Vortex-lattice–vortex-liquid states in anisotropic high-T_c superconductors

B. I. Ivlev,* N. B. Kopnin,* and M. M. Salomaa
Low Temperature Laboratory, Helsinki University of Technology, SF-02150 Espoo, Finland
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We investigate vortex-lattice structures in highly anisotropic superconductors for a situation where the vortices are slightly inclined with respect to the easy plane of the crystal. The equilibrium configuration is found to be a vortex lattice with a rhombic unit cell highly compressed along the direction perpendicular to the easy plane. The elastic modulus for a shear deformation along this direction is exponentially small, which should result in melting of the vortex lattice, already at quite low temperatures. We discuss the implications of our theoretical results for experiments on the angular dependence of the torque for anisotropic high-T_c superconductors in an applied magnetic field.

I. INTRODUCTION

The discovery[1] of high-temperature superconductors served to revive interest in anisotropic and layered superconducting materials. In high-temperature superconductors, the behavior of quantized vortices governs practically all the relevant magnetic properties of these materials. It proved that many peculiarities of anisotropic superconductors in the mixed state had not been studied in sufficient detail. Recently, new interesting properties of vortices in anisotropic and layered compounds were pointed out. For example, it was predicted theoretically[2–4] and experimentally confirmed[5] that there should exist an intrinsic pinning due to the interaction between vortices and the crystal structure of a layered superconductor.

This can be observed not only for layered compounds but also for materials which may be considered as three-dimensional anisotropic superconductors, like the Y-Ba-Cu-O-type high-T_c compounds at temperatures not very far from T_c.[6] This can be seen experimentally, for example, via the angular dependence of the torque acting on the superconductor in a magnetic field,[7–9] or through the angular dependence of the magnetization.[10]

The anisotropy also modifies the interaction between the vortices themselves,[11,12] which should result in a modification of the vortex lattice in anisotropic superconductors, depending on the magnetic field and on its orientation with respect to the crystallographic axes. For example, it has been pointed out[13] that in highly anisotropic (layered) superconductors with $m_c \gg m_{ab}$, the vortices have to lie almost in the plane of the layers (i.e., perpendicular to c, the anisotropy axis) when the magnetic field H is close to the lower critical field H_{c_1}. Here m_c and m_{ab} denote the effective Ginzburg-Landau masses along the anisotropy axis and in the easy plane, respectively. They are connected with the penetration depths λ_c and λ_{ab} and with the coherence lengths ξ_c and ξ_{ab} through

$$\left(\frac{m_{ab}}{m_c}\right)^{1/2} = \frac{\xi_c}{\xi_{ab}} = \frac{\lambda_{ab}}{\lambda_c} \ . \tag{1}$$

In the present paper we consider the vortex-lattice structure for magnetic fields much larger than H_{c_1} and for an orientation in which vortices are slightly inclined with respect to the easy plane (the *ab* plane) of the crystal. We show that for a rather broad region of magnetic fields, the vortex lattice can be in states which have practically the same energies for distinct unit cells with different discrete locations of the lattice sites along one of the crystallographic directions (along the easy plane) and which are separated from each other by energy barriers. At the same time, these states can continuously transform into each other through displacements of vortices in the perpendicular direction. A deformation along this direction is related with a very low shear modulus, which means that the vortex lattice should, in fact, have melted at moderate temperatures.

The ground state of the lattice is a rhombic unit cell with an equilibrium ratio of the intervortex distances in the direction of the easy plane and in the perpendicular direction, x_0/z_0, considerably larger than the ratio $\lambda_c/\lambda(\theta)$, where

$$\lambda(\theta) = (\lambda_{ab}^2 \cos^2\theta + \lambda_c^2 \sin^2\theta)^{1/2} \ , \tag{2}$$

and θ is the angle between the vortices and the *ab* plane of the crystal. This is distinct from the usual case for superconductors with low anisotropy, where $x_0/z_0 \sim \lambda_c/\lambda(\theta)$. Therefore, one may conclude that the lattice has a very small shear modulus for deformations along the direction in which it is highly compressed. A similar effect has previously been pointed out[14] for a vortex lattice in layered superconductors. The results obtained also suggest that the torque acting on highly anisotropic superconductors has a narrow peak for small tilting angles.

Section II outlines the theoretical model and presents the expression for the free energy of vortex lattices in anisotropic superconductors. In Sec. III we calculate the free energy for various types of lattices. Section IV discusses the results and their physical implications for the vortex states observed in anisotropic high-T_c superconductors.

II. THEORETICAL FRAMEWORK

Our approach is based on the London model of the mixed state.[15] One can consider vortices to be rectilinear in an anisotropic homogeneous medium, in contrast to layered superconductors.[16] The free energy is

$$\mathcal{F} = \frac{1}{8\pi}\int dV \left[\mathbf{H}^2 + \lambda_{ab}^2\left(\frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y}\right)^2 + \lambda_{ab}^2\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right)^2 + \lambda_c^2\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)^2\right] . \quad (3)$$

Here $\mathbf{H}$ is the microscopic magnetic field in the superconductor. The coordinate system (x, y, z) has the z axis along the crystal anisotropy c axis, and the easy plane coincides with the (x, y) plane. The vortices are in the (y, z) plane making the angle θ with the y axis. Therefore, let us introduce a new coordinate frame (x', y', z'), with the axis y' parallel to the vortices, see Fig. 1, and the x' axis coinciding with the original x axis. The vortex lattice is described by the interception points of the vortices with the (x', z') plane.

Making use of the generalized London equation, one can easily obtain[13] from Eq. (3) the free-energy density F:

$$F = \frac{B^2}{8\pi}\sum_{\mathbf{b}}\left[1 + \lambda_c^2 b_{x'}^2 + \lambda^2(\theta) b_{z'}^2\right]^{-1}\left(\cos^2\theta + \sin^2\theta\,\frac{1 + \lambda_c^2(b_{x'}^2 + b_{z'}^2)}{1 + \lambda_{ab}^2(b_{x'}^2 + b_{z'}^2)}\right) . \quad (4)$$

Here the summation runs through the reciprocal-lattice vectors.

One can find out, after quite tedious algebra (which we omit here for the sake of brevity), that orientations of the vortex lattice in the (x', z') plane with one of the unit-cell vectors parallel either to the x' axis or to the z' axis are most favorable. Therefore, in order to describe the vortex lattices, we shall employ the two alternative representations illustrated in Figs. 2(a) and 2(b). Class (a) has one of the unit-cell vectors parallel with the z' axis. The reciprocal lattice vectors have the form

$$b_{x'} = \frac{m + q_1 n}{\ell}\sqrt{\frac{z_0}{x_0}} , \quad b_{z'} = \frac{n}{\ell}\sqrt{\frac{x_0}{z_0}} , \quad (5)$$

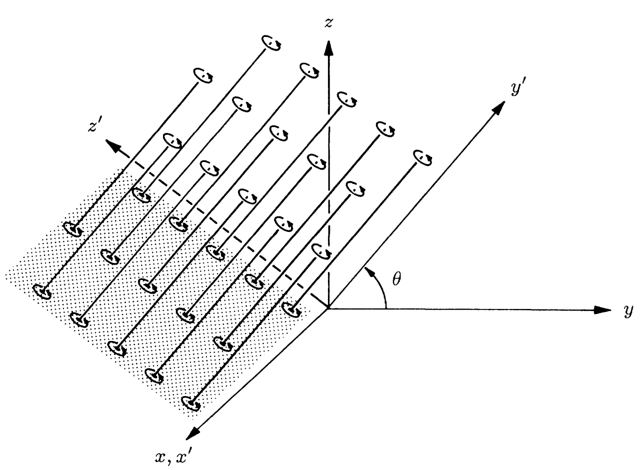

FIG. 1. Vortices are described in an inclined system of coordinates (x', y', z'), with the y' axis (along the vortices) tilted with respect to the easy plane (x, y) of the crystal through the angle θ. The z axis is along the crystalline anisotropy axis (the c axis).

where $\ell = (2\pi)^{-1}\sqrt{\phi_0/B}$ (here ϕ_0 denotes the flux quantum). Class (b) has one of the unit-cell vectors parallel to the x' axis. In this case we have

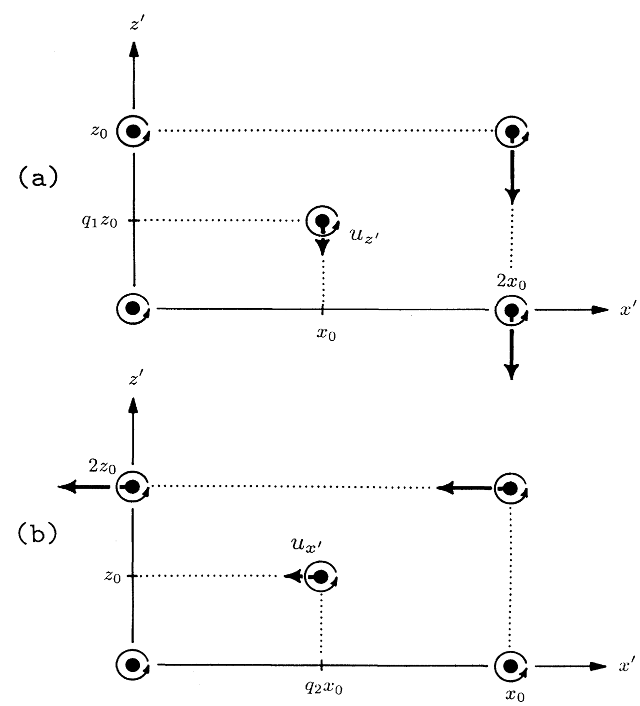

FIG. 2. Two representations for vortex lattices in the (x', z') plane. (a) One of the unit-cell vectors is parallel with the z' axis. The period along z' is z_0. The other unit-cell vector has the coordinates $(x_0, q_1 z_0)$. Variation in q produces a shear distortion $u_{z'}$ along the z' axis. (b) One of the unit-cell vectors is parallel with the axis x'. The period along x' equals x_0; the other unit-cell vector has the coordinates $(q_2 x_0, z_0)$. The specific vortex arrangement shown here corresponds to $q_1 = q_2 = \frac{1}{2}$.

$$b_{x'} = \frac{n}{\ell}\sqrt{\frac{z_0}{x_0}} , \quad b_{z'} = \frac{m + q_2 n}{\ell}\sqrt{\frac{x_0}{z_0}} . \tag{6}$$

In Eqs. (5) and (6), m and n are integers. The orientation of the other unit-cell vector and the type of the vortex lattice is determined by the parameters q_i (for $i = 1, 2$). A rectangular lattice has $q = 0$ and a rhombic (triangular) lattice has $q = \frac{1}{2}$. Variations in q for fixed x_0 and z_0 correspond to shear deformations of the lattice. For a class (a) lattice shown in Fig. 2(a), the shear is along the z' axis. For a class (b) lattice [cf. Fig. 2(b)], the shear is along the x' axis. The unit-cell parameters x_0 and z_0 obey the magnetic flux quantization condition: $x_0 z_0 = \phi_0/B$.

We shall express the free energy for a given magnetic field induction B and tilting angle θ as a function of two variables: one is q and the other (p) is proportional to the ratio of the unit-cell parameters x_0 and z_0.

We consider vortex lattices inclined slightly with respect to to the crystal ab plane for $\theta \ll 1$. We employ the parameter

$$\gamma = \theta \frac{\lambda_c}{\lambda_{ab}} , \tag{7}$$

which, owing to the large anisotropy, $\lambda_c \gg \lambda_{ab}$, can either be less than or larger than unity. We assume that the magnetic induction B in the superconductor satisfies the condition

$$H_{c_1}(\theta) \ll B \ll H_{c_1}(\theta)\left(\frac{\lambda_c}{\lambda_{ab}}\right)^2 , \tag{8}$$

provided that $\gamma \sim 1$. Here

$$H_{c_1}(\theta) = \frac{\phi_0 \lambda(\theta)}{4\pi\lambda_{ab}^2\lambda_c} \ln\kappa \tag{9}$$

denotes the lower critical field[13] for given inclination θ of the vortices with respect to the ab plane; $\kappa = \lambda_{ab}/\xi_{ab}$ is the Ginzburg-Landau parameter. The region of magnetic fields specified by Eq. (8) exists only for highly anisotropic materials with $\lambda_c \gg \lambda_{ab}$.

A. Free energy; class (a)

We first calculate the free energy for a class (a) unit cell, with the reciprocal-lattice vectors **b** from Eq. (5). The summation over m and n in Eq. (4) can be carried out using the relation

$$\sum_{m=-\infty}^{+\infty} \frac{1}{(m+a)^2 + b^2} = \left(\frac{\pi}{b}\right) \frac{\sinh(2\pi b)}{\cosh(2\pi b) - \cos(2\pi a)} . \tag{10}$$

The calculation yields the result

$$F = \frac{B^2}{8\pi} + \frac{\phi_0 B \lambda(\theta)}{32\pi^2\lambda_{ab}^2\lambda_c}\left[\ln\left(\frac{\alpha H_{c_2}(\theta)}{B}\right) + g(p_1, q_1)\right] , \tag{11}$$

where the upper critical field[17] is

$$H_{c_2}(\theta) = \frac{\phi_0}{2\pi\xi_{ab}^2[\sin^2\theta + (m_{ab}/m_c)\cos^2\theta]^{1/2}} \tag{12}$$

and α is of order unity. The function $g(p_1, q_1)$ in Eq. (11) is defined as

$$g(p_1, q_1) = G(p_1, q_1) - \frac{p_1}{6}\frac{\gamma^2}{1+\gamma^2} , \tag{13}$$

where $G(p_1, q_1)$ was first introduced in Ref. 14:

$$G(p, q) = \sum_{n=1}^{\infty} \frac{2}{n}\left(\frac{\sinh(pn)}{\cosh(pn) - \cos(2\pi q n)} - 1\right) - \ln p + \frac{p}{6} . \tag{14}$$

The parameter p_1 in Eq. (13) is the normalized ratio of the unit-cell dimensions:

$$p_1 = 2\pi \frac{\lambda(\theta)}{\lambda_c} \frac{x_0^{(a)}}{z_0^{(a)}} . \tag{15}$$

For the case of vortices strictly parallel to the ab plane [$\gamma = 0$, cf. Eq. (7)], Eq. (13) yields the conventional expression for the free energy. The latter term in this equation is a result of the interaction between the anisotropic medium and the inclined vortices.

B. Free energy; class (b)

For a class (b) unit cell, illustrated in Fig. 2(b), the reciprocal-lattice vectors are given by Eq. (6). By performing the summation in Eq. (4), we get

$$F = \frac{B^2}{8\pi} + \frac{\phi_0 B \lambda(\theta)}{32\pi^2\lambda_{ab}^2\lambda_c}\left[\ln\left(\frac{\alpha H_{c_2}(\theta)}{B}\right) + f(p_2, q_2)\right] , \tag{16}$$

where

$$f(p_2, q_2) = G(p_2, q_2) - \frac{4\gamma^2}{1+\gamma^2}\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)\frac{1}{p_2 + h[1 - \cos(2\pi q_2 n)]} . \tag{17}$$

Here the parameter p_2 is defined by

$$p_2 = 2\pi \frac{\lambda_c}{\lambda(\theta)} \frac{z_0^{(b)}}{x_0^{(b)}} \ . \tag{18}$$

Above, we also defined the reduced magnetic field

$$h = \frac{\lambda_c \lambda_{ab}^2}{\pi \ell^2 \lambda(\theta)} \ , \tag{19}$$

which is on the order of $h \approx B/H_{c_1}(\theta) \gg 1$ according to Eq. (8). The second term in Eq. (17) is due to the strong anisotropy; in conjunction with the last term in Eq. (13) it is essential only for magnetic fields satisfying the upper inequality equation (8). The second term is absent for isotropic superconductors or for large deflection angles $\theta \approx 1$.

III. VORTEX-LATTICE ENERGIES

The function $G(p,q)$, which would be the sole contribution to the free energy of an isotropic superconductor, and also for $\gamma = 0$, has degenerate minima at $p = \pi\sqrt{3}$ and at $p = \pi/\sqrt{3}$ for $q = \frac{1}{2}$; they correspond to a hexagonal vortex lattice on the plane $(x', z'\lambda_c/\lambda(\theta))$ with two orientations [class (a) or class (b)]. As can be seen from Eq. (14), the function $G(p,q)$ is even and periodic in q with period 1; it is invariant under the transformation $q \rightarrow 1-q$.

The rational values $q = \frac{M}{N}$, where M and N are integers, correspond to lattices which simultaneously belong to both of the classes (a) and (b). With $q = \frac{1}{N}$, for example, the vortex-lattice periods $x_0^{(a)}$ and $z_0^{(a)}$ in the class (a) representation, and the periods $x_0^{(b)}$ and $z_0^{(b)}$ in the class (b) representation are mutually related through

$$z_0^{(a)} = \frac{1}{q} z_0^{(b)} = N z_0^{(b)} \ , \tag{20}$$

$$x_0^{(a)} = q x_0^{(b)} = \frac{1}{N} x_0^{(b)} \ , \tag{21}$$

and $q_1 = q_2 = \frac{1}{N}$. On the other hand, for such a lattice, one can transpose the coordinate axes $x' \leftrightarrow z'$ and use $q x_0^{(b)}$ as the new coordinate z_0 along the new z' axis and $\frac{1}{q} z_0$ as the new period x_0. This would result in the transformation $p \rightarrow 4\pi q^2/p$. It is evident, however, that the energy would be invariant under such a change. This physical circumstance can mathematically be expressed in terms of the identity

$$G\left(p, \frac{1}{N}\right) = G\left(\frac{4\pi^2}{N^2 p}, \frac{1}{N}\right) \ . \tag{22}$$

A. Vortex lattices; class (a)

Let us first consider the class (a) lattices. Analytical solutions can be more easily obtained for large $\gamma \gg 1$. In this case the characteristic values of p_1 corresponding to the minima of $g(p_1, q_1)$ are also large. Using the asymptotic expression of $G(p,q)$ for large $p \gg 1$:

$$G(p,q) = 4e^{-p}\cos(2\pi q) - \ln p + \frac{p}{6} \ , \tag{23}$$

we find that the minima of $g(p_1, q_1)$, as functions of p, are at

$$p_{1,\min} = 6(1+\gamma^2) + 4e^{-6(1+\gamma^2)}\cos(2\pi q) \ ; \tag{24}$$

they equal

$$g_{\min}(p_{1,\min}, q_1) = -\ln\left(\frac{6(1+\gamma^2)}{e}\right) + 4e^{-6(1+\gamma^2)}\cos(2\pi q_1) \ . \tag{25}$$

The values of $p_{1,\min}$ obtained from Eq. (24) correspond to lattices with

$$\frac{x_0^{(a)}}{z_0^{(a)}} = \frac{3}{\pi}(1+\gamma^2)\frac{\lambda_c}{\lambda(\theta)} \gg \frac{\lambda_c}{\lambda(\theta)} \ . \tag{26}$$

As one can see from Eq. (25), the free energy displays its minimum at $q_1 = \frac{1}{2}$; a rhombic lattice. However, this minimum is quite shallow, and the corresponding shear modulus $C_{66}^{(z')}$, defined through

$$\delta F = \tfrac{1}{2} C_{66}^{(z')} \left(\frac{\partial u_z}{\partial x}\right)^2 \ , \tag{27}$$

is found to equal

$$C_{66}^{(z')} = \frac{9}{2\pi^2}\frac{\phi_0 B \lambda_c}{\lambda_{ab}^3}(1+\gamma^2)^{3/2} e^{-6(1+\gamma^2)} \ . \tag{28}$$

It is exponentially small already for moderate γ. For example, at $\gamma = 2$ the exponential factor is on the order of 10^{-13}.

Our approach holds within the logarithmic approximation for the large-wavelength cutoff in the free energy of Eq. (4): the total logarithm in Eq. (11), including the contribution from Eq. (25), should be large. This imposes the following limitation on the tilting angle:

$$\gamma^3 \ll \frac{H_{c_2}(\pi/2)}{B}\frac{\lambda_c}{\lambda_{ab}} \ .$$

B. Vortex lattices; class (b)

Consider now the class (b) vortex arrays; the free energy is given by Eqs. (16) and (17). Due to the large magnitude of h, the second term in Eq. (17) gives large negative contributions for rational values of $q_2 = M/N$ whenever $\cos(2\pi q n)$ turns to unity for certain n. We consider in more detail the case $q_2 = \frac{1}{N}$. As we have discussed, such values of q_2 correspond to lattices which simultaneously belong to both of the classes, (a) and (b), with the same $q_1 = q_2 = \frac{1}{N}$.

Within the leading approximation in h we have for $q_2 = \frac{1}{N}$

$$f\left(p_2, \frac{1}{N}\right) = G\left(p_2, \frac{1}{N}\right) - \frac{\gamma^2}{1+\gamma^2}\frac{2\pi^2}{3N^2 p_2} \ . \tag{29}$$

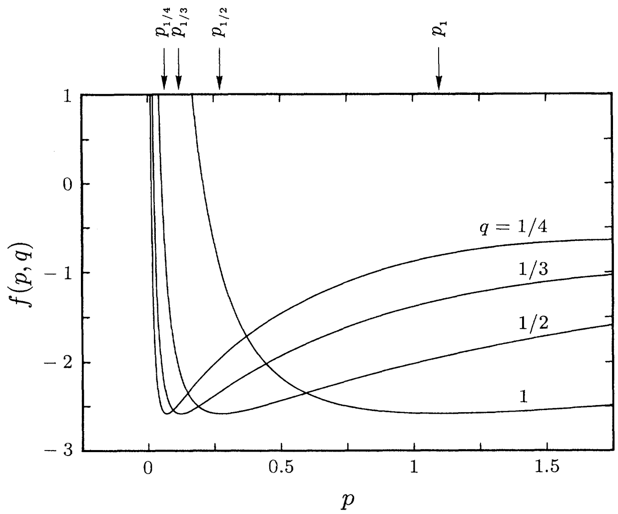

FIG. 3. Vortex-lattice free energy $f(p,q)$ of Eq. (17) as a function of p for chosen fractional values of $q = 1/N$, where N=1, 2, 3, and 4. The special values $p_{1/N}$, indicated in the figure with arrows, correspond to the minima of $f(p, 1/N)$: $p_{1/4} \approx 0.069$, $p_{1/3} \approx 0.122$, $p_{1/2} \approx 0.274$, and $p_1 \approx 1.097$.

The behavior of $f(p, \frac{1}{N})$ in Eq. (29) is shown in Fig. 3 as a function of p for N=1, 2, 3, and 4, and for $\gamma^2 = 5$. For $N = 2$, the minimum of f is at $p = 0.2742$. The differences in the depths of the minima in f for different N are very small and are, in fact, indistinguishable.

For large γ, one can obtain analytical expressions for $f(p,q)$. Characteristic values of p now turn out to be small. Using Eq. (22) and the asymptotic expression, Eq. (23), we find

$$p_{2,\min} = \frac{2\pi^2}{3N^2(1+\gamma^2)} - \frac{16\pi^2}{N^2} e^{-6(1+\gamma^2)} \quad . \tag{30}$$

The ratio of lattice parameters is

$$\frac{x_0^{(b)}}{z_0^{(b)}} \approx \frac{3N^2}{\pi}(1+\gamma^2) \quad . \tag{31}$$

The corresponding minima of f coincide with the right-hand side of Eq. (25) with $q_1 = \frac{1}{N}$, i.e.,

$$f_{\min}\left(p_{2,\min}, \frac{1}{N}\right) = g_{\min}\left(p_{1,\min}, \frac{1}{N}\right) \quad . \tag{32}$$

This is due to the fact that the lattice with $q = \frac{1}{N}$ simultaneously belongs to both of the classes (a) and (b). One can easily see that the lattice parameters $x_0^{(a)}$, $z_0^{(a)}$ and $x_0^{(b)}$, $z_0^{(b)}$ from Eqs. (26) and (31) satisfy the conditions (20) and (21).

The deepest free-energy minimum of the vortex lattices as functions of p for fixed q_2 is at $q_2 = \frac{1}{2}$. The minima of f for various $q = \frac{1}{N}$ differ only by exponentially small values, but they are separated by finite energy barriers. For example, in Fig. 4 we display the behavior of $f(p,q)$ from Eq. (17) as a function of q for $p = 0.2742$, i.e., for the value of p corresponding to the minimum of $f(p,q)$ with $q = \frac{1}{2}$. One clearly finds the minima at $q = \frac{1}{N}$, and also further minima for rational values of $q = M/N$. The minimum value at $q = \frac{1}{2}$ is $f = -2.6039$. The other extrema in Fig. 4 are not true minima of $f(p,q)$ as functions of the two variables, but rather they are minima of f for the fixed value of p.

It is interesting to compare this curve with the behavior of $G(p,q)$ as a function of q for small p studied in Ref. 14, where the function $G(p,q)$ had maxima for rational q's. Now these extrema have been turned into minima by the negative contributions of the second term in Eq..(17) for large values of γ.

IV. DISCUSSION

The results obtained above allow us to draw some interesting conclusions about the vortex state in an anisotropic superconductor for small tilting angles of the magnetic field with respect to the easy plane of the crystal.

It is instructive to first consider the possible configurations of vortex lattices. The ground state corresponds to $q = \frac{1}{2}$. One of the interesting features is the presence of the metastable energy minima for rational $q_2 = \frac{M}{N}$ for the class (b) vortex lattices, displayed by the fractal curve in Fig. 4. As distinct from vortex-glass states,[18] however, the states with $q \neq \frac{1}{2}$ do not correspond to energy minima with respect to a shear along the z' axis, as can be seen from Eq. (25). Therefore, if a lattice with $q \neq \frac{1}{2}$ were formed, its evolution would undergo a shear deformation with lattice-site displacements along the z' axis until the ground state with $q = \frac{1}{2}$ (a rhombic lattice) is reached. The scenario for such a reconfiguration of the lattice with $q = \frac{1}{N}$, $N = 3$, is illustrated in Fig. 5.

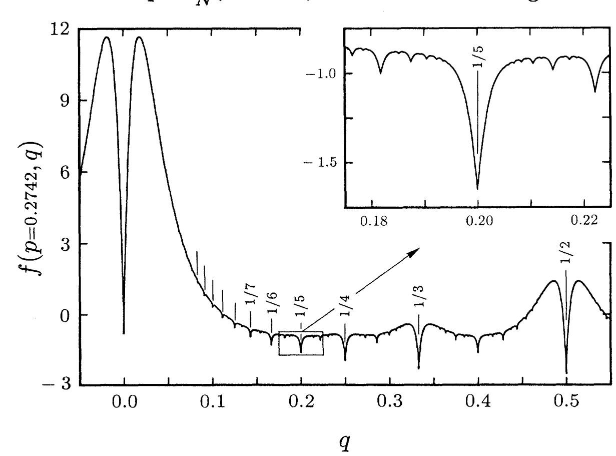

FIG. 4. Vortex-lattice free energy $f(p_{1/2}, q)$ for $\gamma^2 = 5$, $h = 100$, and fixed $p = p_{1/2} \approx 0.2742$ as a function of q. One observes a dense distribution of minima, corresponding to rational values of $q = M/N$. The neighboring minima for $q = 1/N$ are separated by energy barriers. The inset corresponds to the tiny rectangle indicated in the figure; it illustrates the fractal structure, *ad infinitum*, of this curve.

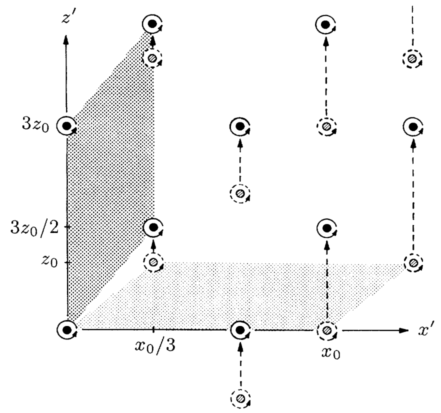

FIG. 5. Vortex lattice undergoing a shear deformation: reconfiguration of a vortex lattice with $q = \frac{1}{3}$ into the stable vortex lattice with $q = \frac{1}{2}$ involves the sliding of vortices along the (y', z') plane. The old and new unit cells for the vortex lattice are indicated with light and dark shadings, respectively.

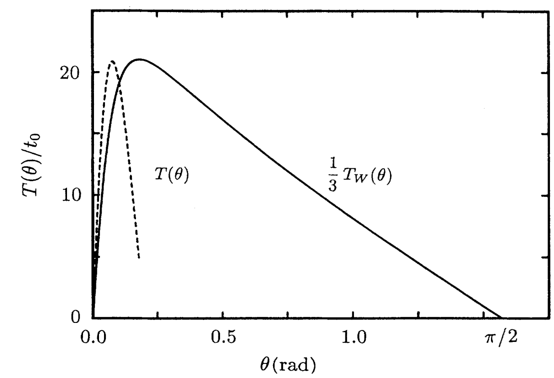

FIG. 6. Torque $T(\theta)$ acting on a highly anisotropic ($\lambda_c \gg \lambda_{ab}$) superconductor as a function of the tilting angle θ between the magnetic field and the easy plane of the crystal, according to Eq. (33) (dashed). Also displayed is the dependence $T_W(\theta)$ from Eq. (34) (solid curve). The torque is normalized as $T(\theta)/t_0$, where $t_0 = B\phi_0/64\pi^2\lambda_{ab}\lambda_c$. The parameters in Eqs. (33) and (34) are $\lambda_c/\lambda_{ab} = 10$ and $\alpha H_{c_2}(\pi/2)/B = 50$. Note the factor of 3 difference in the scales for $T(\theta)$ and $T_W(\theta)$.

However, one has to bear in mind that the shear modulus $C_{66}^{(z')}$ is very small. Such softening of the shear mode should result in vortex-lattice melting.[19] Due to the very small value of $C_{66}^{(z')}$, a vortex-liquid state can persist down to quite low temperatures. A similar effect has also been predicted for layered superconductors.[14]

Such an exponential softening takes place for vortex displacements along the direction of the strong compression of the equilibrium vortex lattice, i.e., along the z' axis and is related with the large value of $p_{1,\min} \propto \gamma^2$, as obtained from Eq. (24). The equilibrium vortex lattice can in this case be visualized as a set of parallel planes containing the closely spaced vortices. These planes are separated from each other by large spacings. Such a picture is similar to that discussed in Ref. 12 for the onset of vortex penetration into superconductors when $H \rightarrow H_{c_1}$. When the planes of the vortices slide along each other, the restoring force is small due to a small modulation of the interaction energy between vortices belonging to different planes. In the present case, it is the modulus $C_{66}^{(z')}$, corresponding to the "hard" direction of displacement (according to Ref. 20), which is exponentially softened. This softening is entirely due to the strong anisotropy $\lambda_c \gg \lambda_{ab}$ and vanishes for $\theta \rightarrow 0$.

The behavior of the equilibrium free energy as a function of the tilting angle θ gives rise to quite an unusual angular dependence of the torque acting on the anisotropic superconductor in a magnetic field. This follows from Eqs. (11) and (25) and (16) and (32). The density of the torque is

$$T(\theta) = \frac{\partial F}{\partial \theta} = \frac{\phi_0 B}{32\pi^2\lambda_{ab}^2\lambda_c}\frac{\partial \lambda(\theta)}{\partial \theta} \times \left(\ln\frac{\alpha H_{c_2}(\theta)}{B} + g_{\min} - 3\right) . \qquad (33)$$

The angular dependence of the torque from Eq. (33) is shown in Fig. 6 for $\lambda_c/\lambda_{ab} = 10$ and $\alpha H_{c_2}(\pi/2)/B = 50$. The dashed line in Fig. 6 terminates where the logarithmic approximation breaks down. For comparison, we also reproduce in Fig. 6 the angular dependence of a torque for a weakly anisotropic superconductor,[8]

$$T_W(\theta) = \frac{\phi_0 B}{32\pi^2\lambda_{ab}^2\lambda_c}\frac{\partial \lambda(\theta)}{\partial \theta}\left[\ln\left(\frac{\alpha H_{c_2}(\theta)}{B}\right) + G_{\min} - 1\right] \qquad (34)$$

[where $G_{\min}(p, \frac{1}{2}) = -0.804$], which it would have for the same values of the parameters. One has to note that Eq. (33) is valid for $\theta > \lambda_{ab}/\lambda_c$; for smaller angles it has to be considered as an interpolation. The exact dependence for $\theta < \lambda_{ab}/\lambda_c$ can be obtained directly from Eq. (11). One can see that the torque on a highly anisotropic superconductor has quite a narrow peak for small tilting angles. A similar angular dependence of $T(\theta)$ has been observed experimentally[9] for a single crystal of $YBa_2Cu_2O_7$, although prior to comparing these results one has to bear in mind the substantial irreversibility pointed out in Ref. 9.

In Ref. 9, jumps in the torque acting on the superconductor have also been observed. These were interpreted to result from reconfigurations of the lattice in response to the field rotation. It follows from our results that the anisotropy of the equilibrium lattice in the (x', z') plane strongly increases for increasing tilting angles. Such a reconfiguration of the lattice requires considerable displacements of the vortices which can be hindered by pinning forces. The jumps in the θ dependence of T could arise when the lattice overcame the pinning.

In conclusion, we have studied vortex states in anisotropic superconductors when the magnetic field is slightly inclined with respect to the easy plane of the crystal. We have found that the equilibrium vortex lattice has a rhombic unit cell, highly compressed along the direction perpendicular to the easy plane. This compression is much larger than what one would expect from the ratio $\lambda_c/\lambda(\theta)$. The elastic modulus for a shear deformation along this direction is exponentially small. This should result in vortex-lattice melting at quite low temperatures.

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*Permanent address: L. D. Landau Institute for Theoretical Physics, U.S.S.R. Academy of Sciences, 117334 Moscow, U.S.S.R.

[1] J. G. Bednortz and K. A. Müller, Z. Phys. B **64**, 189 (1986); Rev. Mod. Phys. **60**, 585 (1988).
[2] M. Tachiki and S. Takahashi, Solid State Commun. **70**, 291 (1989).
[3] B. I. Ivlev and N. B. Kopnin, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 678 (1989) [JETP Lett. **49**, 780 (1989)]; J. Low Temp. Phys. **77**, 413 (1989).
[4] B. I. Ivlev and N. B. Kopnin, Phys. Rev. Lett. **64**, 1828 (1990).
[5] See, for example, B. Roas, L. Schultz, and G. Saemann-Ischenko, Phys. Rev. Lett. **64**, 479 (1990).
[6] L. P. Gor'kov and N. B. Kopnin, Usp. Fiz. Nauk **156**, 117 (1988) [Sov. Phys. Usp. **31**, 850 (1988)].
[7] D. E. Farrell, C. M. Williams, S. A. Wolf, N. P. Bansal, and V. G. Kogan, Phys. Rev. Lett. **61**, 2805 (1988).
[8] D. E. Farrell, S. Bonham, J. Foster, Y. C. Chang, P. Z. Jiang, K. G. Vandervoort, D. J. Lam, and V. G. Kogan, Phys. Rev. Lett. **63**, 782 (1989).
[9] T. R. Chien, Z. Z. Wang, and N. P. Ong, in *Proceedings of the Stanford Conference on High-Temperature Superconductivity, 1989* [Physica C **162-164**, Pt. I, 343 (1990)].
[10] M. Tuominen, A. M. Goldman, Y. Z. Chang, and P. Z. Jiang, Phys. Rev. B **42**, 412 (1990).
[11] V. G. Kogan, Phys. Rev. Lett. **64**, 2192 (1990).
[12] A. I. Buzdin and A. Yu. Simonov, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 168 (1990) [JETP Lett. **51**, 191 (1990)].
[13] A. V. Balatskii, L. I. Burlachkov, and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **90**, 1478 (1986) [Sov. Phys. JETP **63**, 866 (1986)].
[14] B. I. Ivlev, N. B. Kopnin, and V. L. Pokrovsky, J. Low Temp. Phys. **80**, 187 (1990).
[15] P. G. deGennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).
[16] B. I. Ivlev, Yu. N. Ovchinnikov, and V. L. Pokrovsky, Europhys. Lett. **13**, 187 (1990).
[17] E. I. Kats, Zh. Eksp. Teor. Fiz. **56**, 1675 (1969) [Sov. Phys. JETP **29**, 897 (1969)].
[18] C. A. Murray, P. L. Gammel, D. J. Bishop, D. B. Mitzi, and A. Kapitulnik, Phys. Rev. Lett. **64**, 2312 (1990).
[19] D. R. Nelson and H. S. Seung, Phys. Rev. B **39**, 9153 (1989).
[20] V. G. Kogan and L. J. Campbell, Phys. Rev. Lett. **62**, 1552 (1989).