- and Tl-based cuprates. The difference is easy to understand if one compares the extremely large conduction anisotropy of Bi- and Tl-based compounds (nearly 10^5)[12] with that in Y-Ba-Cu-O ($<10^2$).[13] In other words, nearest-neighboring Cu-O planes in Bi and Tl compounds are insulated while in Y-Ba-Cu-O they are electrically connected.

B. Bi- and Tl-based superconducting cuprates

The Bi and Tl compounds of general formula $Bi_2Sr_2Ca_{(n-1)}Cu_nO_y$ and $Tl_{2\,(1)}Ba_2Ca_{n-1}Cu_nO_y$ contain $n=1,2,3,\ldots$ Cu-O single planes per unit cell. Taking into account the large anisotropy in their resistivity, we will consider each of these planes as a metallic sheet insulated from the others. Using the structural data available for these compounds,[14,15] we can calculate the corresponding value of δ [see Eq. (4.9)] for each of the three series [Bi (2,2,$n-1$,n), Tl (2,2,$n-1$,n), and Tl(1,2,$n-1$,n)] and deduce the ratio $\rho=(T_c^{(3)}-T_c^{(1)})/(T_c^{(2)}-T_c^{(1)})$ from Eqs. (4.4) and (4.5). In Table I we list some structural data (L being the size of the unit cell along the c axis and Λ the shortest distance between Cu-O planes), critical temperatures,[16] the δ values, and the calculated and the experimental values of ρ. In this kind of compound it is very hard to obtain an (n) pure phase, especially for $n=1$ which is often polluted by $n>1$ phases. In consequence, the measure of $T^{(1)}$ is an upper limit as is

TABLE I. Bi- and Tl-based cuprates. Structure data (L=period; Λ=shortest distance between Cu-O single planes), critical temperatures T_c, reduced insulating thickness δ, predicted and measured values of the ratio $(T_c^{(3)}-T_c^{(1)})/(T_c^{(2)}-T_c^{(1)})$: ρ_{theory} and ρ_{expl}, calculated value of the effective uniform c component ϵ_c of the dielectric tensor. (The asterisk indicates input values for the calculation of ΔT.)

Compound	n	T_c	L	Λ	δ	ρ_{theor}	ρ_{expl}	ΔT	ϵ_c
$Bi_2Sr_2Ca_{n-1}Cu_nO_y$	1	10							
	2	85*	15.80	3.20	2.93	1.190	1.333	250	37.88
	3	110*							
	4								
$Tl_2Ba_2Ca_{n-1}Cu_nO_y$	1	83							
	2	110*	14.70	3.20	2.59	1.200	1.444	133	71.19
	3	122*							
	4								
$TlBa_2Ca_{n-1}Cu_nO_y$	1	<30							
	2	90*	12.74	3.13	2.07	1.195	1.333	246	39.35
	3	110							
	4	122*							

the estimation of ρ. In this context we can say that the model is in good agreement (10 %) with the observed data.

Finally, from the $T_c^{(n)}$ data we can extract the value of ϵ_c the effective uniform component along the c axis of the dielectric tensor. To do that we use the expression of ΔT that follows from Eqs. (4.1) and (4.2), and we get

$$\epsilon_c = \frac{e^2 \ln 2}{48 k_B \epsilon_0 \, \Delta T \, \Lambda} \, . \tag{4.10}$$

Results for the Bi and Tl compounds are included in Table I together with the estimated ΔT. For Y-Ba-Cu-O we find $\epsilon_c \approx 34$. It is interesting to notice that the maximum dielectric constant corresponds to the most anisotropic compound.

A final remark is needed at this point about the significance of T_{c0} and r_0 that, in our present formalism, appear as arbitrary parameters. The bare critical temperature T_{c0} is related to the Josephson coupling J between adjacent grains, and r_0 to the size of a typical grain. In addition, they are related to each other through Eq. (4.2). An upper critical limit for r_0 can be calculated, starting from the trivial fact that $T_{c0} \geq T_c^{\text{bulk}}$; inserting that in Eq. (4.2) we get

$$r_0 \leq \frac{\Lambda}{\ln 2} \, . \tag{4.11}$$

For Y-Ba-Cu-O this means $r_0 \leq 17$ Å. The thickness of the Y-Ba-Cu-O bilayer being about 3 Å, we can deduce an upper limit for ξ_{ab}, the typical linear size of the superconducting island in the a-b plane, containing the same volume as the ideal sphere of radius r_0: $\xi_{ab} \leq 83$ Å. This limiting value is too small compared with twin spacings. However, it should be pointed out that our estimation of ξ_{ab} agrees with that of Morgenstern *et al.*[19] based on Monte Carlo simulations of the magnetic properties of HTCS within the superconducting-glass model.

V. CONCLUSIONS

In this work we have presented a model that can account for the T_c behavior in high-T_c layered superconductors. The model considers the layered superconductor as a set of metallic layers embedded in a dielectric medium; each metallic layer behaves as a 2D XY system (array of weak links) subjected to charging effects that renormalizes its bare BKT transition temperature. Quantum-phase fluctuations induced by charging effects are inhibited by Coulomb screening along the c axis due to the presence of adjacent metallic planes.

The model describes correctly the T_c behavior of two different kinds of samples: the artificial Y-Ba-Cu-O/Pr-Ba-Cu-O superlattices and the Bi- and Tl-based cuprates containing different numbers of Cu-O planes per unit cell depending on their stoichiometry.

Two kinds of universal relationships are predicted by the model and confirmed by experiment. The most trivial one (that does not depend on the nature of the interlayer coupling) is that, for $n \geq 2$, the T_c is an average over individual metallic layers. This means that each metallic layer undergoes its own BKT transition depending just on its local environment. More interesting are the predictions about the relative values of the T_c's for fixed $d \gg \Lambda$ and varying n. One of them (from which the others can be deduced by averaging) is $(T_c^{(2)} - T_c^{(1)})/(T_c^{(\infty)} - T_c^{(1)}) = 1/2 \ln 2$, which follows directly from the Coulombic nature of the coupling.

If the picture emerging from our work is correct, it should be a guide for further experimental projects aiming to improve superconductivity in this kind of material. This can be achieved by decreasing the distance between metallic planes or increasing the polarizability of the dielectric.

ACKNOWLEDGMENTS

We would like to thank J. M. Triscone, L. Antognazza, and Ø. Fischer for providing us with useful data on artificial structures and for interesting suggestions. One of us (D.A.) also thanks A. Junod for pointing out to him relevant properties of Bi- and Tl-based cuprates. This work was supported by the Swiss National Science Foundation.

[1] J. M. Triscone, Ø. Fischer, O. Brunner, L. Antognazza, A. D. Kent, and M. G. Karkut, Phys. Rev. Lett. **64**, 804 (1990).
[2] Q. Li, X. D. Wu, A. Inam, S. Vadlamannati, W. L. McLean, T. Venkatesan, R. Ramesh, D. M. Hwang, J. A. Martinez, and L. Nazar, Phys. Rev. Lett. **64**, 3086 (1990).
[3] N. Gupte and S. R. Shenoy, Phys. Rev. D **33**, 3002 (1986).
[4] J. M. Triscone (private communication).
[5] See, for example, L. Jacbos, J. V. José, M. A. Novotny, and A. M. Goldman, Phys. Rev. B **38**, 4562 (1988).
[6] V. L. Pokrovsky and G. V. Uimin, Phys. Lett. **45A**, 467 (1973).
[7] P. B. Allen, Z. Fisk, and A. Migliori, in *Physical Properties of High Temperature Superconductors I,* edited by D. M. Ginsberg (World Scientific, Singapore, 1989).
[8] See, for example, E. Durand, *Electrostatique II* (Masson, Paris, 1966).
[9] J. M. Triscone, Ø. Fischer, L. Antognazza, O. Brunner, A. P. Kent, L. Méville, and M. G. Karkut, in *Science and Technology of Thin Films Superconductors,* edited by R. D. McDowell and S. A. Wolf (Plenum, New York, in press).
[10] L. Antognazza, J. M. Triscone, O. Brunner, M. G. Karkut, and Ø. Fischer, Physica B **165-166**, 1503 (1990).
[11] D. Ariosa, Ø. Fischer, M. G. Karkut, and J. M. Triscone, Phys. Rev. B **37**, 2415 (1988); **37**, 2421 (1988).
[12] S. Martin *et al.*, Phys. Rev. Lett. **60**, 2194 (1988).
[13] T. Penney *et al.*, Phys. Rev. B **38**, 2918 (1988).
[14] H. W. Zandbergen *et al.*, Physica C **156**, 325 (1988).
[15] P. Haldar *et al.*, Science **241**, 1198 (1988).
[16] S. Theodorakis, Physica C **156**, 795 (1988).

[17]See, for example, J. Clem, Physica C **153-155**, 50 (1988); C. Ebner and D. Stroud, Phys. Rev. B **31**, 165 (1987).
[18]K. A. Müller, M. Takashige, and J. G. Bednorz, Phys. Rev. Lett. **58**, 1143 (1987).
[19]I. Morgenstern, K. A. Müller, and J. G. Bednorz, Z. Phys. B **69**, 33 (1987).
[20]D. H. Lowndes, D. P. Norton, J. D. Budai, S. J. Pennycook, D. K. Christen, B. C. Sales, and R. Feenstra, in *Proceedings of the Spring Meeting of the Material Research Society, Symposium N: Laser Ablation for Material Synthesis,* San Francisco, 1990 (unpublished).