Upper bound on the ratio of Ginzburg-Landau parameters κ_2/κ for layered superconductors near T_c

J. Rammer
Physics Department, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6
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The slope of the generalized Ginzburg-Landau parameter $\kappa_2(T)$ at T_c is calculated in the clean limit using Eliashberg's strong-coupling theory. Fermi surfaces rotationally invariant about the magnetic-field axis are considered, and specific results are presented for the spherical and the cylindrical case. For a given Fermi surface, an upper bound on the normalized κ_2 slope is found for a coupling strength $T_c/\langle\omega\rangle \cong 0.16$, where $\langle\omega\rangle$ is a characteristic frequency in the coupling function. The maximum enhancement factor relative to the weak-coupling value is of the order 1.5. Finally, it is found that for very strong coupling, $T_c/\langle\omega\rangle \gtrsim 0.4$, the slope in κ_2 changes sign.

I. INTRODUCTION

The equilibrium magnetization of a type-II superconductor is largely determined by four parameters: the lower and upper critical fields, H_{c1} and H_{c2}, the thermodynamic critical field H_c, and the generalized Ginzburg-Landau parameter κ_2. All these quantities display characteristic temperature dependences, and they are affected in distinct ways by material properties such as the detailed band structure, impurity-scattering effects, and strong electron-phonon coupling. For conventional type-II superconductors, these effects are believed to be at least qualitatively understood in the framework of Gorkov's[1] and Eilenberger's[2] formulation of the Bardeen-Cooper-Schrieffer (BCS) theory,[3] including strong-coupling effects (Eliashberg[4]). In most cases, good agreement is found between theory and experiment.[5–10]

Out of the four quantities above, it appears that the least amount of work has been devoted to the generalized Ginzburg-Landau parameter κ_2. This parameter determines the slope of the magnetization curve at H_{c2}, and it is defined as follows:[11]

$$-4\pi\mathcal{M}(H \rightarrow H_{c2}, T) = \frac{H_{c2}(T) - H}{\beta_A\,[2\kappa_2^2(T) - 1]} \ . \qquad (1)$$

Here $\mathcal{M}$ is the magnetization, H is the external magnetic field, and β_A is a parameter which depends on the symmetry of the vortex lattice; we have $\beta_A = 1.16$ (1.18) for a triangular (square) lattice of flux lines. For anisotropic superconductors, both H_{c2} and κ_2 may depend on the orientation of the crystal with respect to the external field. Furthermore, it should be mentioned that according to Ginzburg-Landau theory, we have $\kappa_2(T \rightarrow T_c) = \kappa$, where κ is the ordinary Ginzburg-Landau parameter. Following early work for limiting cases,[12–14] Eilenberger[15] calculated the full temperature dependence of κ_2 for arbitrary impurity-scattering times in the weak-coupling limit. Recently, these calculations were generalized to include strong-coupling effects in the framework of Eliashberg's theory.[16] Also anisotropy effects were taken into account within a simple model,[17] and comparison with experiments on transition metals was made.[10]

The present work is motivated by a recent measurement[18] of κ_2 (T) for the high-T_c superconductor $YBa_2Cu_3O_{7-\delta}$ near T_c. It was found in this experiment, which was carried out on a polycrystal, that in the available regime (89–91.5 K), the temperature dependence of κ_2 was much stronger than predicted by conventional theories. In fact, the (asymptotic) normalized slope $\kappa_2'\,T_c/\kappa$ (the prime denotes derivative with respect to temperature, taken at T_c) turned out to be about ten times larger than expected for an isotropic, weak-coupling BCS superconductor. In view of this huge discrepancy it is desirable to establish, within the BCS-Eliashberg model, an upper bound on $\kappa_2'\,T_c/\kappa$ for a *layered* superconductor with an *arbitrarily strong* electron-phonon interaction. This is the scope of the present work.

Since high-T_c superconductors are believed to be in the clean limit, we will neglect impurity-scattering effects in the following [such effects reduce the temperature dependence of κ_2 (Ref. 15)]. Furthermore, in order to simplify the calculation, no anisotropies in the plane perpendicular to the direction of the flux lines will be considered; the appropriate experiment corresponding to the present theory is therefore best realized for the case where the applied field lies along the $\hat{c}$ axis of a single crystal or a sample consisting of oriented grains. Although the calculations are valid for more general rotationally invariant Fermi surfaces, specific results will only be given for spherical and cylindrical Fermi surfaces.

II. UPPER CRITICAL FIELD

Since the first step in the theory of κ_2 involves the calculation of the upper critical field H_{c2}, we will consider this quantity first. Using the notation of Ref. 16 the clean-limit H_{c2} is given by the maximum field B such that the eigenvalue equation

$$X_n = t \sum_{l=0}^{l_{\rm cut}} S^{\Delta}_{nl} \frac{\langle (T_0)_l \rangle}{\omega^0_l} X_l \tag{2}$$

has a nontrivial solution. Here $t = T/T_c$, and magnetic fields are measured in units of

$$H_0 = \frac{\hbar c}{2eR_0^2} = \frac{\Phi_0}{2\pi R_0^2} \quad \text{with} \quad R_0 = \frac{\hbar v_F}{2\pi k_B T_c} = 0.882\,\xi_0, \tag{3}$$

where Φ_0 is the flux quantum and ξ_0 is the clean-limit BCS coherence length at zero temperature. (H_0 is of the order of the zero-temperature H_{c2} for a clean superconductor.) Furthermore, the abbreviation

$$(T_0)_l = \frac{\sqrt{\pi}}{2\sqrt{\beta_l}} \operatorname{erfc}\left(\frac{1}{2\sqrt{\beta_l}}\right) \exp\left(\frac{1}{4\beta_l}\right) \tag{4}$$

with

$$\beta_l = \frac{B \sin^2\theta}{4(\omega^0_l)^2}$$

and

$$\omega^0_l = \omega_l + t \sum_{n=0}^{l_{\rm cut}} S^{\Sigma}_{nl}, \quad \omega_l = t\,(2l+1)$$

has been used. The Matsubara frequencies ω_l are measured in units of $\pi k_B T_c$. The cut off $l_{\rm cut}$ for the Matsubara sums has to be chosen high enough such that the results are independent of the cutoff. In the following, $l_{\rm cut}$ corresponds to ten times the cutoff in $\alpha^2 F(\omega)$. The real symmetric matrices

$$S^{\Delta}_{nl} = \lambda(\omega_n - \omega_l) + \lambda(\omega_n + \omega_l) - 2\mu^*, \tag{5}$$

$$S^{\Sigma}_{nl} = \lambda(\omega_n - \omega_l) - \lambda(\omega_n + \omega_l) \tag{6}$$

with

$$\lambda(\omega_l) = 2 \int_0^\infty d\omega\, \frac{\alpha^2 F(\omega)\, \omega}{\omega^2 + \omega_l^2} \tag{7}$$

contain information on the electron-phonon [Eliashberg function $\alpha^2 F(\omega)$] and the electron-electron (Coulomb pseudopotential μ^*) interactions. Finally, the angular brackets in Eq. (2) denote Fermi surface averages:

$$\langle g(\theta) \rangle = \begin{cases} \int_0^{\pi/2} d\theta\, \sin\theta\, g(\theta) & \text{(spherical FS)} \\ g(\pi/2) & \text{(cylindrical FS)} \end{cases} \tag{8}$$

with θ the angle between the direction of magnetic field and quasiparticle momentum. Equation (2) in an equivalent form was derived by Schossmann and Schachinger[19] (for earlier work see Werthamer and McMillan[20] and Eilenberger and Ambegaokar[21]). Since we are interested in the upper critical field near T_c, we expand Eq. (2) in the small parameter

$$\epsilon = 1 - t \ll 1 ,$$

and neglect terms of higher than linear order in ϵ. We first obtain for $(T_0)_l$

$$(T_0)_l = 1 - \frac{B \sin^2\theta}{2(\omega^0_l)^2} + O(\epsilon^2) = 1 - \frac{B\sin^2\theta}{2\sigma_l^2} + O(\epsilon^2) \tag{9}$$

with

$$\sigma_l = 2l + 1 + \sum_{n=0}^{l_{\rm cut}} (S^{\Sigma}_{nl})^{(0)} .$$

Furthermore, the strong-coupling matrices take on the following form:

$$S^{\Delta}_{nl} = (S^{\Delta}_{nl})^{(0)} + \epsilon\,(S^{\Delta}_{nl})^{(1)} + O(\epsilon^2) , \tag{10}$$

$$S^{\Sigma}_{nl} = (S^{\Sigma}_{nl})^{(0)} + \epsilon\,(S^{\Delta}_{nl})^{(1)} + O(\epsilon^2) , \tag{11}$$

where the zeroth- and first-order contributions are obtained from Eqs. (5) and (6) with

$$\lambda(\omega_l) = \lambda^{(0)}(\omega_l) + \epsilon \lambda^{(1)}(\omega_l) + O(\epsilon^2). \tag{12}$$

For $\lambda^{(0)}$ and $\lambda^{(1)}$ we obtain from Eq. (7)

$$\lambda^{(0)}(\omega_l) = 2 \int_0^\infty d\omega\, \frac{\alpha^2 F(\omega)\,\omega}{\omega^2 + (2l+1)^2} ,$$

$$\lambda^{(1)}(\omega_l) = 4(2l+1)^2 \int_0^\infty d\omega\, \frac{\alpha^2 F(\omega)\,\omega}{[\omega^2 + (2l+1)^2]^2} .$$

The eigenvalue equation for H_{c2}, Eq. (2), now takes on the following simple form, valid for $\epsilon \ll 1$:

$$X_n = \sum_{l=0}^{l_{\rm cut}} \frac{(S^{\Delta}_{nl})^{(0)}}{\sigma_l} \left[1 + \epsilon \left(\frac{(S^{\Delta}_{nl})^{(1)}}{(S^{\Delta}_{nl})^{(0)}} - \frac{\sum_{m=0}^{l_{\rm cut}} (S^{\Sigma}_{ml})^{(1)}}{\sigma_l} - b \frac{\langle \sin^2\theta \rangle}{2\sigma_l^2} \right) \right] X_l, \tag{13}$$

with $b = B/\epsilon$ (corresponding to $h_{c2} = H_{c2}/\epsilon$) and

$$\langle \sin^2\theta \rangle = \begin{cases} \frac{2}{3} & \text{(spherical FS)} \\ 1 & \text{(cylindrical FS)} . \end{cases} \tag{14}$$

In physical units, the normalized upper critical field slope is given by

$$h_{c2} = -\frac{H'_{c2}\, T_c}{H_0}. \tag{15}$$

Equation (13) can easily be solved numerically, and the eigenvector, which will be needed in the next section for the calculation of κ_2, can be written as

$$X_n = X_n^{(0)} + \epsilon X_n^{(1)}. \tag{16}$$

It is interesting to note that in the eigenvalue equation Eq. (13), which determines h_{c2}, both $X_n^{(0)}$ and $X_n^{(1)}$ are independent of the Fermi surface average; only the upper critical field slope h_{c2} is affected. In particular, for a cylindrical Fermi surface h_{c2} is smaller by a factor $\frac{2}{3}$ than for a spherical Fermi surface with the same v_F and T_c. This result is independent of the strength of the electron-phonon coupling, and it is in very good numerical agreement with the weak-coupling calculation of Pint[22] for $YBa_2Cu_3O_{7-\delta}$ in the limit $T \to T_c$. Finally, it is straightforward to employ the weak-coupling approximation in Eq. (13). The eigenvalue problem reduces to an algebraic equation for h_{c2}, and we obtain

$$h_{c2} = \frac{8\langle \sin^2\theta\rangle^{-1}}{7\zeta(3)} = \begin{cases} 12/[7\zeta(3)] = 1.4261 & \text{(spherical FS)} \\ 8/[7\zeta(3)] = 0.9507 & \text{(cylindrical FS)}\ . \end{cases} \tag{17}$$

Here $\zeta(n)$ is Riemann's zeta function with

$$\left(1 - \frac{1}{2^n}\right)\zeta(n) = \sum_{l=0}^{\infty} \frac{1}{(2l+1)^n}. \tag{18}$$

III. THE GENERALIZED GINZBURG-LANDAU PARAMETER κ_2

The general expression for κ_2 within Eliashberg's theory was derived recently.[16] Since this result is the starting point of the present calculation, we will reproduce the necessary equations in the following. For a clean high-κ superconductor, we have

$$\left(\frac{\kappa_2}{\kappa_0^b}\right)^2 = \beta_A^{-1}\,\frac{7\zeta(3)}{9}\,\frac{\{K_4, t, H_{c2}\}}{[\partial E_0(t, H_{c2})/\partial B]^2}, \tag{19}$$

where κ_0^b is the (fictitious) bare Ginzburg-Landau parameter for a clean, isotropic and weak-coupling superconductor:

$$(\kappa_0^b)^{-2} = \frac{7\pi\zeta(3)}{9} N^b(0) \left(\frac{e}{\hbar c}\right)^2 \frac{(\hbar v_F^b)^4}{(\pi k_B T_c)^2}. \tag{20}$$

The corresponding dressed quantity κ_0 is obtained in terms of the dressed electronic density of states at the Fermi level $[N(0) = N^b(0)(1+\lambda)]$ and the dressed Fermi velocity $[v_F = v_F^b/(1+\lambda)]$ as follows:

$$\kappa_0 = \kappa_0^b\,(1+\lambda)^{3/2}.$$

Furthermore,

$$\frac{2}{t}\{K_4, t, H_{c2}\} = \sum_{m=0}^{l_{\text{cut}}} \frac{X_m^4}{(\omega_m^0)^3} \left(\langle (T_0)_m\,(T_1)_m \rangle + (\beta_A - 1) \left\langle \sum_{i,k=0}^{\infty} (-1)^i \gamma_m^{i+k}\, \frac{(T_{2k+i+1})_m\,(T_i)_m}{(2k+i+1)!\; i!} \right\rangle \right) - \sum_{n,m=0}^{l_{\text{cut}}} \frac{X_n^2 X_m^2}{(\omega_n^0)^2 (\omega_m^0)^2} S_{nm}^{\Sigma} [I_n\, I_m + (\beta_A - 1)\, J_n\, J_m] \tag{21}$$

with

$$I_l = \langle (T_1)_l \rangle \tag{22}$$

and

$$J_l = \left\langle \left((T_1)_l + \frac{\gamma_l}{3!}(T_3)_l + \frac{\gamma_l^2}{5!}(T_5)_l + \cdots \right) \right\rangle . \tag{23}$$

Here

$$\gamma_l = \begin{cases} H_{c2}\,\pi \sin^2\theta\,/[\sqrt{3}\,(\omega_l^0)^2] & \text{(triangular lattice)} \\ H_{c2}\,\pi \sin^2\theta\,/[2\,(\omega_l^0)^2] & \text{(square lattice)} \end{cases} \tag{24}$$

and

$$(T_i)_l = \int_0^{\infty} dt\; t^i \exp(-t - \beta_l\, t^2). \tag{25}$$

This integral can be solved analytically. Finally, we have

$$\frac{\partial E_0(t, H_{c2})}{\partial B} = \frac{t}{2} \sum_{l=0}^{l_{\text{cut}}} \frac{X_l^2}{(\omega_l^0)^3} \langle \sin^2\theta\,(T_2)_l \rangle. \tag{26}$$

Our goal is to calculate the slope of κ_2 at T_c. Therefore, we write

$$\frac{\kappa_2}{\kappa_0} = \frac{\kappa}{\kappa_0}\left(1 - \epsilon\,\frac{\kappa_2'\,T_c}{\kappa}\right) + O(\epsilon^2)\ , \tag{27}$$

or

$$\frac{\kappa_2}{\kappa} = 1 - \epsilon\,\frac{\kappa_2'\,T_c}{\kappa} + O(\epsilon^2). \tag{28}$$

Here κ is the ordinary (measurable) Ginzburg-Landau parameter, which may be obtained from the identity $\kappa = \kappa_2(T_c)$. In order to calculate the ratio κ/κ_0 for an arbitrary rotationally invariant Fermi surface (FS) and arbitrary electron-phonon coupling, we have to perform the limit $T \to T_c$ in Eq. (19). Since in this limit $(T_i)_l \to 1$ and $\gamma_l \to 0$, we obtain immediately

$$\frac{\kappa}{\kappa_0} = \frac{\kappa_2(t=1)}{\kappa_0} = \frac{\langle \sin^2\theta\rangle^{-1}}{X_{23}} \left(\frac{7\zeta(3)}{18} Y\right)^{1/2} \tag{29}$$

with

$$Y = X_{43} - \sum_{n,m=0}^{l_{\rm cut}} Y_{nm}, \tag{30}$$

$$X_{ij} = \sum_{n=0}^{l_{\rm cut}} X_n^{ij}, \quad X_n^{ij} = \frac{(X_n^{(0)})^i}{\sigma_n^j}, \tag{31}$$

and

$$Y_{nm} = X_n^{22}\, X_m^{22}\, (S_{nm}^{\Sigma})^{(0)}. \tag{32}$$

Note that the ratio in Eq. (29) is independent of the symmetry of the vortex lattice, and that its weak-coupling limit in the isotropic (cylindrical) case is given by $\kappa/\kappa_0 = 1$ $(\frac{2}{3})$. In performing this limit, Eq. (18) has been used.

In order to calculate the quantity $\kappa_2' T_c/\kappa$ [see Eq. (28)], Eq. (19) has to be expanded up to first order in ϵ. Using the representation

$$(T_i)_l = \sum_{k=0}^{\infty} (-1)^k \beta_l^k \frac{(i+2k)!}{k!} = 1 - (i+2)!\,\frac{h_{c2}\sin^2\theta}{4\sigma_l^2}\,\epsilon + O(\epsilon^2) \tag{33}$$

for $(T_i)_l$, and expanding the numerator and the denominator in Eq. (19) to first order in ϵ, we finally obtain

$$-\frac{\kappa_2' T_c}{\kappa} = \frac{1}{2Y}\left[\sum_{n=0}^{l_{\rm cut}} X_n^{43}\left(4\frac{X_n^{(1)}}{X_n^{(0)}} - 3\tau_n\right) - \sum_{n,m=0}^{l_{\rm cut}} Y_{nm}\left(4\frac{X_n^{(1)}}{X_n^{(0)}} - 4\tau_n + \frac{(S_{nm}^{\Sigma})^{(1)}}{(S_{nm}^{\Sigma})^{(0)}}\right) - X_{43}\right] - \frac{1}{X_{23}}\sum_{n=0}^{l_{\rm cut}}\left(2X_n^{13}\,X_n^{(1)} - 3X_n^{24}\,\tau_n\sigma_n\right) + Z - \tfrac{1}{2} \tag{34}$$

with

$$\tau_n = \frac{1}{\sigma_n}\sum_{m=0}^{l_{\rm cut}} (S_{nm}^{\Sigma})^{(1)} \tag{35}$$

and

$$Z = \langle \sin^2\theta\rangle\, h_{c2}\left(3a_1\frac{X_{25}}{X_{23}} - \frac{X_{45}}{Y} + \frac{3c_1}{2Y}\sum_{n,m=0}^{l_{\rm cut}} \frac{Y_{nm}}{\sigma_n^2}\right). \tag{36}$$

Here

$$a_1 = \frac{\langle \sin^4\theta\rangle}{\langle \sin^2\theta\rangle^2} = \begin{cases} 6/5 & \text{(spherical FS)} \\ 1 & \text{(cylindrical FS)} \end{cases} \tag{37}$$

and

$$c_1 = \begin{cases} 1 - 2\pi\, 0.16/(3\sqrt{3})(1.16) = 0.833 & \text{(triangular lattice)} \\ 1 - \pi\, 0.18/(3)(1.18) = 0.838 & \text{(square lattice)}. \end{cases} \tag{38}$$

Several features in the final result Eq. (34) should be emphasized. First, we notice that the information on both the Fermi surface and the symmetry of the vortex lattice enters only through the parameter Z. In fact, since the product $\langle \sin^2\theta\rangle h_{c2}$ is independent of the Fermi surface [see Eq. (13)], the ratio Eq. (37) is the only relevant anisotropy parameter for the normalized κ_2 slope. A second observation concerns the effect of the vortex lattice symmetry. The relevant parameter, c_1, which is defined in Eq. (38), takes on very similar values for the two symmetries considered. Furthermore, since the matrix elements Y_{nm} become very small for weak-coupling superconductors, the effect of the lattice symmetry on the κ_2 slope drops out in the weak-coupling limit. [Note, however, that according to Eq. (1), the magnetization near H_{c2} is still slightly affected by the lattice symmetry.]

Using Eq. (17) for the weak-coupling H_{c2}, as well as Eq. (18), the weak-coupling approximation in Eq. (34) may be performed. One obtains

$$-\frac{\kappa_2' T_c}{\kappa} = \frac{62}{49}\frac{\zeta(5)}{\zeta^2(3)}\left(3\frac{\langle \sin^4\theta\rangle}{\langle \sin^2\theta\rangle^2} - 1\right) - 1 = \begin{cases} 1.361 & \text{(spherical FS)} \\ 0.816 & \text{(cylindrical FS).} \end{cases} \tag{39}$$

This result can be compared with previous work. We find numerical agreement both with Neumann and Tewordt's[13] isotropic weak-coupling result, and with the extrapolated value obtained by Rammer and Pesch[17] in the cylinder symmetrical case. The general form of Eq. (39) also allows one to establish a *lower* limit on $-\kappa_2' T_c/\kappa$ for a *weak-coupling* superconductor with an arbitrary rotationally invariant Fermi surface: since $\langle \sin^4\theta\rangle/\langle\sin^2\theta\rangle^2 \geq 1$, the lower limit is obtained for the cylindrical Fermi surface.

IV. DISCUSSION

Equation (34) represents the final result of this work. In this equation, the normalized slope of κ_2 at T_c, $-\kappa_2' T_c/\kappa$, is explicitly written in terms of the solution of the eigenvalue equation Eq. (13), with $X_n^{(0)}$ and $X_n^{(1)}$ given by Eq. (16). The calculation of the normalized κ_2 slope for an arbitrary electron-phonon interaction and (rotationally invariant) Fermi surface is therefore reduced to a standard numerical problem, which can be solved with little computational effort. In this section, typical results will be presented.

In view of the experimental situation described in the Introduction, we are mainly interested in establishing an upper bound on $-\kappa_2' T_c/\kappa$ within Eliashberg's theory. Since the optimum choice for the electron-phonon coupling function $\alpha^2F(\omega)$ for calculating upper bounds is a δ function[23–25] ("Einstein spectrum"), we will restrict ourselves to this special case in the following numerical investigation. Furthermore, it is known that results based on an Einstein spectrum are representative of other classes of spectra as well; the essential parameter, which largely determines the size of strong-coupling effects, is $T_c/\langle\omega\rangle$, where $\langle\omega\rangle$ is a characteristic frequency in $\alpha^2F(\omega)$.

In Figs. 1 and 2, the results of the present numerical study are displayed. Figure 1, which refers to a spherical Fermi surface, shows the normalized H_{c2} and κ_2 slopes at T_c, as well as the ratio κ/κ_0 as functions of T_c/ω_E, where ω_E is the frequency of the Einstein phonon in $\alpha^2F(\omega)$. We have chosen $\mu^* = 0$ in these calculations. (It turns out that μ^* has only a very small effect on these quantities; for the effect on H_{c2}, see Ref. 24.) We first notice that in the weak-coupling limit, $T_c/\omega_E \to 0$, the correct values [Eqs. (17) and (39)] are reproduced. For increasing coupling strength, both $-H_{c2}' T_c/H_0$ and $-\kappa_2' T_c/\kappa$ initially increase, and reach maxima at $T_c/\omega_E \cong 0.16$. For coupling strengths exceeding this value, we observe a monotonic decrease, and both functions eventually drop below their respective weak-coupling values. The position as well as the height of the maximum in the normalized H_{c2} slope agree with previous numerical work.[24]

As can also be seen from Fig. 1, the maximum of $-\kappa_2' T_c/\kappa$ is about 2.1, as compared with the weak-coupling value of 1.36. The height of this maximum is essentially unaffected by the symmetry of the vortex lattice (compare dashed line with solid line). It is furthermore remarkable that the normalized κ_2 slope $-\kappa_2' T_c/\kappa$ becomes *negative* for very strong coupling, $T_c/\omega_E \gtrsim 0.4$. This means that the slope of the magnetization curve with respect to the applied field at H_{c2} actually *increases* with decreasing temperature.

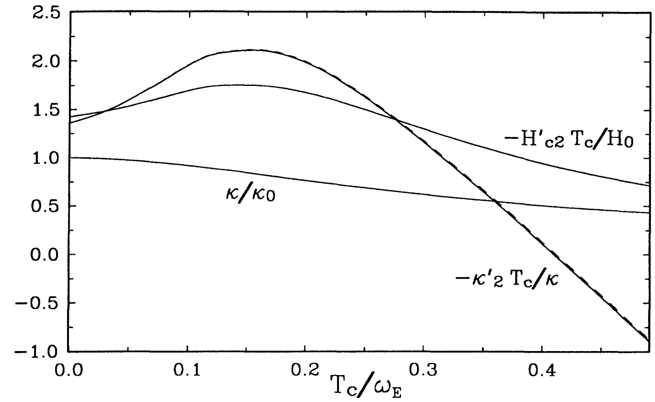

FIG. 1. Normalized slopes of H_{c2} and κ_2 at T_c for a spherical Fermi surface, as a function of the coupling strength for a δ-function α^2F spectrum and $\mu^* = 0$; H_0 [see Eq. (3)] is of the order of the zero-temperature upper critical field for a clean superconductor, and κ is the ordinary Ginzburg-Landau parameter. The solid (dashed) line for $-\kappa_2' T_c/\kappa$ refers to a triangular (square) lattice of flux lines. Also shown is the dependence of the Ginzburg-Landau parameter on the coupling strength [Eq. (29)].

Qualitatively similar results are obtained for a cylindrical Fermi surface, as can be seen in Fig. 2. In this case, the maximum in $-\kappa_2' T_c/\kappa$ is about 1.4, as compared with the corresponding weak-coupling value of 0.82. The κ_2 slope changes sign at $T_c/\omega_E \cong 0.37$, which is somewhat smaller than in the isotropic case.

The obtained maximum values for the normalized κ_2 slope are still almost an order of magnitude smaller than observed by Zhou *et al.*[18] on polycrystalline $YBa_2Cu_3O_{7-\delta}$. Since in the present theory it is assumed that the magnetic field lies along the symmetry axis of the Fermi surface, a direct comparison with this exper-

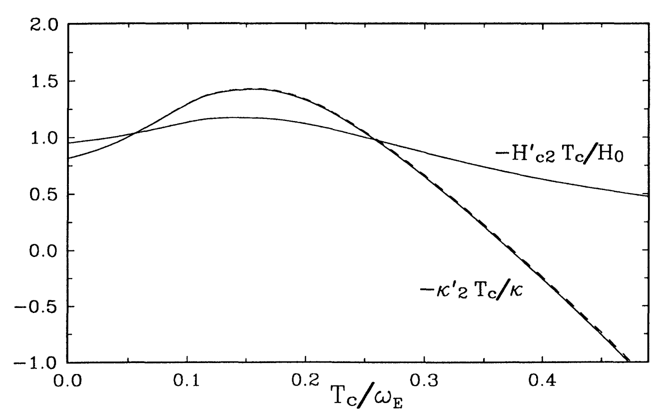

FIG. 2. Same as in Fig. 1, for a cylindrical Fermi surface.

iment may not be very conclusive, however. It is, e.g., conceivable that the temperature dependence of κ_2/κ is very different if the vortices lie in the $\hat{\mathbf{a}}$-$\hat{\mathbf{b}}$ planes of a single crystal, as compared to the geometry considered here. Indeed, the discrepancies in $\kappa_2(T)$ for the two fitting procedures found in Ref. 18 may indicate such effects, which are outside the scope of the present work. In order to avoid these complications, it is suggested that $\kappa_2(T)$ should be measured in single-crystalline or grain-aligned samples with $\mathbf{H} \parallel \hat{\mathbf{c}}$. In this configuration, the normalized κ_2 slope $-\kappa_2' T_c/\kappa$ should not exceed the value 1.4 (Fig. 2), if the BCS-Eliashberg formalism is applicable, independent of the symmetry properties of the vortex lattice.

In conclusion, using Eliashberg's strong-coupling theory, I have calculated the slope of $\kappa_2(T)$ at T_c in the clean limit for a Fermi surface which is rotationally invariant about the magnetic-field axis. The generalized Ginzburg-Landau parameter κ_2 describes the magnetization of a type-II superconductor near the upper critical field. An upper bound on the dimensionless quantity $-\kappa_2' T_c/\kappa$ is reached at $T_c/\langle\omega\rangle \cong 0.16$, where $\langle\omega\rangle$ is a characteristic frequency in the coupling function. For a cylindrical (spherical) Fermi surface, the maximum in $-\kappa_2' T_c/\kappa$ is larger by a factor ~ 1.7 (~ 1.5) than the corresponding weak-coupling value. These enhancement factors are much smaller than the one recently observed[18] in polycrystalline $YBa_2Cu_3O_{7-\delta}$, which indicates a strongly anomalous temperature dependence of κ_2 for $\mathbf{H} \perp \hat{\mathbf{c}}$. Finally, it is found that for very strong coupling, $T_c/\langle\omega\rangle \gtrsim 0.4$, the slope in κ_2 at T_c changes sign.

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