Spin-wave modes in layered magnetic sandwich structures

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A calculation of spin-wave propagation through a nearly nonmagnetic film sandwiched between two magnetic films is given as a function of the thickness of the nonmagnetic film and of a parameter that describes the decay of the exchange through this film and can be related to structural features such as interface roughness or spin-polarization penetration depth. The comparison with experiment enables us to determine this exchange decay parameter for a series of experiments in Permalloy-copper-Permalloy sandwiches prepared in an UHV system.

INTRODUCTION

Recently, many spin-wave-resonance experiments have been performed in thin sandwiches where a nonmagnetic film is inserted between two magnetic films,[1–3] in order to study their coupling from a dynamical point of view. For all these samples, it has been observed that, when the nonmagnetic film is thin enough, exchange spin waves propagate through the nonmagnetic film in between. This has been demonstrated by comparing the spin-wave spectra obtained for given thicknesses of the magnetic films as a function of the thickness of the nonmagnetic film.[2] This defines the first goal of this paper: to find a model which can follow this transition from coupling to decoupling. A second point which appears as an experimental evidence is the effective unpinning at the internal surfaces when the coupling of the two magnetic films is very weak but nonzero.[2] The understanding of this experimental feature is the other goal of this paper.

The problem of exchange spin-wave modes in "sandwiches" can be considered as a one-dimensional problem where the relevant dimension is defined by the common normal to the sandwich. As a matter of fact, the nonhomogeneity of the interfilm can be accounted for in a layered way which describes the diffusion profile of the spins through the nonmagnetic interlayers, or the exchange process through this interfilm. For instance, the spacing of these fictitious layers of spins in the nonmagnetic interlayer can be taken as equal to the spacing of layers, i.e., successive planes, in the magnetic regions. In this layered picture of the central part of the sandwich, the simple model used here consists of introducing two exchange parameters $J_{i-1,i}$ and $J_{i,i+1}$ which couple the layer i to the layers $i-1$ and $i+1$, respectively, when coupling occurs only between nearest layers. These exchange parameters can be estimated from the local exchange parameter J and the densities of spins in layers $i-1$, i, and $i+1$. In other words, these $J_{i,i+1}$ mainly depend on the diffusion profiles, and are symmetric, i.e., $J_{i,i+1}=J_{i+1,i}$. These exchange parameters through the interfilm region can also be defined more directly from electronic properties, through this interfilm. From the phenomenological point of view used here, the interfilm is still characterized by the set of exchange constants $J_{i,i+1}$ where i names a layer of the interfilm.

This one-dimensional problem can be solved by means of an overall method, i.e., the method of the dynamical matrix D,[4] with the facilities of the Green-function method.[5] This approach was used in the seventies for spin-wave resonance in thin films.[6–11] An alternative approach is that of a local method, namely, the transfer-matrix method.[12–15] This method is founded upon the local character of the spin-wave equation which, in the case of a layer-to-nearest-layer coupling, means that if the wave amplitude is known on two neighboring layers, this wave amplitude is known everywhere. Another, more local version consists in knowing the amplitude and the gradient of amplitude on one layer and then deriving both the amplitude and its gradient everywhere, for a standing wave. With this very local definition, the method of transfer matrix was developed in mechanics by Poincaré[16] and has been used in many fields since then.[17,18] However, it must be acknowledged that for numerical applications, if there is any approximation, i.e., any error, it can be quite strongly amplified far from the initial layer. This remark encouraged the use of a more global method such as that of the dynamical matrix. Yet, analytical checks of the accuracy can be derived and used, and with care, the method of transfer matrix can be quite useful, since it is more easily tractable than an overall method. Moreover, in the case of interest for us, this limit of accuracy can be used to demonstrate a possible decoupling of the two films separated by the interfilm as will be seen in Sec. III. As a matter of fact, only such a local analysis can demonstrate the occurrence of a bad coupling or of a decoupling since when using an overall method, the existence of the coupling is assumed from the very beginning. Furthermore, these comments about weak coupling, partial decoupling, and complete decoupling point out the part of the physical noise for such standing waves, and this will be useful in the discussion on experimental results.

The main point of the propagation through the interfilm, i.e., a highly inhomogeneous medium, consists in observing a profile of the $J_{i,i+1}$'s. It can be easily assumed that these exchange parameters decrease as their sites become more remote from a magnetic film. This leads to the definition of a decay rate α per layer and results in a hyperbolic cosine for the profile of the $J_{i,i+1}$'s. As already said such a profile can come from purely structural features or from purely electronic features or from a mixture of both. The transfer-matrix method is a useful tool to determine the eigen modes of this set of spins, since the overall problem is quite complex. One main result both analytical and numerical, is that, as the decay rate α is reduced to 0, the decoupling between the two films leads to a situation of effective unpinning for each of the surfaces of the two magnetic films, i.e., spins in the last layer of the magnetic film have a large deviation amplitude. Of course, this effective unpinning at the inner surfaces is due to the weak exchange coupling through the nonmagnetic region. And qualitatively this effective unpinning is well understood since on one side there is a weak exchange stiffness, the mechanical analogy suggests the existence of a large oscillation amplitude at this surface. The numerical computations provide a description of the full variation of α from 0 to 1. This unpinning on the internal surface leads to selection rules for spin waves which are well observed at the experimental level. This feature is interesting since without any coupling a surface may be expected to be a pinning condition to the spin waves.[19] Thus the transition from no coupling at all to very weak coupling but mainly decoupled, is expected to be a rather abrupt transition from pinning to unpinning. This also occurs experimentally.

In Sec. I, the spin Hamiltonian is defined and the equation of motion of spins are derived from it, while Sec. II deals with the transfer matrices and the characteristic equations for standing spin waves. Finally the computations of field spectra, spin-wave mode patterns and spin-wave-resonance (SWR) line intensities are given in Sec. III.

I. THE SPIN HAMILTONIAN AND THE EQUATIONS OF MOTION OF SPINS

A. The spin Hamiltonian

Four kinds of interaction for spins are usually distinguished: the exchange interaction between spins, which is strong for permalloy ($Ni_{0.8}Fe_{0.2}$) and short ranged; the anisotropy interaction which is rather weak in this case and short ranged, being mainly due to intra-atomic spin orbit coupling; the magnetostatic interaction which is long ranged and the Zeeman effect with the external field. In this complex case of propagation of spin waves through an inhomogeneous system, it is very necessary to have a simple configuration and to avoid any subsidiary complications. Such a simple case occurs when the external magnetic field is normal to the sample, then the magnetic field required for resonance is rather large and all the spins can be considered statically as parallel to the external field. This avoids the complex problem of the detailed magnetic structure and rearrangement.[7] Further the anisotropic interactions can be neglected except for the surface anisotropy which is responsible for the spin-wave pinning at the external surfaces. In this calculation, the spin-wave pinning or unpinning at the external surfaces can be introduced easily at a later stage in the calculation, so we neglect all anisotropies. Magnetostatic interactions are weak and long ranged, they are responsible for magnetostatic modes, but these modes can be separated from the exchange modes by using appropriate radiofrequencies. Consequently, we will assume that magnetostatic effect occurs only via a uniform magnetic field, the demagnetizing field, and not at all with a detailed local picture. Then the spin Hamiltonian reduces to two parts, the exchange part $\mathcal{H}_{ex}$ and the Zeeman part $\mathcal{H}_{Zeeman}$, with

$$\mathcal{H}_{ex} = -(1/2)\sum_{f,g} J_{fg}\mathbf{S}_f\cdot\mathbf{S}_g \tag{1}$$

and

$$\mathcal{H}_{Zeeman} = -\mu \sum_{z} H_z S_f^z , \tag{2}$$

where $\mathbf{S}_f$ is the spin vector of the electron located at site f, J_{fg} is the exchange integral for sites f and g and z is the axis normal to the film, while H_z is the intensity of the effective magnetic field in the z direction. This effective field accounts for the magnetostatic correction.

With the previous considerations, the axis z is collinear with the equilbrium orientation of all $\mathbf{S}_f$. We change from spin-$\frac{1}{2}$ operators $\mathbf{S}_f$ to Pauli operators b_f with

$$S_f^x = \tfrac{1}{2}(b_f^\dagger + b_f) , \tag{3a}$$

$$S_f^y = \frac{i}{2}(b_f^\dagger - b_f) , \tag{3b}$$

$$S_f^z = \tfrac{1}{2} - b_f^\dagger b_f . \tag{3c}$$

B. The equations of motion of spins

From the commutation relations between Pauli operators, the equations of motion of the Pauli operators are easily derived when considering the restricted Hamiltonian $\mathcal{H}_Q$ which is quadratic in Pauli operators and neglects interactions via more than two Pauli operators. This is a low temperture approximation which is quite usual for spin-wave resonance. Then the equation of motion reads:[7]

$$i\frac{d}{dt}(b_g) = hb_g + \tfrac{1}{2}\sum_f J_{fg}(b_g - b_f) \tag{4}$$

with the time dependence of a standing wave in $\exp(-i\omega t)$, the secular equation is derived:

$$\omega b_g = hb_g + \tfrac{1}{2}\sum_f J_{fg}(b_g - b_f) \tag{5}$$

when summing over all the spins g which belong to the same layer l, it reads

$$\sum_g (\omega-h)b_g=\tfrac{1}{2}\sum_{g,f\in l-1} J_{fg}(b_g-b_f)+\tfrac{1}{2}\sum_{g,f\in l+1} J_{fg}(b_g-b_f)\ , \tag{6}$$

where the symmetric terms with g and f in the same layer l disappear. It reduces to a one-dimensional problem which can be written as:

$$2(\omega-h)b_l=(b_l-b_{l-1})J_{l,l-1}+(b_l-b_{l+1})J_{l,l+1}\ , \tag{7}$$

where the b_l's are averaged over the layers and the $J_{l,l\pm1}$ are also averaged over the two layers.

With this definition of the averaged Pauli operators which is valid for spin-wave resonance since there is a selection rule for propagation only normal to the film, in the bulk of a ferromagnetic film,

$$J_{l,l\pm1}=J\beta\ , \tag{8}$$

where J is the bulk exchange integral β is the number of nearest neighbors in a nearest layer. The general equation (7) links the spin deviations of a third layer to the spin deviations in the two previous layers, and thus leads to a transfer-matrix form by means of two by two matrices.

II. THE TRANSFER MATRICES

A. Definition of the transfer matrix

The previous reference to the Poincaré's transfer matrices[16] leads us to define as a vector, the set (b_n, b_n-b_{n-1}) of the spin-wave amplitude b_n and its difference between two neighboring layers. With this remark, the equation of motion [Eq. (7)] of spin deviations reads for the layer n:

$$\begin{bmatrix} b_{n+1} \\ b_{n+1}-b_n \end{bmatrix}=\underline{T}_n\begin{bmatrix} b_n \\ b_n-b_{n-1} \end{bmatrix} \tag{9}$$

if both n and $n+1$ belong to the multilayered system. Starting from the layer l and finishing at the layer N defines the complete transfer matrix $\underline{\tau}$:

$$\begin{bmatrix} b_N \\ b_N-b_{N-1} \end{bmatrix}=\underline{\tau}\begin{bmatrix} b_1 \\ b_1-b_0 \end{bmatrix}\ , \tag{10a}$$

$$\underline{\tau}=\underline{T}_{N-1}\cdots\underline{T}_2\underline{T}_1=\prod_{i=1}^{N-1}\underline{T}_i\ , \tag{10b}$$

where b_0 has been formally introduced. The equations of motion for the external layers 1 and N define the spin pinning conditions, in the Spark's approach:[5]

$$b_1+\lambda(b_1-b_0)=0 \tag{11}$$

instead of

$$b_1+\lambda c\frac{\partial b_1}{\partial x}=0\quad (\text{Ref. }5) \tag{11'}$$

and similarly:

$$b_N+\mu(b_N-b_{N-1})=0\ . \tag{12}$$

B. The characteristic equations

When fulfilling both the pinning conditions at the external surfaces, the characteristic equation is obtained, as an equation on h for a given ω:

$$\lambda\mu\tau_2^1-\mu\tau_2^2+\lambda\tau_1^1-\tau_1^2=0 \tag{13}$$

with the classical examples

—both perfect pinning

$$\lambda=\mu=0 \quad\text{and}\quad \tau_1^2=0 \tag{14}$$

—both perfect unpinning

$$\lambda^{-1}=\mu^{-1}=0 \quad\text{and}\quad \tau_2^1=0 \tag{15}$$

perfect pinning and perfect unpinning

$$\lambda=\mu^{-1}=0 \quad\text{and}\quad \tau_2^2=0\ . \tag{16}$$

C. The realistic transfer matrices, and their properties

1. The bulk transfer matrix of a ferromagnetic film

In the bulk, the sample is homogeneous and $J_{n,n-1}=J_{n,n+1}=J$ which is the bulk exchange integral, the transfer matrix $\underline{T}_n$ no longer depends on n and may be written as defined by equations (9),

$$\underline{T}_n=\underline{T}=\begin{bmatrix} 2a+1 & 1 \\ 2a & 1 \end{bmatrix}\ , \tag{17}$$

where

$$a=(h-\omega)/J\ . \tag{17a}$$

Then $\det\underline{T}=1$, $\underline{T}$ is a unitary matrix and can be easily diagonalized with the eigenvalues $a+1\pm x$, where $x^2=a(a+2)$. The calculation of $\underline{T}^N$ leads to the Kittel's modes for perfect pinning of one film[17] and to other classical modes for more complex pinnings of one film:

$$\underline{T}^N=\begin{bmatrix} (a+x)(a+x+1)^N-(a-x)(a-x+1)^N & (a+x+1)^N-(a-x+1)^N \\ (x^2-a^2)[(a+x+1)^N-(a-X+1)^N] & (x-a)(a+x+1)^N+(a+x)(a-x+1)^N \end{bmatrix}\ . \tag{18}$$

2. The transfer matrices in the interfilm

In this case, each transfer matrix is no longer unitary, which means there is an asymmetry in the propagation, with:

$$\det \underline{T}_n = \frac{J_{n,n-1}}{J_{n,n+1}} . \tag{19}$$

As a consequence, if the spin wave starts from a given ferromagnetic film I of P layers and, after an interfilm of k layers, goes through a ferromagnetic film similar to I, the overall transfer matrix $\underline{\tau}'$ of the interfilm is unitary

$$\det \underline{\tau}' = \frac{J_{P+k+1,P+k}}{J} \cdots \frac{J_{P+1,P}}{J_{P+2,P}} \frac{J}{J_{P+1,P}} = 1 , \tag{20}$$

and the overall propagation through the interfilm remains symmetric.

In order to introduce the chosen parameter of the problem, i.e., the decay rate α, the geometry of the bifilm must be defined precisely. The first film has P layers labeled from 1 to P. The interfilm has k layers labeled from $P+1$ to $P+k$. The final film has N layers labeled form $P+k+1$ to $P+k+N$.

If k is odd and equal to $2q+1$, the first new transfer matrix $\underline{B}_1$ appears when going from layer P to layer $P+1$, and from the definition of α

$$J_{P,P+1} = \alpha J_{P,P-1} = \alpha J .$$

Thus, $\underline{B}_1$ reads

$$\underline{B}_1 = \begin{bmatrix} \frac{2a}{\alpha} + 1 & \frac{1}{\alpha} \\ \frac{2a}{\alpha} & \frac{1}{\alpha} \end{bmatrix} . \tag{21}$$

More generally, when $0 < i \le q+1$, the transfer matrix $\underline{B}_i$ for the transition from layer $P+i-1$ to layer $P+i$, with $J_{P+i,P+i-1} = \alpha J_{P+i-1,P+i-2} = \alpha^i J$ reads

$$\underline{B}_i = \begin{bmatrix} \frac{2a}{\alpha^i} + 1 & \frac{1}{\alpha} \\ \frac{2a}{\alpha^i} & \frac{1}{\alpha} \end{bmatrix} . \tag{22}$$

The transition from layer $P+q+1$ to layer $P+q+2$, with $J_{P+q+2,P+q+1} = J_{P+q+1,P+q} = \alpha^{q+1} J$ defines the matrix $\underline{C}_{q+1}$ with

$$\underline{C}_{q+1} = \begin{bmatrix} \frac{2a}{\alpha^{q+1}} + 1 & 1 \\ \frac{2a}{\alpha^{q+1}} & 1 \end{bmatrix} . \tag{23}$$

The transition from layer $P+i$ to layer $P+i+1$ when $k \ge i > q+1$, with

$$J_{P+i-1,P+i} = \alpha J_{P+i,P+i+1} = \alpha^{k-i} J$$

defines the transfer matrix $\underline{A}_{k-i}$, with

$$\underline{A}_{k-i} = \begin{bmatrix} \frac{2a}{\alpha^{k-i}} + 1 & \alpha \\ \frac{2a}{\alpha^{k-i}} & \alpha \end{bmatrix} . \tag{24}$$

Finally the complete transfer matrix $\underline{\tau}$ reads

$$\underline{\tau} = \underline{T}^{N-1} \underline{A}_0 \cdots \underline{A}_q \underline{C}_{q+1} \underline{B}_{q+1} \cdots \underline{B}_q \underline{T}^{P-1} . \tag{25}$$

If k is even and equal to $2q$, when $1 \le i \le q+1$ the same transfer matrix $\underline{B}_i$ appears for the transition from layer $P+i-1$ to layer $P+i$, as defined by Eq. (21). And when i runs from $q+2$ to $k+1 = 2q+1$, there is a transfer matrix $\underline{A}_{(k-i+1)}$ which describes the transition from layer $P+i-1$ to layer $P+i$, as defined in Eq. (24). And the complete transfer matrix τ reads here

$$\underline{\tau} = \underline{T}^{N-1} \underline{A}_0 \cdots \underline{A}_q \underline{B}_{q+1} \cdots \underline{B}_1 \underline{T}^{P-1} . \tag{26}$$

The calculation of the spin-wave amplitudes b_i and differences of amplitude $(b_i - b_{i-1})$ throughout the film is obviously obtained by truncating the complete transfer matrix $\underline{\tau}$ of the film. The normalization of the magnons and calculations of the spin-wave-resonance line intensities can be easily achieved.

III. RESULTS OF SPIN-WAVE SPECTRA AND BEHAVIOR OF SPIN WAVES

A. Spin-wave spectra: The partial decoupling

When the decay rate parameter α is decreased from 1 to zero, the spin-wave spectrum goes from that of a $N+k+P$ layer film to the superposition of one film of N layers and one film of P layer. This is the decoupling, but this decoupling depends not only on α but also on k as seen in Fig. 1 where the spin-wave spectrum $h\ (k)$ is given for $N = P = 400$ and, respectively, $\alpha = 0.2$ [Fig. 1(a)]; 0.3 [Fig. 1(b)]; 0.5 [Fig. 1(c)]; while $\omega = 10^4$ G. Here $k+1$ is the number of perturbed interlayers and while k is the number of perturbed layers. For all parts of Figs. 1, both external surfaces are unpinned, as suggested from experimental observation.[2]

B. Spin-wave behavior. The unpinning

The numerical results for the spin-wave patterns are shown in the symmetric case $N = P = 400$, with a normalization of the maximum amplitude, with both external surfaces pinned with, respectively, $\alpha = 0.8$ [Fig. 2(a)] and 0.2 [Fig. 2(b)] while $k = 2$. In Fig. 2(a), α is near enough to 1, and there is nearly no deviation from the spin-wave pattern of a uniform film. Quite differently in Fig. 2(b), α is low enough, and among the eigen modes shown, the even ones show nearly a discontinuity at the interfilm. It can be noticed that both even and odd modes look nearly unpinned on the surfaces of the magnetic films which are adjacent to the nonmagnetic film. This is the effective unpinning effect which can be described in general terms as follows. When the decay parameter α approaches zero, the spin-wave amplitude of the eigen modes goes to a maximum value on the internal surfaces of the films.

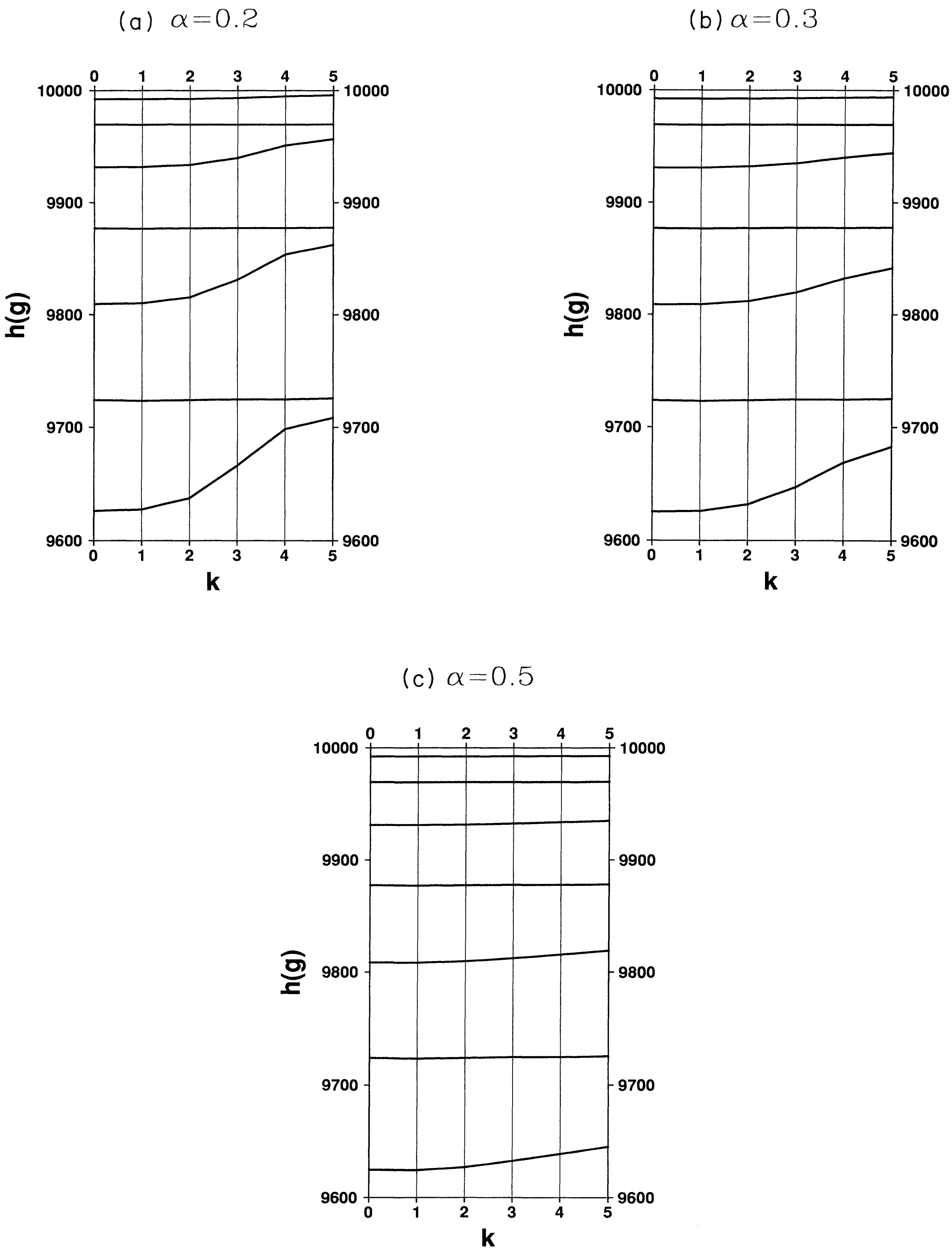

FIG. 1. Spin-wave spectra h (k) with the resonant field h as a function of the number k of nonmagnetic, $N=P=400$ layers, with $\omega=10^4$ G and $J=10^6$ G, when both the external surfaces of the two magnetic films are unpinned. The decay parameter α takes the respective values, 0.2 (1a), 0.3 (1b), and 0.5 (1c). The transition from perfect coupling which occurs when there is no interlayer, i.e., $k=1$, to more or less weak coupling which occurs for k large, can be seen for odd modes only.

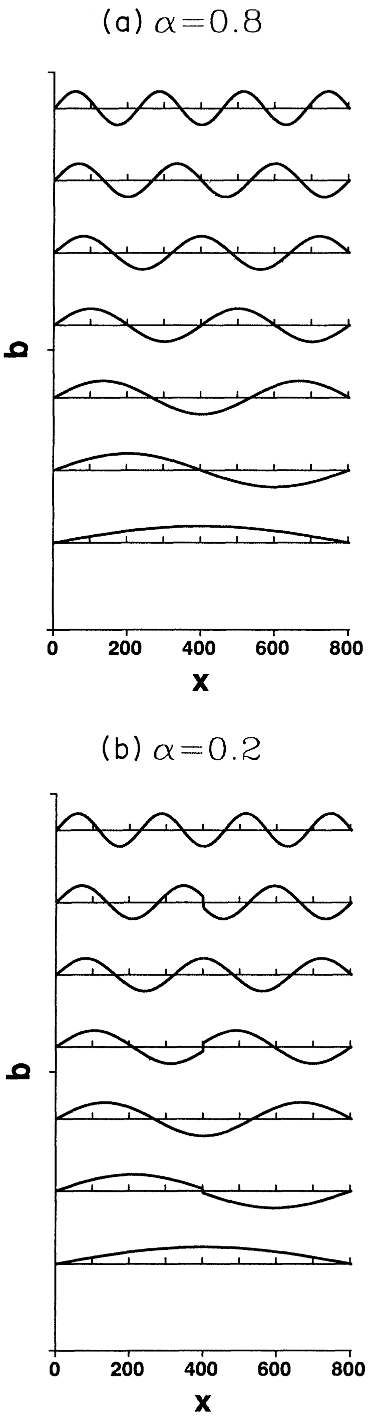

FIG. 2. Spin-wave patterns $b(x)$ with $N=P=400$, $k=2$, $\omega=10^4$ G, and $J=10^6$ G. Both the external surfaces of the magnetic film are pinned. The decay parameter α takes the respective values 0.8 (2a) and 0.2 (2b). The first seven modes are shown from the bottom of the figure, with the same amplitude normalization. In Fig. 2(a), there is a nearly perfect coupling between the two magnetic films, while in 2(b) even modes are badly coupled in the two films. In (b) the rather strong unpinning condition at the internal surfaces can be noticed, as explained in the text.

This unpinning condition can be easily demonstrated to occur, as follows. We must study the behavior of the surface of a magnetic film weakly connected to another magnetic film. The spin-wave amplitude σ and difference of amplitude, $\sigma-\sigma'$ on the surface is described by the product of an $\underline{A}_0$ transfer matrix times the amplitude, difference of amplitude in the interfilm layer. For a Ni-Fe magnetic film and a mode of large enough field as observed, a is very weak and noted as ε. It reads:

$$\begin{bmatrix} \sigma \\ \sigma-\sigma' \end{bmatrix} = \begin{bmatrix} 2\varepsilon+1 & \alpha \\ 2\varepsilon & \alpha \end{bmatrix} \begin{bmatrix} s \\ s-s' \end{bmatrix} . \qquad (27)$$

The amplitude s and difference of amplitude $s-s'$ are not known, but both are of order unity. It comes out obviously that σ is of the same order of magnitude as s, but that $\sigma-\sigma'$ is nearly zero. This probes the effective unpinning character of this weak connection. Conversely on the other surface, a matrix $\underline{B}^{-1}$ must be used, and the same effective unpinning property appears.

Finally, the practical decoupling, more exactly weak or bad coupling can be seen on the numerical results in the inequality of the amplitudes of sinusoidal variations in the two films. Since the transfer matrix through the interface is unitary and thus symmetric, this inequality is due to numerical approximations, and because of the high level of accuracy of this numerical work, it means that the definition of an eigen mode is quite unstable, and that practically the mode will be more or less localized on one side of the interfilm.

C. Experimental determination of α—experimental results

In order to explore the theoretical features outlined in Secs. III A and III B, we have studied Permalloy ($Ni_{80}Fe_{20}$) bilayers separated by films of either carbon,[20] or copper.[2] Here we concentrate on those structures with copper interfilms.

The films were prepared by electron beam evaporation onto Corning 0211 glass substrates at room temperature in a UHV system. RHEED data indicated that all the films were polycrystalline (further details are given in Ref. 2). Investigation of the continuity of the interfilm by Auger spectroscopy indicates that copper grows on Permalloy according to the Stranski-Krastanov growth mode, in which the first two layers grow as complete layers, after which subsequent growth results in the formation of islands. This behavior has also been reported for ion-scattering-spectroscopy measurements of copper grown on nickel.[21] Thus the evidence is against direct contact between the two Permalloy films via pinholes in the copper interfilm.

The spin-wave-resonance (SWR) spectra were measured using a 16.5-GHz reflection spectrometer.

Figure 3 shows the resonant field values of the SWR spectra as a function of the copper interfilm thickness for the magnetic field applied along the normal to the bilayer. For all the bilayers each Permalloy film was 600 Å

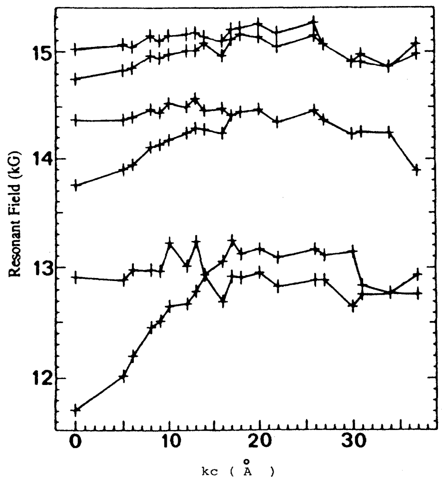

FIG. 3. The resonant field values of the SWR spectra of two Permalloy films separated by a copper interfilm of variable thickness k, when $N=P=238$ and the thickness c of layer is 2.52 Å. The transition between coupling and very weak coupling is seen to occur from $kc=0$ to $kc=14$ Å. Then the transition from very weak coupling to uncoupling is seen to occur from $kc=25$ Å to $kc=36$ Å.

(i.e., $N=P$). It can be seen that the resonant field positions move to higher field values as the copper interfilm thickness decreases from 37 to 26 Å. This behavior can be understood in terms of the induced unpinning of spins at the interfilm interfaces as the coupling between the magnetic films is "turned on" (Sec. III B). If, in the absence of coupling, there is partial pinning of the spins at the two interfilm interfaces then the lowest-order mode will not be the uniform mode (wavevector 0) and other spin-wave modes will be excited as are observed. The introduction of the coupling and the consequent unpinning of the interface spins will cause the values of the wavevector to decrease with the result that the resonant field positions will increase.

Further reduction of the copper interfilm thickness leads to a splitting of the resonance lines into pairs, one of which remains relatively constant whilst the other decreases in field position. This region covers the range of k from ten to zero, the latter limit corresponding to a single film with $N+P=2N=475$ layers. Comparison of the experimental field shifts with theoretical results of the form represented by Fig. 1(b) gives a best fit value of $\alpha \approx 0.3$.

Additional magnetostatic measurements support the onset of exchange coupling as discussed in Ref. 2.

CONCLUDING REMARKS

Firstly the experimental observation of the turning on of the coupling confirms the remark about the effective unpinning and also lies a little beyond the scope of the present model since this model always assumes the existence of such a coupling, even if weak. Secondly this unpinning of spin waves must lead to magnetization effects at finite temperature, i.e., at rather high temperature the magnetization near the interface must be weaker than in the bulk of the film, as suggested by Prinz.[22]

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