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Thermal fluctuations and phase transitions in the vortex state of a layered superconductor

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We study thermal fluctuations in a layered superconductor in the presence of a magnetic field applied orthogonal to the layers. A phase diagram for this case is proposed. In the weak-field region, fluctuations of a vortex lattice are of three-dimensional (3D) nature. This leads to a two-stage melting: When the temperature is raised, a phase transition to the vortex-line liquid occurs, then independent liquid systems of 2D vortices in different layers are formed. For fields larger than the crossover value, both fluctuations and melting of the vortex lattice become of 2D type. We study the effect of vortex-lattice fluctuations on the long-range superconducting order. We demonstrate the existence of a phase transition within the vortex-lattice state: The superconducting coherence between distant layers vanishes at a temperature which is substantially lower than the melting temperature.

I. INTRODUCTION

Thermal fluctuations in high-temperature superconductors are much stronger than in conventional ones due to several reasons: the small value of the coherence length ξ_{ab}, the high value of the transition temperature T_c, and the layered structure of these compounds. The layered structure can lead to quasi-two-dimensional fluctuation behavior. In most layered compounds (Bi-Sr-Ca-Cu-O, Ta-Ba-Ca-Cu-O), the superconducting transition in the absence of the magnetic field is similar to the Berezinskii-Kosterlitz-Thouless transition in two-dimensional systems.[1–4] Some latest experiments indicate the importance of thermal fluctuations in the presence of the magnetic field. In particular, for $Bi_2Sr_2CaCu_2O_8$, pinning is absent in a wide temperature region below T_c. That leads to the suppression of critical current[5,6] and the emergence of a long tail on the resistive transition down to $T \sim 30$–40 K. Such behavior is connected with the destruction of the vortex lattice due to thermal fluctuations. In the presence of the magnetic field, the damping of mechanical oscillations caused by the Bi-Sr-Ca-Cu-O sample changes drastically at the same temperature $T \sim 30$–40 K.[7] This was also interpreted as the manifestation of lattice melting.

In this paper, we study the properties of a layered superconductor in a magnetic field applied along the c axis of the crystal (i.e., orthogonally to layers). The fluctuation behavior is rather complicated in such a field. First, thermal fluctuations cause the melting of the Abrikosov lattice at temperatures well below the superconducting transition temperature $T_c(B)$ (here B is the magnetic induction in the superconductor). There is a characteristic value $B = B_{cr}$ which separates regions of three-dimensional (3D) and two-dimensional (2D) melting.[8] In the strong-field region ($B \gg B_{cr}$), the melting temperature $T_m(B)$ is close to that of a single layer and weakly depends upon B. Second, vortex thermal fluctuations substantially enhance fluctuations of the order parameter. There are contradictory statements about this influence. It was shown by Maki and Takayama[9] and Moore[10] that the mean-square fluctuation of the order parameter diverges logarithmically in the mixed state. These authors[9,10] conclude that vortex fluctuations destroy long-range superconducting order. This statement was questioned by Pelkovits *et al.*[11] because the order parameter is not a gauge-invariant quantity and its divergence does not carry any special physical significance. The opinion of these latter authors[11] is that fluctuations of the gauge-invariant phase

$$\varphi(\mathbf{r}) - \frac{2e}{c}\int_0^{\mathbf{r}} \mathbf{A}(l)\cdot dl$$

are finite and the long-range superconducting order is conserved [here $\varphi(\mathbf{r})$ is the phase of order parameter, $\mathbf{A}(\mathbf{r})$ is the vector potential].

We study the long-range asymptotics of the gauge-invariant correlation function of the order parameter and demonstrate that the long-range superconducting order is absent in the a-b plane at any temperature. Phase coherence along the c axis is conserved up to some definite temperature depending on magnetic field $T_0(B)$. In the strong-field region ($B \gg B_{cr}$), the value of $T_0(B)$ is much smaller than the temperature T_m of the vortex-lattice melting. Moreover, at $T = T_0(B)$, vortex fluctuations remain weak. Such behavior is caused by a nonlocal relation between the disturbance of the order parameter and the displacements of the lattice. Thus, there is a broad region of temperatures and fields within which the long-range superconducting order is absent but the long-range order in the vortex lattice is conserved.

Suppression of the phase coherence across layers manifests itself in some effects connected with the excitation of the superconducting currents along the c axis. Particularly, at $T=T_0(B)$, screening of a weak magnetic field applied parallel to the a-b plane vanishes.

II. VORTEX LATTICE ELASTIC ENERGY FOR A LAYERED SUPERCONDUCTOR

Consider a layered superconductor in the magnetic field $\mathbf{H}$ applied orthogonal to the layers. We restrict ourselves with the case $H_{c1} \ll H \ll H_{c2}$. Within this interval, the magnetic induction B inside the sample almost coincides with the strength of the magnetic field $\mathbf{H}$. Equilibrium phase distribution and phase fluctuations for a layered superconductor are determined by energy:

$$E=\sum_n \int d^2\mathbf{r} \left\{ \frac{1}{2} J \left[\nabla\varphi_n - \frac{2e}{c}\mathbf{A}_\parallel \right]^2 + E_J \left[1-\cos\left[\varphi_{n+1}-\varphi_n - \frac{2e}{c}\int_{z_n}^{z_{n+1}} A_z dz \right] \right] \right\} + \int d^3\mathbf{r} \frac{B^2}{8\pi} . \qquad (1)$$

(We fix the gauge by the condition div $\mathbf{A}=0$.) Here

$$J=\frac{\phi_0^2 d}{\pi(4\pi\lambda_{ab})^2}$$

is the stiffness constant characterizing phase fluctuations in a single layer,

$$E_J=\frac{\phi_0^2}{\pi(4\pi\lambda_c)^2 d}$$

is the energy of Josephson coupling between layers $[\lambda_{ab}(T)$ and $\lambda_c(T)$ are the components of the London penetration depth], and the z axis coincides with the c axis. The anisotropy parameter $\gamma=\lambda_c/\lambda_{ab}$ is assumed to be large, $\gamma \gg 1$. Expression (1) is valid if the gauge-invariant phase varies along layers on a scale that is larger than the coherence length, i.e.,

$$\left| \nabla\varphi_n - \frac{2e}{c}\mathbf{A}_\parallel \right| \ll 1/\xi_{ab} .$$

It is well known that, at low temperatures in the range of the fields $H > H_{c1}$, the triangular vortex lattice corresponds to thermodynamic equilibrium. In a layered superconductor, the Abrikosov flux line consists of "sections." Each "section" is a vortex excitation in a single superconducting 2D layer. Below we shall call these excitations 2D vortices. The energy of a layered superconductor in a mixed state is a function of the positions of the 2D vortices in each layer.

The fluctuation behavior of a vortex lattice is determined by its elastic energy at small deviations $\mathbf{u}(\mathbf{R},n)$ of the 2D vortices from equilibrium positions (n is the layer's number, $\mathbf{R}$ is the vortex position in a layer). The elastic energy for a three-dimensional isotropic superconductor was calculated by Brandt[12] (the generalization for the anisotropic case was made, e.g., in Ref. 13). Because of the weak interlayer coupling, there is a wide interval of fields where fluctuations are determined by the whole range of wave vectors k_z across the layers. [The elastic energy for the arbitrary dependence of 2D vortices displacements $\mathbf{u}(\mathbf{R},n)$ on the layer's number n is obtained in the Appendix.] On the other hand, the wavelengths of vortex displacements in the a-b plane are usually quite long. If the inequality $a \ll \lambda_{ab}$ holds, then the bulk modulus C_{11} is much greater than the shear modulus C_{66} (a is the vortex-lattice parameter). Hence, the fluctuation behavior is basically determined by the transversal lattice deformations $\mathbf{u}_t(\mathbf{R},n)$ ($\mathrm{div}_{\mathbf{R}}[\mathbf{u}_t(\mathbf{R},n)]=0$). We shall only consider this type of deformation. In this case the elastic energy can be written down in a simplified form:

$$\varepsilon_{el}=\frac{1}{2}\int_{-\pi/d}^{\pi/d}\frac{dk_z}{2\pi}\int_{k_\parallel<K_0}\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}[C_{66}\mathbf{k}_\parallel^2+C_{44}^{\mathrm{eff}}(k_z,\mathbf{k}_\parallel)\tilde{k}_z^2]|\mathbf{u}_t(k_z,\mathbf{k}_\parallel)|^2 ,$$

$$C_{44}^{\mathrm{eff}}(k_z,\mathbf{k}_\parallel)=\frac{B^2/4\pi}{1+\lambda_c^2k_\parallel^2+\lambda_{ab}^2\tilde{k}_z^2}+\frac{B\phi_0}{2(4\pi\lambda_{ab})^2}\left[\frac{1}{\gamma^2}\ln\frac{k_{\max}^2}{K_0^2+(\tilde{k}_z/\gamma)^2}+\frac{1}{(\lambda_{ab}\tilde{k}_z)^2}\ln(1+\tilde{k}_z^2/K_0^2)\right] , \qquad (2)$$

$$\tilde{k}_z \equiv 2\sin(k_zd/2)/d . \qquad (2a)$$

Here[14]

$$C_{66}=\phi_0B/(8\pi\lambda_{ab})^2 ,$$

$$k_{\max}\sim 1/\xi_{ab}(1+T/T_1)^{-1/2} ,$$

$$T_1\sim\frac{2\phi_0^2\xi_{ab}^2}{(4\pi)^2\lambda_c^2 d} .$$

For the sake of simplicity we approximate the 2D Brillouin zone by a circle with radius

$$K_0=(4\pi B/\phi_0)^{1/2}\sim 1/a$$

and use the quadratic in the $\mathbf{k}_\parallel$ expansion within the whole zone. Such an approximation slightly modifies only the numerical factors. The effective tilt modulus

$C_{44}^{\text{eff}}(k_z,\mathbf{k}_\parallel)$ consists of two terms. The first term corresponds to a nonlocal contribution to the tilt energy,[12] the second one is caused by the tension of separate vortex lines. This "single-vortex" contribution is usually omitted,[12,13,15] but it exceeds the nonlocal contribution for

$$\mathbf{k}_\parallel^2 \gg K_0^2 \Big/ \ln\frac{k_{\max}^2}{K_0^2+(\tilde{k}_z/\gamma)^2} , \tag{3}$$

i.e., in the largest part of the 2D Brillouin zone. Note that the tilt stiffness of the vortex lattice is preserved due to the magnetic interaction between vortices in different layers even in the absence of the interlayer Josephson coupling[3] ($\lambda_c,\gamma\rightarrow\infty$). Magnetic interaction is given by the "single-vortex" contribution in (2a). It exceeds the Josephson contribution if the anisotropy is sufficiently large, $\gamma>\lambda_{ab}/d$. In the following sections we use expression (2) to study the fluctuation behavior within the field interval $H_{c1}<B<H_{c2}$.

III. EFFECT OF THERMAL FLUCTUATIONS ON THE VORTEX LATTICE

A. Region of small fluctuations

The main characteristic of fluctuation behavior of the vortex lattice is the correlation function of displacements:

$$\langle \mathbf{u}(R,n)\mathbf{u}(\mathbf{0},0)\rangle \approx T\int_{-\pi/d}^{\pi/d}\frac{dk_z}{2\pi}\int_{\mathbf{k}_\parallel<K_0}\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}\frac{\exp(ik_z nd+i\mathbf{k}_\parallel\mathbf{R})}{C_{66}\mathbf{k}_\parallel^2+C_{44}^{\text{eff}}(k_z,\mathbf{k}_\parallel)\tilde{k}_z^2} . \tag{4}$$

The fluctuation behavior of layered superconductors is substantially different in the regions of weak and strong magnetic fields. The characteristic magnetic field separating these regions (crossover field) B_{cr} can be estimated from the relation

$$C_{66}K_0^2 \sim C_{44}^{\text{eff}}(\pi/d)^2 .$$

If the Josephson coupling dominates over the magnetic one in the tilt energy (i.e., $\lambda_c<\lambda_{ab}^2/d$), then the crossover field is estimated as

$$B_{\text{cr}} \approx 2\pi\frac{\phi_0\ln(\gamma k_{\max}d)}{\gamma^2 d^2} . \tag{5}$$

In the weak fields, $B \ll B_{\text{cr}}$, the relative displacement of 2D vortices in adjacent layers is much smaller than the fluctuation displacement of the vortex itself,

$$\langle[\mathbf{u}(\mathbf{R},n+1)-\mathbf{u}(\mathbf{R},n)]^2\rangle \ll \langle \mathbf{u}^2\rangle .$$

In this case an usual concept of a vortex lattice as an array of well-defined flux lines penetrating through superconducting layers is valid. The main contribution to $\langle\mathbf{u}^2\rangle$ is given by the wave-vector region $k_z\ll\pi/d$, $0<k_\parallel<K_0$, and because of the large logarithmic factor in (3), one can neglect the nonlocal tilt modulus in comparison with "single-vortex" contribution. Thus, we obtain the following estimation:

$$\langle \mathbf{u}^2\rangle \approx \frac{16\pi^{3/2}}{\ln^{1/2}(k_{\max}a)}\frac{T\lambda_c\lambda_{ab}}{\phi_0^{3/2}B^{1/2}} . \tag{6}$$

In Refs. 13 and 15 the estimation for $\langle\mathbf{u}^2\rangle$ was obtained without a "single-vortex" contribution to the tilt energy. This leads to an overestimation of $\langle\mathbf{u}^2\rangle$ in the logarithmic factor $\ln^{1/2}(k_{\max}a)$.

In the strong-field region $B\gg B_{\text{cr}}$, an interaction between 2D vortices in the same layer is stronger than the interaction between 2D vortices belonging to the same vortex line. This leads to quasi-two-dimensional behavior of fluctuations. In a strictly 2D case, the long-wave soft excitations induce logarithmic divergency in the mean-square fluctuation displacement $\langle\mathbf{u}^2\rangle$. The coupling between vortex lattices in adjacent layers suppresses fluctuations with small wave vectors $k_\parallel^2\ll(\gamma da)^{-1}$. That is why fluctuation displacements are finite,

$$\langle \mathbf{u}^2\rangle = \frac{T}{4\pi dC_{66}}\ln\left[\frac{\gamma d}{a}\right] , \tag{7}$$

and long-range order in the vortex lattice is conserved at low temperatures.

B. Vortex-lattice melting

The increase of temperature causes the destruction of the long-range order in the lattice. The melting temperature T_m depends on the magnetic field B. As it was mentioned in a number of papers,[13,15,16] a high-transition temperature, small coherence length, and high anisotropy of high-T_c superconductors lead to a substantial reduction of $T_m(B)$ in respect to the mean-field transition temperature $T_c(B)$ for the wide range of applied magnetic fields.

The character of lattice melting depends on the relation between the magnetic induction B and the crossover field B_{cr}. At $B\ll B_{\text{cr}}$, the vortex lattice is similar to a lattice of strings. Due to this reason, one can expect that, at the melting point, only the shear stiffness disappears but the tilt modulus C_{44} remains finite. The estimation of the melting temperature valid for this range of fields was obtained in Refs. 13 and 15 from the Lindemann criterion

$$\langle \mathbf{u}^2\rangle^{1/2} = c_L a . \tag{8}$$

Expression (2a), which takes into account the linear strain energy of separate vortices, allows us to determine more precisely the field dependence of the melting temperature[17] $T_m(B)$:

$$T_m(B) \approx \frac{c_L^2}{2^{3/2}}\frac{d\phi_0^2}{(4\pi\lambda_{ab})^2}\left[\frac{B_{\text{cr}}}{B}\right]^{1/2} . \tag{9}$$

Melting corresponds to the transition from a vortex lattice to a liquid of vortex lines.[15] In the latter phase, the tilt modulus $C_{44}\neq 0$. A further increase of temperature leads to the destruction of the vortex lines and the vanishing of the tilt modulus C_{44}. The temperature corresponding to this destruction, $\tilde{T}_m(B)$, can be estimated from the relation

$$\langle [\mathbf{u}(\mathbf{0},n+1)-\mathbf{u}(\mathbf{0},n)]^2\rangle \lesssim a^2 , \tag{10}$$

which, with help of (4), gives

$$\tilde{T}_m(B) \sim T_m(B)\left[\frac{B_{\rm cr}}{B}\right]^{1/2} > T_m(B) . \tag{11}$$

At $B \gg B_{\rm cr}$, the vortex lattice in a layered superconductor behaves as a quasi-two-dimensional object, consisting of weakly interacting vortex lattices in different layers. In this region, the melting temperature is close to the one for a single superconducting layer $T_m^{\rm 2D}$. The latter is field independent:[18]

$$T_m^{\rm 2D} = \frac{A}{8\pi 3^{1/2}}\frac{d\phi_0^2}{(4\pi\lambda_{ab})^2}, \quad A \lesssim 1 . \tag{12}$$

Both moduli C_{44} and C_{66} drop to zero simultaneously at the melting point. The phase emerging at $T > T_m^{\rm 2D}$ is usually called the vortex hexatic because it conserves long-range orientational order; the true liquid is realized at a somewhat higher temperature.[19] Note that criterion (8) in the quasi-2D region determines the boundary of the fluctuation region.

For a rough estimate of the field dependence of the melting temperature in the entire interval of magnetic fields, one can use a modified Lindemann criterion:

$$\langle [\mathbf{u}(\mathbf{a},0)-\mathbf{u}(\mathbf{0},0)]^2\rangle^{1/2} = c^L a . \tag{13}$$

[In the region $B \ll B_{\rm cr}$, criterions (8) and (13) differ from each other only by the definitions of the Lindemann constants c_L and c^L.] Equations (12) and (13) give an estimate $c^L \lesssim 1/2\pi\sqrt{2}$. The function $T_m(B)$ obtained from Eq. (13), assuming weak field dependence of c^L, is plotted in Fig. 1.

The exact asymptotic of the deviation of $T_m(B)$ from $T_m^{\rm 2D}$ at $B \gg B_{\rm cr}$ can be obtained as follows. In a wide region of temperatures $T > T_m^{\rm 2D}$, vortex hexatics in different layers are independent. Fluctuations in a two-dimensional hexatic are characterized by a correlation length $\xi^{\rm 2D}$ that diverges in the vicinity $T_m^{\rm 2D}$ as[19]

$$\xi^{\rm 2D}(T) \approx a \exp[bT_m^{\rm 2D}/(T_m^{\rm 2D}-T)]^{\nu} , \tag{14}$$

$\nu = 0.37$, b is a numerical constant of order 1. The two-dimensional character of transition is violated if the Josephson energy related to the area $\sim[\xi^{\rm 2D}(T)]^2$ becomes of the order of temperature:

$$E_J^{\rm eff}(T)[\xi^{\rm 2D}(T)]^2 \sim T . \tag{15}$$

This relation determines the B-dependent difference between the melting temperature T_m and limiting value $T_m^{\rm 2D}$. Thermal fluctuations suppress the energy of the Josephson coupling in comparison with the bare value E_J. This suppression is studied in the next section. Using Eqs. (14), (15), and (31), one finds

$$T_m(B) = T_m^{\rm 2D}\left[1+\frac{b^{1/\nu}}{\ln^{1/\nu}(B/B_{\rm cr})}\right] . \tag{16}$$

The $T_m(B)$ dependence was also obtained in Ref. 8. The difference of a numerical factor is caused by taking into account here the suppression of E_J by fluctuations.

We conclude this section with a remark considering the microscopic inhomogeneity of the internal magnetic field. A magnetic flux quantum "attached" to a single vortex line corresponds to a field that is distributed over the area λ_{ab}^2. At $B \gg B_{c1}$, the condition $a/\lambda_{ab} \ll 1$ holds and areas λ_{ab}^2 belonging to different vortices overlap. That is why the modulation of the magnetic field is small. (The remaining effect of the flux quantization for each vortex line is that the flux through the elementary cell in a vortex lattice is quantized.) In the absence of fluctuations, 2D vortex lattices in different layers are positioned exactly on top of each other. Hence, the in-plane periodic modulation of the magnetic field has the same phase in all planes and field $\mathbf{B}$ has only a z component. Fluctuations produce a relative dephasing of this modulation in different planes. As a consequence of the magnetic flux conservation law, a nonzero $B_{\parallel}$ component of the field should appear. The stronger the fluctuations in the vortex positions, the larger the rms of this component is. However, $B_{\parallel}$ remains small even after the 2D melting because of the condition $d, a \ll \lambda_{ab}$. An estimation based on the London equations gives

$$\langle B_{\parallel}^2\rangle^{1/2} \sim (da/\lambda_{ab}^2)B$$

and shows that the component $B_{\parallel}$ may be neglected.

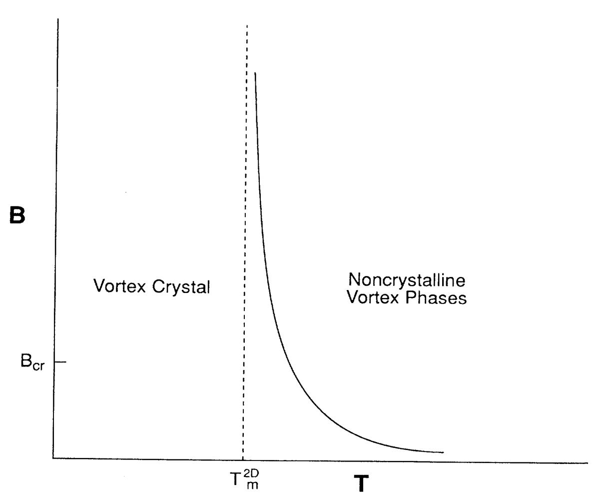

FIG. 1. The melting line on the B-T plane for a layered superconductor obtained from criterion (13) in the case $T_m^{\rm 2D} \ll T_c$.

IV. FLUCTUATIONS OF PHASE AND LONG-RANGE SUPERCONDUCTING ORDER

A. Region of small fluctuations

In Refs. 9 and 10, a statement about the absence of the long-range superconducting order in a vortex-lattice state was made. This statement was based on the logarithmic divergency of a mean-square fluctuation of the order parameter $\langle(\psi-\langle\psi\rangle)^2\rangle$. This divergency, however, has no physical significance because the order parameter is not a gauge-invariant quantity. In fact, the long-range superconducting order is determined by the behavior of the gauge-invariant correlation function

$$G(\mathbf{r},\mathbf{r}')=\left\langle \psi(\mathbf{r})\psi^*(\mathbf{r}')\exp\left[-i\frac{2\pi}{\phi_0}\int_{\mathbf{r}'}^{\mathbf{r}} dl\cdot\mathbf{A}\right]\right\rangle \tag{17}$$

at large distances between $\mathbf{r}$ and $\mathbf{r}'$. The integration in the exponent should be performed along the straight line connecting points $\mathbf{r},\mathbf{r}'$ and not intersecting vortex cores. For small fluctuations it is convenient to rewrite the last expression in the following form:

$$G(\mathbf{r},\mathbf{r}')=G_0(\mathbf{r},\mathbf{r}')\exp[-S(\mathbf{r},\mathbf{r}')]\ , \tag{18}$$

$$S(\mathbf{r},\mathbf{r}')=\frac{1}{2}\left\langle\left[\int_{\mathbf{r}'}^{\mathbf{r}} dl\cdot\left[\nabla\tilde{\varphi}(\mathbf{r})-\frac{2\pi}{\phi_0}\tilde{A}\right]\right]^2\right\rangle . \tag{18a}$$

Here $G_0(\mathbf{r},\mathbf{r}')$ is an equilibrium value of the correlation function, $\tilde{\varphi}$ and $\tilde{A}$ are fluctuations of the phase and the vector potential. The suppression of long-range order is determined by the asymptotic behavior of the phase correlation function $S(\mathbf{R})$ at $R\to\infty$ ($\mathbf{R}=\mathbf{r}-\mathbf{r}'$). Below we study this behavior for different orientations of $\mathbf{R}$.

One can relate the fluctuations of the gradient of phase to the long-wave transversal lattice distortions

$$\mathbf{u}(\mathbf{r})=\int\int\frac{dk_z d\mathbf{q}}{(2\pi)^3}\frac{\mathbf{n}_z\mathbf{q}}{q}u_t(k_z,\mathbf{q})\exp(ik_z z+i\mathbf{q}\cdot\mathbf{r}_\parallel)\ . \tag{19}$$

In the linear approximation[20]

$$\nabla_z\tilde{\varphi}(\mathbf{k})=2\pi\frac{B}{\phi_0}\sum_{\mathbf{K}}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\delta(\mathbf{q}-\mathbf{k}-\mathbf{K})\times\frac{1+\lambda_c^2k^2}{1+\lambda_c^2k_\parallel^2+\lambda_{ab}^2k_z^2}\frac{k_z(\mathbf{q}\cdot\mathbf{k}_\parallel)}{k^2q}u_t\ , \tag{20a}$$

$$\nabla_\parallel\tilde{\varphi}(\mathbf{k})=2\pi\frac{B}{\phi_0}\sum_{\mathbf{K}}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\delta(\mathbf{q}-\mathbf{k}-\mathbf{K})\times\left[-\frac{\mathbf{q}}{q}+\frac{1+\lambda_{ab}^2k^2}{1+\lambda_c^2k_\parallel^2+\lambda_{ab}^2k_z^2}\times\frac{\mathbf{k}_\parallel(\mathbf{q}\cdot\mathbf{k}_\parallel)}{k^2q}\right]u_t\ . \tag{20b}$$

Perturbations of vector potential components are proportional to the components of $\nabla\tilde{\varphi}$:

$$\tilde{A}_z(\mathbf{k})=\frac{\phi_0}{2\pi}\frac{\nabla_z\tilde{\varphi}}{1+\lambda_c^2k^2}\ , \tag{21a}$$

$$\tilde{A}_\parallel(\mathbf{k})\frac{\phi_0}{2\pi}\frac{\nabla_\parallel\tilde{\varphi}}{1+\lambda_{ab}^2k^2}\ . \tag{21b}$$

The parallel to layers component of the phase gradient $\nabla_\parallel\tilde{\varphi}$ contains a singular part: a sum of δ functions localized on vortex cores. [This part corresponds to a term $-\mathbf{q}/q$ in the large parentheses in (20b).] The singular part, however, does not contribute to the $S(\mathbf{R})$ function because the path of integration in (17) avoids vortex cores. The corresponding part in the vector potential is not localized on cores and that is why it should be taken into account. This can be done with a certain simplification: because of the condition $a/\lambda_{ab}\ll 1$, the only relevant term in the lattice sum for $\tilde{A}(\mathbf{k})$ [see expressions (20a), (20b), (21a), and (21b)] is the term with $\mathbf{K}=0$. The part with $\mathbf{K}=0$ in $\nabla_\parallel\tilde{\varphi}$ tends to zero at $k_z=0$, i.e., transverse lattice deformations only cause a weak perturbation of the magnetic field. With the above-mentioned simplifications, one finds[21]

$$S(\mathbf{R})=\left[2\pi\frac{B}{\phi_0}\right]^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}(1-\cos\mathbf{k}\cdot\mathbf{R})\left[\frac{k_\parallel(\mathbf{k}\cdot\mathbf{n})(1+\lambda_c^2k^2)-k_\parallel k_z n_z+k_z^2(\mathbf{k}_\parallel\cdot\mathbf{n}_\parallel)/k_\parallel}{(\mathbf{k}\cdot\mathbf{n})k^2(1+\lambda_{ab}^2k_z^2+\lambda_c^2k_\parallel^2)}\right]^2\frac{T}{C_{66}k_\parallel^2+C_{44}^{\mathrm{eff}}(\mathbf{k})k_z^2}\ , \tag{22}$$

here $\mathbf{n}=\mathbf{R}/R$.

It is possible to neglect screening in a wide interval of distances, i.e., to omit part of $S(\mathbf{R})$ connected with the vector potential and to use an expression

$$C_{44}^{\mathrm{eff}}(\mathbf{k})\approx\frac{B^2}{4\pi\lambda_c^2k_\parallel^2}$$

for the tilt modulus. These approximations lead to a logarithmically increasing $S(\mathbf{R})$ function:

$$S(\mathbf{R})=\left[\frac{\pi B}{16\phi_0}\right]^{1/2}\frac{(4\pi)^2\lambda_{ab}\lambda_c T}{\phi_0^2}\times\ln\left[\frac{(K_0R_\parallel)^4+(\gamma K_0R_z)^2/4}{1+(\gamma K_0 d/2\pi)^2}\right]\ . \tag{23}$$

Dependence (23) corresponds to a power-law decay in the order-parameter correlation function (in accordance with Refs. 9 and 10).

The asymptotic of the $S(\mathbf{R})$ function as $R\to\infty$ depends significantly on the vector $\mathbf{R}$ orientation. We start with the asymptotic for the direction in a-b plane ($\mathbf{R}_z=0$). As was mentioned above, perturbations of the parallel component of the vector potential caused by transversal distortions of the vortex lattice are small. That is why the only effect of the screening for the parallel direction is the renormalization of the tilt modulus that occurs on the scale $R_\parallel\gtrsim\lambda_c$. This renormalization leads to a change from the logarithmic increase of $S(\mathbf{R})$ to a linear one at distances $R_\parallel\gtrsim\lambda_c$:

$$S(\mathbf{R})=\left[\frac{\pi B}{\phi_0}\right]^{1/2}\frac{(4\pi)^2\lambda_{ab}T}{\phi_0^2}R_{\parallel}\text{at}\quad R_z=0,\ R_{\parallel}>\lambda_c\ ,\tag{24}$$

and to an exponential decay of the order-parameter correlation function (18).

In the transversal direction ($R_{\parallel}=0$) at $R_z\to\infty$, the $S(R_z)$ function approaches finite value due to screening. The logarithmic increase of $S(R_z)$ saturates at distances $R_z\sim R_{z\max}$ that can be estimated as follows. The main contribution in $S(R_z)$ originates from the region of wave vectors $k_{\parallel}^2\gtrsim 2\pi K_0/\gamma R_z$. On the other hand, screening becomes important under the condition $\lambda_c^2k_{\parallel}^2\lesssim 1$. Hence, a logarithmic increase of $S(R_z)$ stops at $R_z\sim R_{z\max}$, $R_{z\max}\sim 2\pi K_0\lambda_c\lambda_{ab}$, and $S(R_z)$ approaches the value

$$S(R_z\to\infty,R_{\parallel}=0)=\left[\frac{\pi B}{4\phi_0}\right]^{1/2}\frac{(4\pi)^2\lambda_{ab}\lambda_c T}{\phi_0^2}\times\ln\left[\frac{(K_0\lambda_c)^4}{1+(\gamma K_0 d/2\pi)^2}\right].\tag{25}$$

So, the lattice fluctuations at low temperatures do not destroy the long-range order in the direction of the magnetic field.

B. Vanishing of the long-range superconducting order in the direction of the field

The destruction of the long-range superconducting order in the magnetic field direction occurs as a result of a phase transition. Here we show that, at $B\gg B_{\mathrm{cr}}$, the temperature of this transition is well below the melting temperature of the vortex lattice. To study the phase transition under discussion, it is convenient to introduce a simplified Hamiltonian H_φ that does not include screening and describe phase fluctuations on the scale $d<R_z<\lambda_c\lambda_{ab}/a$, $a<R_{\parallel}<\lambda_c$:

$$H_\varphi=\sum_n\int d^2r\left[\frac{D}{2}(\Delta\tilde{\varphi}_n)^2+E_J[1-\cos(\tilde{\varphi}_{n+1}-\tilde{\varphi}_n)]\right].\tag{26}$$

Here

$$D=\frac{d\phi_0^3}{(4\pi)^4B\lambda_{ab}^2}.$$

At low temperatures, the Hamiltonian (26) allows us to reproduce the result (23) for $S(\mathbf{R})$. The suppression of the interplanar Josephson coupling is determined by a mean-square fluctuation:

$$\langle(\tilde{\varphi}_{n+1}-\tilde{\varphi}_n)^2\rangle=2\left[\frac{\pi B}{\phi_0}\right]^{1/2}\frac{(4\pi)^2\lambda_{ab}\lambda_c T}{\phi_0^2}.\tag{27}$$

The temperature $T_0(B)$ of the destruction of the phase coherence along the field direction is estimated from the relation

$$\langle(\tilde{\varphi}_{n+1}-\tilde{\varphi}_n)^2\rangle\lesssim 1:$$
$$T_0(B)\approx T_m^{2\mathrm{D}}(B_{\mathrm{cr}}/B)^{1/2}.\tag{28}$$

At $B\gg B_{\mathrm{cr}}$ this temperature is substantially lower than the melting temperature in a single layer $T_m^{2\mathrm{D}}$ (12).

To demonstrate the existence of the phase transition, we show that the power-law decay of the correlation function $G(\mathbf{R})$, (19), changes to an exponential one at $T\gg T_0$. In this temperature region, the high-temperature expansion can be used which is similar to the one for the XY model.[22] It leads to the following result for the correlations in the z direction:

$$\langle\exp\{i[\varphi(N)-\varphi(0)]\}\rangle\approx\left[\frac{2\pi^2T_0^2}{T^2\ln(T/T_0)}\right]^N.\tag{29}$$

This expression is valid on all scales of $R_z=Nd$ and screening does not change the exponential asymptotic. Thus, at $T\sim T_0$, the system undergoes a phase transition that is accompanied by the destruction of phase coherence in the field direction. The corresponding line is plotted in Fig. 3.

The transition under discussion is similar in a number of features to the Berezinskii-Kosterlitz-Thouless transition in a degenerate two-dimensional system. At temperatures $T<T_0$, the system with the Hamiltonian (26) possesses with a transversal stiffness that determines the increase in free energy F caused by a small change of the phase across the layers (i.e., in the z direction)

$$F=\int d^3r\frac{J_z}{2}\left[\frac{\partial\varphi}{\partial z}\right]^2.\tag{30}$$

In the range $T\ll T_0$ the value of J_z is given by $J_z=E_Jd^2$. The existence of this stiffness leads to the screening (on the length scale λ_c) of a weak magnetic field $\tilde{B}$ applied along the layers (the considered geometry is presented in Fig. 2). We suggest that, in an analogy with the Berezinskii-Kosterlitz-Thouless transition,[23] the phase stiffness jumps to zero at the transition point. Experimentally, the jump in the stiffness should manifest itself as a vanishing of the screening of the field $\tilde{B}$.

In the temperature region $T>T_0(B)$, the effective interplanar Josephson coupling energy

$$E_J^{\mathrm{eff}}=E_J\langle\cos(\varphi_{n+1}-\varphi_n)\rangle$$

is also significantly suppressed,

$$E_J^{\mathrm{eff}}=E_J\frac{2\pi^2T_0^2}{T^2\ln(T/T_0)}.\tag{31}$$

C. The influence of pinning on the long-range superconducting order

The large-R behavior of the order-parameter correlation function is determined by the small-k elastic energy of the vortex lattice. That is why even weak pinning strongly affects the $S(\mathbf{R})$ asymptotic. Pinning lifts the translational degeneracy of the lattice. The increase of energy δE_p under a small homogeneous shift $\mathbf{u}$ of the lat-

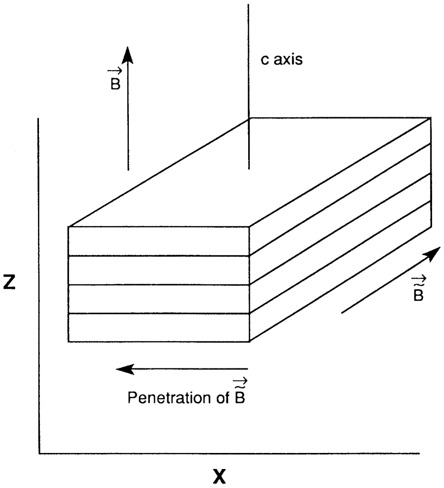

FIG. 2. Proposed experiment for detecting the transition between coherent and incoherent states. The strong-field B forms a vortex lattice in a layered superconductor. In a coherent phase the probing weak-field $\tilde{B}$ penetrates in the X direction on a finite depth $\lambda_c^{\rm eff}$. In the incoherent phase field, $\tilde{B}$ is not screened.

tice from the equilibrium position can be estimated as

$$\delta E_p \approx \int d^3\mathbf{r} \frac{Bj_c}{2cr_p} \mathbf{u}^2 , \tag{32}$$

where j_c is the critical depinning current, r_p is the radius of the random potential action. Comparing δE_p with the elastic energy of the vortex lattice, one finds that pinning suppresses lattice fluctuations with wave vectors $k_\parallel < 1/R_c$, $k_z < 1/L_c$, where

$$R_c = \left[\frac{cC_{66} r_p}{Bj_c} \right]^{1/2} , \quad L_c = R_c \left[\frac{B^2 R_c^2}{4\pi C_{66} \lambda_c^2} \right]^{1/2}$$

are the correlation radii of the collective pinning theory[24] for the directions across and along the magnetic field correspondingly. (We assume the requirements $a \ll R_c \ll \lambda_c$ and $d \ll L_c \ll \lambda_c \lambda_{ab}/a$ to be fulfilled.)

Hence, the earlier obtained formula (23) for the correlation function $S(\mathbf{R})$ is preserved in the presence of pinning up to the scale $R_\parallel \lesssim R_c$, $R_z \lesssim L_c$ only. For larger R, the correlation function approaches the limiting value

$$S(\infty) = \left[\frac{\pi B}{16\phi_0} \right]^{1/2} \frac{(4\pi)^2 \lambda_{ab} \lambda_c T}{\phi_0^2} \times \ln \left[\frac{(K_0 R_c)^4}{1+\gamma(K_0 d/2\pi)^2} \right] . \tag{33}$$

The random distribution of the pinning centers leads to the violation of the regularity of the vortex equilibrium positions.[24] This causes additional random oscillations in the order-parameter correlation function $G(\mathbf{r},\mathbf{r}')$ that are typical for the glassy state. Therefore, expression (33) describes, in fact, the suppression of the order parameter by thermodynamic fluctuations in a statically disordered vortex-glass state.

V. CONCLUSIONS

The results obtained in this paper allow us to propose a phase diagram for a layered superconductor in a magnetic field applied orthogonally to the layers (Fig. 3). The typical feature related to the layered structure is the presence of crossover that separates the region of quasi-two-dimensional behavior of fluctuations from the three-dimensional one. The layered structure also leads to the existence of a specific noncoherent phase in which the absence of long-range superconducting order coexists with the long-range order in the vortex lattice. The transition to this phase is accompanied by the divergency of the effective penetration depth $\lambda_c^{\rm eff}$ for a probing weak magnetic field applied in the a-b plane (Fig. 2). The incoherent phase exists at fields $B > B_{\rm cr}$.

In the weak-field regime ($B < B_{\rm cr}$), the vortex-lattice melting occurs in two stages because of the layered structure. First, a liquid of vortex lines appears[16] in which the tilt modulus C_{44} remains finite. At a higher temperature $\tilde{T}_m(B)$ (11), vortex lines are destroyed and a liquid of unbound 2D vortices is formed, the tilt modulus turns to zero. At $B > B_{\rm cr}$, such a phase is formed just after crystal melting (here we neglect the existence of this intermediate phase of the vortex hexatic[19]). Thermodynamically, the vortex-liquid state does not differ from the normal one, although 2D vortices remain well defined until the field $B = B_{c2}(T)$.

It is very important for understanding the experimental data to realize the effect of a weak disorder on the vortex motion. In the vortex-lattice state, weak pinning

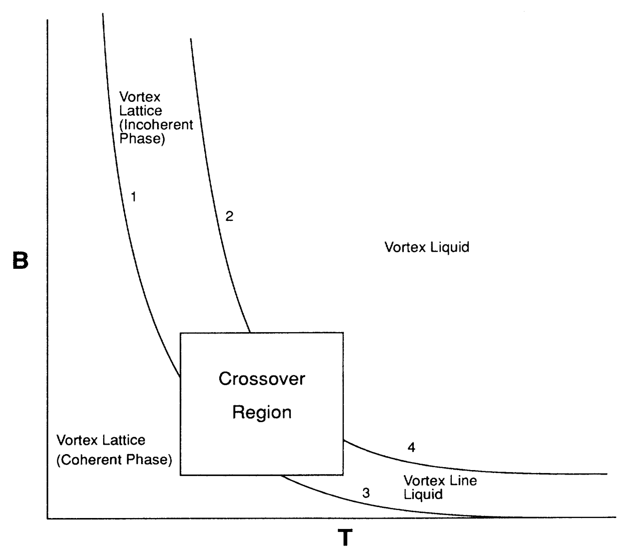

FIG. 3. Phase diagram for a layered superconductor in a magnetic field applied along the c axis. Phase boundaries 1, 2, 3, and 4 are determined by Eqs. (28), (16), (9), and (11), respectively. Note that line 4 determines a resistive superconducting transition in the weak-field region.

leads to the appearance of the vortex-glass state without linear resistance.[25-27] In the vortex-line-liquid state, flux lines move almost independently of one another. However, disorder effectively pins each line and linear resistance is also absent. On the other hand, in the vortex-liquid state, 2D vortices in different layers move independently of one another and can overcome the disorder potential relief due to thermal activation. It means that true superconducting transition takes place at the temperature $\tilde{T}_m(B)$ (11). In the vicinity of T_c, the field $\tilde{B}_m$ at which linear resistance appears depends upon temperature in a linear way:

$$\tilde{B}_m(T)=\alpha_m \frac{\phi_0^3(T_c-T)}{(4\pi\lambda_c)^2 dT_c T} . \tag{34}$$

Here α_m is a universal constant. Using the experimental values $d\tilde{B}_m/dT \approx 10^4$ Oe/K and $\lambda_c \approx 7\times 10^{-5}$ cm for the Y-Ba-Cu-O compound, one can obtain the estimation $\alpha_m \sim 0.1$. With this value of α_m, Eq. (34) gives, for the compound $Bi_2Sr_2CaCu_2O_8$, an estimation $d\tilde{B}_m/dT \approx 20$ Oe/K. The rather low value of this slope explains the existence of the long resistance tail.

We note, finally, that, for the compound $Bi_2Sr_2CaCu_2O_8$ ($d=15$ Å, $\lambda_{ab}=3000$ Å, $\gamma=50$), the 2D melting temperature (12) $T_m^{2D}\approx 30-40$ K is well below T_c. The typical crossover field [see (5)] B_{cr} can be easily achieved because of strong anisotropy, $B_{cr}\sim 1$ T. That is why the predicted features of the phase diagram at $B>B_{cr}$, in principle, allow the experimental validation.

APPENDIX: DYNAMIC MATRIX FOR A LAYERED SUPERCONDUCTOR

Under the condition $d \ll \lambda_{ab}$, one can change the continuous function $\mathbf{A}(z)$ to a discrete one $\mathbf{A}_n$. So, the distribution of fields and currents in a layered superconductor is determined by a set of London equations:

$$\frac{1}{\lambda_{ab}^2}\left[\frac{\phi_0}{2\pi}\nabla_\parallel \varphi_n - A_{\parallel n}\right]+(\Delta_\parallel+\Delta_z)\mathbf{A}_{\parallel n}=0 , \tag{A1a}$$

$$\frac{1}{\lambda_c^2}\left[\frac{\phi_0}{2\pi}\nabla_z \varphi_n - A_{zn}\right]+(\Delta_\parallel+\Delta_z)A_{zn}=0 , \tag{A1b}$$

$$\nabla_\parallel \mathbf{A}_{\parallel n}+\nabla_z \mathbf{A}_{zn-1}=0 . \tag{A1c}$$

Here the symbols $\parallel$ and z correspond to the directions parallel and orthogonal to the layers, respectively, the symbols ∇_z and Δ_z denote the lattice gradient and the lattice Laplace operator:

$$\nabla_z f_n \equiv (f_{n+1}-f_n)/d ,$$

$$\Delta_z f_n \equiv (f_{n+1}+f_{n-1}-2f_n)/d^2 .$$

Equations (A1a)–(A1c) should be completed by a topological relation:

$$\mathrm{rot}_\parallel \nabla_\parallel \varphi_n = 2\pi \mathbf{n}_z \sum_{\mathbf{r}^{(n)}} \delta(\mathbf{r}_\parallel - \mathbf{r}^{(n)}) . \tag{A2}$$

Here $\mathbf{r}^{(n)}$ are the coordinates of the vortex cores in nth layer.

Using Eqs. (A1)–(A2), one can obtain the perturbation of the phase $\tilde{\varphi}$ and vector potential $\tilde{\mathbf{A}}$ under a small displacement $\mathbf{u}$ of the 2D vortex in the nth layer from the equilibrium position. Fourier components of $\tilde{\varphi}(\mathbf{k})$ and $\tilde{\mathbf{A}}(\mathbf{k})$ are given by

$$\tilde{\varphi}(\mathbf{k})=2\pi d \frac{1+\lambda_c^2(k_\parallel^2+\tilde{k}_z^2)}{1+\lambda_c^2 k_\parallel^2+\lambda_{ab}^2\tilde{k}_z^2}\frac{(\mathbf{k}_\parallel\times\mathbf{u})_z}{k^2}\exp(i\mathbf{k}_\parallel\cdot\mathbf{r}+ik_z nd) , \tag{A3a}$$

$$\tilde{A}_z(\mathbf{k})=\frac{\phi_0}{2\pi}\frac{i\tilde{k}_z\tilde{\varphi}}{1+\lambda_c^2(k_\parallel^2+\tilde{k}_z^2)}\exp[i\mathbf{k}_\parallel\cdot\mathbf{r}+ik_z(n+1/2)d] , \tag{A3b}$$

$$\tilde{\mathbf{A}}_\parallel(\mathbf{k})=\frac{\phi_0}{2\pi}\frac{i\mathbf{k}_\parallel\tilde{\varphi}-2\pi d\mathbf{n}_z\times\mathbf{u}}{1+\lambda_{ab}^2(k_\parallel^2+\tilde{k}_z^2)}\exp(i\mathbf{k}_\parallel\cdot\mathbf{r}+ik_z nd) . \tag{A3c}$$

We use the notation $\tilde{k}_z=2\sin(k_z d/2)/d$. Substituting these relations into formula (1), we obtain the following expression for the elastic energy:

$$E_{\mathrm{el}}=\frac{1}{2}\left[\frac{d\phi_0}{B}\right]^2 \sum_{\substack{\mathbf{r},\mathbf{r}',\\ n,n'}} W_{ij}(\mathbf{r}-\mathbf{r}',n-n') \times u_i(\mathbf{r},n)u_j(\mathbf{r}',n') , \tag{A4}$$

where the dynamic matrix $W_{ij}(\mathbf{r},n)$ is determined by relations:

(1) At $(\mathbf{r},n)\neq(\mathbf{0},0)$,

$$W_{ij}(\mathbf{r},n)=\int_{\pi/d}^{\pi/d}\frac{dk_z}{2\pi}\int\frac{d\mathbf{k}_\parallel}{(2\pi)^2}W_{ij}(\mathbf{k})\exp(i\mathbf{k}_\parallel\cdot\mathbf{r}+ik_z nd) ,$$

$$W_{ij}(\mathbf{k})=\frac{B^2}{4\pi}\left[\frac{k_z^2\delta_{ij}}{1+\lambda_c^2k_\parallel^2+\lambda_{ab}^2\tilde{k}_z^2}+\frac{[1+\lambda_c^2(k_\parallel^2+\tilde{k}_z^2)]k_ik_j}{(1+\lambda_c^2k_\parallel^2+\lambda_{ab}^2\tilde{k}_z^2)[1+\lambda_{ab}^2(k_\parallel^2+\tilde{k}_z^2)]}\right] . \tag{A5a}$$

(2) At $(\mathbf{r},n)=(\mathbf{0},0)$

$$W_{ij}(\mathbf{0},0)=-\sum_{(\mathbf{r},n)\neq(\mathbf{0},0)}W_{ij}(\mathbf{r},n) . \tag{A5b}$$

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